This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

Phys 2439.07.5

SCIENCE CENTER LIBRARY

HARVARD COLLEGE LIBRARY

BEQUEST OF
ERASMUS DARWIN LEAVITT
1916

6

TECHNICAL THERMODYNAMICS

BY

DR. GUSTAV ZEUNER

FIRST ENGLISH EDITION

FROM THE FIFTH COMPLETE AND REVISED EDITION OF

"GRUNDZÜGE DER MECHANISCHEN WÄRMETHEORIE"

VOLUME II

THE THEORY OF VAPORS

AUTHORIZED TRANSLATION BY

J. F. KLEIN, D.E.

Professor of Mechanical Engineering, Lehigh University

NEW YORK

D. VAN NOSTRAND COMPANY

23 MURRAY AND 27 WARREN STREETS

1907

Phys 2439.07.5

Harvard College Library
Aug. 14, 1916.
Bequest of
[redacted] Darwin Leavitt.

Copyright, 1907
BY
D. VAN NOSTRAND COMPANY

ROBERT DRUMMOND, PRINTER, NEW YORK

CONTENTS OF THE SECOND VOLUME.

THIRD SECTION.

THEORY OF VAPORS.

PAGE

§ 1. Preliminary remarks. 3

A. The Behavior of Saturated Vapors.

§ 2. The pressure of saturated vapors. 9
§ 3. The heat of the liquid, the total heat and the latent heat. 20
§ 4. Steam heat and the inner and outer latent heat. 26
§ 5. Specific weight of saturated vapors. 33
§ 6. Clapeyron's equation. 38
§ 7. Recent investigations by Battelli on the behavior of the vapor of water. 48
§ 8. Heat equations of mixtures of vapor and liquid (wet vapors). . . . 53
§ 9. The entropy of wet vapors and the transformation of their pressure curves. 61
§ 10. The isothermal and the isodynamic curve of wet vapors. 71
§ 11. The adiabatic curve of wet vapors. 75

APPLICATIONS.

I. REVERSIBLE CHANGES OF STATE OF WET VAPORS.

§ 12. Heating or cooling of wet vapors at constant volume. 89
§ 13. The curve of constant steam weight and zero curve of wet steam. 99
§ 14. Investigation of different hypotheses concerning the expansion of steam in the cylinder of the steam engine. 106
§ 15. Continuation of the investigation of different curves of expansion. 116

II. NON-REVERSIBLE CHANGE OF STATE OF WET VAPORS.

§ 16. Expansion of wet vapors under different circumstances. 120
§ 17. Mixture of wet vapors of the same kind. 129

PAGE
§ 18. Theory of surface condensers 135
§ 19. Theory of the jet condenser 138
§ 20. Calorimetric investigation of the Giffard steam jet pump (injector) 142

III. The Flow and the Efflux of Wet Vapors.

§ 21. Fundamental formulas for the flow of wet vapors 153
§ 22. Efflux of highly heated liquids from simple orifices under constant pressure 160
§ 23. Efflux of dry saturated vapors from simple orifices under constant pressure 167
§ 24. Experiments on the efflux of steam 181
§ 25. Efflux experiments with the help of the steam jet pump 189
§ 26. Recent investigations on the flow of vapors 196
(De Laval Nozzle.)

B. The Behavior of Superheated or Unsaturated Vapors.

§ 27. Extension of the propositions concerning the limit curve and the critical temperature 202
§ 28. The equation of condition of dry saturated, superheated vapors. 208
§ 29. Older approximate forms for the equation of condition of vapors . 223
§ 30. Superheated steam 229
§ 31. The heat equations of superheated vapors 243
§ 32. The vapor of ammonia 252
§ 33. The vapor of sulphurous acid 256
§ 34. The vapor of carbonic acid 262

APPLICATIONS.

Reversible and Non-reversible Changes of Superheated Vapor of Water.

§ 35. The isothermal, isodynamic, and adiabatic curve for superheated steam 274
§ 36. Generation of superheated steam under constant pressure 281
§ 37. The flow of steam from one vessel to another, and the throttling of the steam 293

C. Atmospheric Air as Liquid and Vapor.

§ 38. Linde's machine for the liquefaction of air 303
§ 39. Extended equation of condition for atmospheric air 313

D. On the Behavior of Mixtures.

(a) Mixtures of Atmospheric Air and Steam.

PAGE
§ 40. The general behavior of the mixture of air and wet steam..... 320
§ 41. Isothermal changes of state of the mixture of air and wet steam. 323
§ 42. Isothermal change of state for the mixture of air, steam, water, and ice..... 329
§ 43. Adiabatic change of state of the mixture of air and wet steam... 333
§ 44. Behavior of the mixture of air and unsaturated vapor 338
§ 45. Behavior of the mixture of air, steam, and ice 350

(b) Mixtures of Different Kinds of Vapors.

§ 46. General remarks on mixtures of different kinds of liquids and their vapors..... 353
§ 47. Recent researches on the behavior of "saturated and superheated steam"..... 357

APPLICATIONS.

TECHNICAL PART.

Theory of Steam Engines.

§ 48. Preliminary remarks..... 368

A. Steam Engines (Hot-vapor Engines).

(a) Engines for Saturated Vapors.

§ 49. The Carnot cycle of the steam engine..... 371
§ 50. Investigation of the perfect steam engine and calculation of its dimensions..... 381
§ 51. Cycle of the actual steam engine and loss of work due to imperfection of the cycle..... 396
§ 52. Loss of work due to incomplete expansion..... 406
§ 53. Loss of work due to clearance. Action of compression..... 410
§ 54. Loss of work due to the action of the cylinder walls. Calorimetric investigation of steam engines..... 419
§ 55. Discussion of the equations for the heat exchange between steam and cylinder walls..... 428

(b) Engines for Superheated Steam.

§ 56. The indicated work and the thermal efficiency of the steam engine. The standard cycle and the corresponding efficiency. 441
§ 57. Addendum. Theory of fireless steam engines..... 449

B. Cold-vapor Engines.

PAGE
§ 58. Refrigerating machines in general. 459
§ 59. Cycle of the theoretically perfect cold-vapor engine. 463
§ 60. Cycle of a perfect cold-vapor engine according to Lorenz 474
§ 61. Cycle of actual cold-vapor engines. 478
§ 62. Influence of superheating in the compressor of actual cold-vapor engines. 484
§ 63. Calculation of the dimensions of the compressor. 487

APPENDIX.

Tables of the Appendix (French Units).

Saturated vapor of water. II
Saturated vapor of ether. VI
Saturated vapor of alcohol. VIII
Saturated vapor of acetone. X
Saturated vapor of chloroform. XII
Saturated vapor of chloride of carbon. XIV
Saturated vapor of bisulphide of carbon. XVI
Saturated vapor of ammonia. XVIII
Saturated vapor of sulphurous acid. XX
Saturated vapor of carbonic acid XXII
First principal table for water vapor XXIV
Second principal table for water vapor. XXVII

Tables of the Appendix (English Units).

Saturated vapor of water. XXX
Saturated vapor of ether. XXXIV
Saturated vapor of alcohol. XXXVI
Saturated vapor of acetone. XXXVIII
Saturated vapor of chloroform XL
Saturated vapor of chloride of carbon. XLII
Saturated vapor of bisulphide of carbon. XLIV
Saturated vapor of ammonia. XLVI
Saturated vapor of sulphurous acid. XLVIII
Saturated vapor of carbonic acid. L
First principal table for water vapor. LII
Second principal table for water vapor. LV
Auxiliary table (English units) corresponding to pp. 62 and 63. . LVIII
Auxiliary table for conversion of pounds per square inch into atmospheres. LX

THIRD SECTION.

Theory of Vapors.

TECHNICAL THERMODYNAMICS.

§ 1. PRELIMINARY REMARKS.

According to old usage we designate as "vapors" the air-like bodies which can be transformed into liquids and even under certain circumstances into solids by cooling, generally accompanied by a diminution of volume, and conversely can be generated from liquid or solid bodies by the addition of heat.

In the introductory remarks to the investigations of gases (p. 89) it was brought out that, in the sense indicated, all kinds of gases without exception might justly be regarded as vapors; at the same time (p. 93) the reasons were given for maintaining in Thermodynamics the distinction between gases and vapors.

In the following investigations, which are intended exclusively for technical needs, we will in general consider only those vapors which, comparatively speaking, do not require too great a lowering of temperature for their liquefaction or condensation. The technically most important vapor is steam; the vapors of carbonic acid, ammonia, sulphurous acid and ether are also important, the latter on account of their application in refrigerating engines; the vapors mentioned fulfill the prescribed condition, and so do a few others which are of physical interest because certain observations are available for their more thorough investigation.

In more closely considering vapors two essentially different states must be distinguished from each other; namely, if there exists in an enclosed space a certain kind of vapor, then the behavior of this vapor is very different according to whether there is present in the same space the liquid from which the vapor was

formed, or whether the steam exists there alone without any liquid.

In the first case the vapor is said to be "moist" or "saturated," in the other case to be "non-saturated" or "superheated," and this distinction will be maintained in all the following investigations; in the first case the assumed mixture of liquid and vapor may have the liquid completely separated from the vapor, so that the space will be divided into a liquid space and a steam space, as is in fact the case in ordinary steam boilers; but it may happen that a part of the liquid, and under certain circumstances the whole of the liquid mass (particularly if its weight is small in comparison with that of the vapor), may hover in the vapor in the form of little vesicles of water; in this latter case the vapor, which is usually as invisible as ordinary air, will become perceptible to the eye as fog. Such vapor is in technical circles said to be "wet"; for the sake of brevity of expression, however, we will hereafter use the designation "wet steam" mostly for a mixture of liquid and vapor, even though the liquid is separated from the vapor in the same vessel.

Let the cylinder AB (Fig. 1) have a cross-section of 1 sq. m. [1 sq. ft.] and contain 1 kg. [1 lb.] of the liquid between the cylinder head A and piston K when the piston occupies the position K_1; let its temperature then be 0° C. [32° F.] and its volume σ_0; if it is now subject to the external pressure p, the point a will represent the condition of the liquid mass at this instant, the volume σ_0 being represented by the abscissa and the pressure p by the ordinate. If, during the operation to be immediately discussed, the pressure p, acting on the piston from the outside, is kept constant and the liquid receives heat from the outside, then its temperature will rise from 0° C. [32° F.

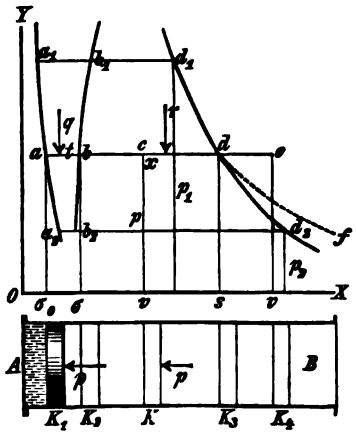

Fig. 1.

until evaporation begins at a certain temperature t; let us suppose that at this instant the volume has increased from σ_0 to σ; now the point b of the diagram corresponds to the instantaneous state of the liquid.

A remarkable phenomenon now occurs; the temperature t at which evaporation begins depends only upon the pressure p, the greater or smaller value of the pressure p corresponding respectively to the higher or lower value of the temperature t.

If the supply of heat is continued beyond the state b, the liquid will gradually pass into steam, the piston will yield, making room, and in doing this the corresponding temperature t will remain unchanged so long as the external pressure p that is overcome is kept constant, and this will continue until the last liquid element has become steam at the point d; let s designate now and hereafter the volume of the steam for this state.

Along the whole path $b\,c\,d$ (Fig. 1) the steam is in contact with the liquid and is therefore called "saturated," and the remarkable thing about this condition is that the temperature of the steam rises and falls with the pressure p so that a particular temperature corresponds to a particular value of the pressure; "the temperature t of the saturated steam is *only* a function of the pressure p"; it is not at the same time dependent on the volume of the mixture consisting of liquid and vapor, nor does it depend upon the proportion of the mixture existing at the instant, that is, on the ratio of the kilograms of liquid to the kilograms of steam present in a unit of weight of the mixture.

At any point c of the path $b\,c\,d$ (Fig. 1) a certain proportion of the mixture will exist which is completely determined by the weight x kg. [lb.] of the steam present at the instant; as we assume the mixture to have a unit of weight, it follows that the weight of the liquid mass present must be $(1-x)$ kg. [lb.]; the magnitude x may be called "specific steam weight" and $(1-x)$ the "specific liquid weight"; instead of these long terms, however, we will designate by x and $1-x$ respectively the "quality" and the "wetness" of the mixture. At the point b (Fig. 1) only liquid is present and therefore the steam quality $x=0$; at the

other limit d we have $x=1$, and this means that the last liquid element has passed into the condition of steam; steam in this condition is said to be "*dry* saturated"; the specific volume of this steam, already designated by s, and the specific volume σ of the liquid are only to be regarded as a function of the pressure p or of the temperature t.

At any intermediate point c the volume of the steam present is xs, according to the given notation, and the volume of the liquid present is $(1-x)\sigma$; consequently the specific volume v of a unit of weight of mixture of liquid and steam is

$$v=xs+(1-x)\sigma,$$

or if, for the sake of simplicity, we substitute

$$u=s-\sigma, \quad \ldots\ldots\ldots \quad (1)$$

we get

$$v=xu+\sigma, \quad \ldots\ldots\ldots \quad (2)$$

and this equation might be called the equation of condition of wet steam. The magnitude u, as well as s and σ, is a function of the pressure p or a function of the temperature t only; it is represented in Fig. 1 by the distance bd and gives the increment of volume which occurs when 1 kg. [lb.] of liquid, of the pressure p and corresponding temperature t, is transformed at constant pressure p into dry saturated steam.

If from the point d on, the steam is still heated under constant pressure p, there will occur a further increment of volume from d to the state e (Fig. 1), but now the heat supplied is expended in raising the temperature because there is no more liquid on hand; consequently increment of volume under constant pressure is connected with increase of temperature; on the way from d to e, and beyond, the steam is "superheated"; hence follows the proposition: Superheated steam has always a higher temperature than saturated steam *of like pressure.*

Let v be the specific volume of the superheated steam in the state e (Fig. 1), then this volume will always be greater than the

volume s of dry steam of the same pressure; and finally, because the specific weight γ of a body is simply the reciprocal of the specific volume, we can say, in comparing superheated steam with *dry* saturated steam of the same pressure, that in the dry saturated condition the specific weight of steam is a maximum, but its specific volume and its temperature are each a minimum.

On the other hand if from the piston position K_3 and beyond the point d expansion takes place under constant temperature t with a corresponding supply of heat, then this will be accompanied by a diminution of pressure somewhat like the course of the dotted curve df; conversely, superheated steam compressed at constant temperature until such saturation occurs will show an increase of pressure; therefore, comparing dry saturated steam with superheated steam of the same temperature, the former will possess the maximum pressure; a further increase of pressure by compression under constant temperature is not possible because the isothermal curve fd passes into the horizontal dcb at the point d.

In the foregoing discussion there was assumed in Fig. 1 a particular external pressure p against the piston, and the point a represented by its coordinates σ_0 and p a kilogram [lb.] of liquid of the temperature 0° C. [32° F.]. Now if in a second and third case the pressure p is at one time greater, namely p_1, and at the other time smaller, namely p_2, then the two points a_1 and a_2 will show by their coordinates the state of this liquid at the temperature 0° C. [32° F.]; or still more generally, the curve a_1aa_2 gives, for any pressure whatever at the temperature 0° C. [32° F.], the specific volume of the liquid measured by the abscissa corresponding to the pressure.

Now if, starting from the point a_1, heat is imparted under constant pressure p_1, evaporation will begin at the point b_1 when the temperature t_1 is reached, and it will end at the point d_1; and similarly starting with the point a_2 and heating under the constant pressure p_2, the beginning of evaporation will fall at b_2 and its end at d_2. Consequently in the curves b_1bb_2 and d_1dd_2 we have two new curves of which the first, lying to the left, can be characterized as the lower limit, and the other, lying to the right,

as the upper limit; both curves play a prominent rôle in the theory of vapors. Here for the present we will only emphasize the following.

Let us suppose that for a particular kind of vapor the two limiting curves are known by their equations

$$p=\psi(\sigma) \quad \text{and} \quad p=\phi(s), \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

where ψ and ϕ indicate functions, and suppose both formulas are represented graphically by two curves, then the constitution of the steam and liquid mixture is given as soon as we know the pressure p and the volume v of the unit of weight.

If we lay off in Fig. 1 the volume v as abscissa and the pressure p as ordinate, and if the corresponding point c falls between the limiting curves, the steam is wet, that is, mixed with liquid, is saturated; if the point falls into the space to the right of the upper limiting curve, this steam is superheated; but if it falls into the area to the left of the lower limiting curve, then only liquid is present and no steam. If the point falls on the lower limiting curve b_1b_2, then liquid alone is on hand, but of such a nature that the least supply of heat will start the evaporation. Finally if the point falls on the upper limiting curve d_1d_2, then the steam present is dry saturated.

It has already been remarked that equation (2) can be regarded as the equation of condition of wet steam; as the values σ and u occurring in it must be regarded as functions of the pressure p, the equation embraces the three variables, pressure p, volume v, and steam quality x. On the other hand if the steam is superheated, the place of the latter variable is taken by the temperature T; the general expression, therefore, for the equations of condition of superheated steam is given by

$$T=f(p, v). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3b)$$

If, as was formerly assumed, superheated vapors behave like gases, then the place of the foregoing equation would be taken by the equation of condition of gases used in Volume I; but further investigations, in which the discussion of wet steams will be kept separate from that of the superheated ones, will show that the assumption mentioned is not an appropriate one.

A. THE BEHAVIOR OF SATURATED VAPORS.

§ 2. THE PRESSURE OF SATURATED VAPORS.

The expansive force or the pressure of wet steam is, as we have already emphasized, independent of the volume v and the quality x, and is to be regarded solely as a function of the temperature t.

But the efforts to develop from a general law the relation between pressure and temperature of wet (or saturated) steam of any sort have so far not succeeded (we will call attention later to the experiments which have been made in this direction); we have been obliged to content ourselves with the setting up of empirical formulas which agree more or less closely with the results of observation.

A large number of such empirical formulas have been proposed, particularly for the vapor of water, and new ones are constantly being presented even at the present time.

There was formerly used in technical circles the formula given by Young, namely,

$$p=\left(\frac{\alpha+t}{\beta}\right)^m, \quad \ldots\ldots\ldots \quad (4)$$

in which formula α, β, and m are coefficients of experience derived from experiment.

The older formulas of Dulong and Arago, of Tredgold, of Mellet, of Pambour, of Coriolis, of the Artizan-Club were of the preceding form,[1] and they differ only in the values of the constants α, β, and m and in the limits of their application. Another much-used formula comes from Roche and was employed for the more exact physical investigations:

$$\log\frac{p}{p_0}=\frac{\beta t}{\alpha+t}. \quad \ldots\ldots\ldots \quad (5)$$

[1] A discussion and collection of the older formulas is given by Egen, "Über die Expansivkraft des Wasserdampfes." Poggendorff's Annalen, Vol. 27, 1833, p. 9.

Here p_0, α, and β mean constant quantities; the formula was used by August and by Magnus. The latter determined the constants for the vapor of water from his famous experiments on its expansive force. Holtzmann[1] calculated the constants of the formula from the experiments of Avogadro, Marx, and Bunsen for the vapors of mercury, bisulphide of carbon, sulphurous acid, cyanogen, and ammonia; even in recent times recourse has often been had to the form given. We will here omit giving the very different numerical values of the constants of the preceding formulas, for, in the principal investigations that are to follow, we will exclusively employ the relation used by Regnault and before him by Prony and Biot. The formula reads:

$$\log p = a + b\alpha^{\tau} + c\beta^{\tau}, \quad \ldots\ldots \quad (6)$$

where

$$\tau = t - t_0 \quad \ldots\ldots\ldots \quad (6a)$$

and a, b, c, α, β, and t_0 are constant quantities. Regnault has observed, for a whole series of different kinds of vapors in the saturated condition, the pressures corresponding to the most varied temperatures, and has determined for each kind of vapor the constants of the preceding equation (6) and grouped them in tabular form. (Compare reference, Vol. I, p. 97.[2])

The logarithm employed is that of Briggs, and the pressure p is measured in millimeters of mercury; t is the temperature according to Celsius; t_0 is a constant quantity which was arbitrarily chosen by Regnault and is the temperature of that one of five experiments which corresponds to the smallest pressure. The analytical determination of the five constants of equation (6) is only possible when the five values of p, underlying the determination of the constants, are chosen for equal intervals of temperature, which was an easy thing for Regnault to do because of the graphical representation of his experimental results.

---

[1] C. Holtzmann, "Über die Wärme und Elasticität der Gase und Dämpfe." Mannheim, 1845.

[2] Rel. II, 651.

In a series of such experiments let t_0 be the temperature of the experiment corresponding to the smallest pressure, and t_1 the temperature of the following experiment, then we have generally, for equal temperature differences,

$$\tau = n(t_1 - t_0),$$

wnere $n = 0, 1, 2, 3$, etc.; substitution in equation (6) gives

$$\log p = a + b\alpha^{n(t_1-t_0)} + c\beta^{n(t_1-t_0)},$$

or, if we temporarily assume

$$\alpha_1 = \alpha^{t_1-t_0} \quad \text{and} \quad \beta_1 = \beta^{t_1-t_0}, \quad \ldots\ldots \quad (7)$$

$$\log p = a + b\alpha_1^n + c\beta_1^n. \quad \ldots\ldots \quad (8)$$

Now for the sake of simplicity let us put $\log p = y$, and distinguish the five experimental values of $\log p$ by the symbols

$$y_0\ y_1\ y_2\ y_3\ y_4.$$

If we take in order $n = 0, 1, 2, 3, 4$, then equation 8 for the five experiments becomes

$$\begin{aligned} y_0 &= a + b + c, \\ y_1 &= a + b\alpha_1 + c\beta_1, \\ y_2 &= a + b\alpha_1^2 + c\beta_1^2, \\ y_3 &= a + b\alpha_1^3 + c\beta_1^3, \\ y_4 &= a + b\alpha_1^4 + c\beta_1^4. \end{aligned}$$

Let us, moreover, introduce for the differences of adjacent values the following symbols:

$$\begin{aligned} y_1 - y_0 &= \delta y_0, \\ y_2 - y_1 &= \delta y_1, \\ y_3 - y_2 &= \delta y_2, \\ y_4 - y_3 &= \delta y_3, \end{aligned}$$

then by a suitable combination of the given formulas we first of all get for the determination of the constants α_1 and β_1 the two equations:

$$\alpha_1+\beta_1=\frac{\delta y_1\cdot\delta y_2-\delta y_0\cdot\delta y_3}{(\delta y_1)^2-\delta y_0\cdot\delta y_2},$$

$$\alpha_1\beta_1=\frac{(\delta y_2)^2-\delta y_1\cdot\delta y_3}{(\delta y_1)^2-\delta y_0\cdot\delta y_2},$$

and then follows

$$b=\frac{\beta_1\cdot\delta y_0-\delta y_1}{(\alpha_1-1)(\beta_1-\alpha_1)},$$

$$c=\frac{\delta y_1-\alpha_1\cdot\delta y_0}{(\beta_1-1)(\beta_1-\alpha_1)},$$

$$a=y_0-b-c,$$

and finally, according to equation (7),

$$\log\alpha=\frac{1}{t_1-t_0}\log\alpha_1 \quad \text{and} \quad \log\beta=\frac{1}{t_1-t_0}\log\beta_1.$$

In the manner here indicated [1] the constants in Regnault's formulas were determined and are the ones used in his writings. I have omitted giving here all of these constants, preferring to give the values, for different vapors, of $\log(b\alpha^\tau)$ and $\log(c\beta^\tau)$ as functions of the temperature t, and this necessitates indicating the sign in front of the second and of the third term of equation (6); the tabulation on page 14 also contains the values of a for different kinds of vapors, the whole facilitating the work of numerical computation.

The tabulation contains only a selection from the different vapors investigated by Regnault; only such kinds are there inserted as have already found technical application or could

[1] I have not regarded it as superfluous to give here the formulas for computing the five constants, because in Regnault's papers (Mémoires de l'Académie, Vol. XXI, p. 596) there are typographical errors in the two formulas for $\alpha_1+\beta_1$ and $\alpha_1\beta_1$; Moritz called attention to them in the Bulletin de l'Académie de St. Pétersbourg, Vol. XIII, p. 43. They are also pointed out by Regnault himself in the second volume of "Relation," etc.

possibly be thus used; the table also contains a few vapors whose physical behavior in other particulars was examined by Regnault and which therefore seemed suited to explain the behavior of the other vapors which were not so fully subjected to experiment.

With the help of the expressions given in the table there can now be computed the pressures for given temperatures in the case of the eleven kinds of vapors enumerated; the pressure is here given in millimeters of mercury, and this is also done for these vapors in the first seven tables of the appendix of this book; these computations are in part due to Regnault and in part due to myself.

If we divide these pressure values by 760 we get for the corresponding temperature t the pressure p "in atmospheres," and if we multiply the values thus obtained by 10333 we get "the specific pressure," i.e., its pressure expressed in kilograms per square meter. If we now divide the latter value by ten thousand, we get the pressure expressed in kilograms per square centimeter, a value which in common parlance is generally *also* designated as being expressed in atmospheres; to avoid confusion many speak of new atmospheres; but in the present treatise we understand once for all by the pressure of one atmosphere the average pressure of the atmospheric air at the level of the sea, which is assumed to be 760 millimeters of mercury and corresponds to a pressure of 10333 kg. on 1 sq. m. or of 1.0333 kg. on 1 sq. cm. [2116.3 lb. per sq. foot].

When we adjust ourselves to the newer measurement we will not use the term "atmospheres," but simply say the pressure amounts to so and so many kilograms, it being then understood that one square centimeter of area is meant. It is hardly necessary to say, because it seems scientifically self-evident, that the absolute pressure is always meant, never the so-called *excess* or *gauge* pressure, that is, the pressure remaining after the back pressure of one atmosphere is subtracted.

Now the application of Thermodynamic laws to the investigation of the behavior of vapors demands not only a knowledge of

Elements of the Formula $\log p = a + b\alpha^\tau + c\beta^\tau$.

Saturated Vapors of	Values of a	Sign of $b\alpha^\tau$	Sign of $c\beta^\tau$	Values of $\log(b\alpha^\tau)$	Values of $\log(c\beta^\tau)$
1. Water[1] ..... 0—100°	4.7393707	−	+	+0.6117408 − 0.003274463 t	−1.8680093 + 0.006864937 t
100—200°	6 2640348	−	−	+0.6593123 − 0.001656138 t	+0.0207601 − 0.005950708 t
2. Ether.....	5.0286298	−	+	+0.4414317 − 0.0031223 t	−3.3497030 + 0.0145775 t
3. Alcohol.....	5.4562028	−	+	+0.6390301 − 0.0029143 t	−2.4949325 − 0.0590515 t
4. Acetone[2].....	5.3085419	−	−	+0.5312766 − 0.0026148 t	−0.9645222 − 0.0215592 t
5. Chloroform.....	5.2253893	−	−	+0.5219943 − 0.0025856 t	−0.9111383 − 0.0131824 t
6. Chloride of carbon.....	12.0962331	−	−	+0.9550683 − 0.0002880 t	+0.1934744 − 0.0050220 t
7. Bisulphide of carbon.....	5.4011662	−	−	+0.4918860 − 0.0023372 t	−0.7200368 − 0.0088003 t
8. Mercury.....	5.6640459	−	+	+0.8890208 − 0.0012438 t	−0.4179605 − 0.0119062 t
9. Carbonic acid.....	5.6771989	−	+	+0.2175359 − 0.0052911 t	−0.3948491 − 0.0089594 t
10. Ammonia.....	5.5582655	−	Zero	+0.3124815 − 0.0035023 t	
11. Sulphurous acid[3].....	5.6604443	−	−	+0.4026078 − 0.0027084 t	−1.1733846 − 0.0120780 t

(t Temperature according to Celsius.)

[For the pressure p in lbs. per sq. in. and temperature Fahrenheit, simply subtract 1.7135937 from a and put $\frac{5}{9}(t-32)$ for t.]

[1] The constants for water and the temperature limits 0—100° C. were newly computed by Moritz (Bulletin de l'Académie de St. Pétersbourg, Vol. XIII, p. 43), because an error had crept into the corresponding computations of Regnault.

[2] I have newly determined the constants for Acetone, because the values given by Regnault proved to be inaccurate. (Relation II, 474.)

[3] The constants for sulphurous acid are likewise incorrectly given by Regnault (Rel. II, 590 and 651); the table there given is approximately correct and was therefore probably computed with the help of other constants.

the relation between the pressure p and the temperature t, but it is also necessary to know how the differential coefficient $\frac{dp}{dt}$, appearing in the most important formulas, is connected with the temperature. This relation, however, can be easily derived from Regnault's formula.

If we multiply the equation

$$\log p = a + b\alpha^\tau + c\beta^\tau$$

on both sides by the natural logarithm of 10, namely by 2.302585093, which value we will designate by k, we will get in the left member the natural logarithm of p; we thus obtain

$$\log_e p = ka + kb\alpha^\tau + kc\beta^\tau.$$

If we differentiate this equation and consider in so doing that $\tau = t - t_0$, where t_0 is a constant quantity, there follows

$$\frac{dp}{pdt} = kb \log_e \alpha \cdot \alpha^\tau + kc \log \beta \cdot \beta^\tau,$$

or if we replace the natural logarithms of the right member by the Briggs, or common, logarithms,

$$\frac{dp}{pdt} = (k^2 b \log \alpha)\alpha^\tau + (k^2 c \log \beta)\beta^\tau.$$

Now, because from the earlier data the constants b, c, α, and β are known for the different vapors, the two coefficients of α^τ and β^τ can be computed; let us designate them by m and n, so if we put

$$m = k^2 b \log \alpha \quad \text{and} \quad n = k^2 c \log \beta, \quad . \quad . \quad . \quad . \quad (9)$$

there follows

$$\frac{dp}{pdt} = m\alpha^\tau + n\beta^\tau. \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In the following tabulation I have not given the values of m and n, but have given, for more convenient use, the values of $\log (m\alpha^\tau)$ and $\log (n\beta^\tau)$ as functions of t; as a consequence I had to

introduce into the first two columns the signs belonging to the two terms of the right member of equation (10).

With the help of the expressions of the following tabulation (p. 17), the values of

$$\frac{dp}{pdt}$$

in column 5, tables 1–9 of the Appendix were computed. These values were placed in the tables because they are to play an important part, as will become evident later; if we multiply them by p, there will result for the different temperature values the corresponding values of the differential coefficient

$$\frac{dp}{dt},$$

which values are inserted in column 4 of said tables. This differential coefficient is there given in millimeters of mercury; in using it in the formulas which are to follow it must be expressed in kilograms per square meter [pounds per square inch].

We will now illustrate by an example the use of the two auxiliary tables.

Example. The pressure of saturated acetone vapor is to be found for the temperature $t=100°$ C. [212° F.]; here we get, according to table on p. 14,

$$\log (b\alpha^\tau) = +0.5312766 - 0.0026148\, t = 0.2697966,$$
$$\log (c\beta^\tau) = -0.9645222 - 0.0215592\, t = -3.1204422$$
$$= 0.8795578 - 4,$$

from which follows

$$b\alpha^\tau = 1.8612155,$$
$$c\beta^\tau = 0.0007578.$$

Now because $a=5.3085419$ [$a=3.5949482$], we get from equation (8), taking account of the signs in front of $b\alpha^\tau$ and of $c\beta^\tau$,

$$\log p = 3.4465686$$
$$[\log p = 1.7329749]$$

and

$$p = 2796.20 \text{ mm.}$$
$$[p = 54.0723 \text{ lb. per sq. in.}]$$

a result which can also be found in Table 4*a* of the Appendix.

Elements for the Formula $\frac{dp}{pdt} = m\alpha^\tau + n\beta^\tau$.

Saturated Vapors of		Signs of $m\alpha^\tau$	Signs of $n\beta^\tau$	Values of log $(m\alpha^\tau)$	Values of log $(n\beta^\tau)$
1. Water. .	0—100°	+	+	$-1.1486877-0.003274463\,t$	$-3.3069414+0.006864937\,t$
	100—200°	+	+	$-1.3971597-0.001656138\,t$	$-1.4802398-0.005950708\,t$
2. Ether		+	+	$-1.3396623-0.0031223\,t$	$-4.4616396+0.0145775\,t$
3. Alcohol		+	−	$-1.1720041-0.0029143\,t$	$-2.9992701-0.0590515\,t$
4. Acetone		+	+	$-1.3268535-0.0026148\,t$	$-1.9064582-0.0215592\,t$
5. Chloroform		+	+	$-1.3410130-0.0025856\,t$	$-2.0667124-0.0131824\,t$
6. Chloride of carbon		+	+	$-1.8611078-0.0002880\,t$	$-1.3812195-0.0050220\,t$
7. Bisulphide of carbon		+	+	$-1.4339778-0.0022372\,t$	$-2.0511078-0.0088003\,t$
8. Mercury		+	−	$-1.2917974-0.0012438\,t$	$-1.6177651-0.0119062\,t$
9. Carbonic acid		+	−	$-1.3344869-0.0052911\,t$	$-1.7181390-0.0089584\,t$
10. Ammonia		+	zero	$-1.4187347-0.0035023\,t$	
11. Sulphurous acid		+	+	$-1.4402478-0.0027084\,t$	$-2.3669587-0.0120780\,t$

(t Temperature according to Celsius).

[For English units and temperature Fahrenheit, substitute $\frac{5}{9}(t-32)$ for t and add arithmetically 0.2552725 to the first term in both columns]:

Example. For sulphurous acid we get, according to the preceding auxiliary table, for $t = 0°$ C. [32° F.]:

$$\log(m\alpha^\tau) = -1.4402478 = 0.5597522 - 2 \quad [0.3044797 - 2],$$
$$\log(n\beta^\tau) = -2.3669587 = 0.6330413 - 3 \quad [0.3777688 - 3].$$

Hence

$$m\alpha^\tau = 0.0362871 \quad \text{and} \quad n\beta^\tau = 0.0042958,$$

and therefore

$$\frac{dp}{pdt} = 0.0405829 \quad [0.022546].$$

According to the data of the foregoing auxiliary table the pressure p of sulphurous acid for $t = 0°$ C. [32° F.] is equal to 1165.05 mm. [48.8695 in.] of mercury, or, divided by 760 [29.922], it becomes 1.5330 atmospheres or 15840 kg. to the square meter [22.53 lb. per sq. in.]; therefore finally we get

$$\frac{dp}{dt} = 47.281 \text{ mm. or } 642.83 \text{ kg.}$$

$$\left[\frac{dp}{dt} = 1.03145 \text{ in. or } 0.508 \text{ lb. per sq. in. and per degree F.}\right]$$

The values introduced into the first ten tables of the Appendix proceed by equal intervals of temperature; but for the technically highly important vapor of water there are added two

additional tables, 11 and 12, the values of which are computed for equal intervals of pressure. Biot's formula (6) used by Regnault is not reversible, like the other formulas (4) and (5), in the sense that the temperature can be computed for the given pressure; consequently the determination of temperature was found by interpolation; this was also done in the following auxiliary table, which ought to render good service in certain comparisons.[1]

Temperature t (Celsius) for Saturated Vapors of:

Pressure in Atmospheres (10333 kg. per sq. m.).	Water.	Ether.	Alcohol.	Acetone.	Chloroform.	Chloride of Carbon.	Bisulphide of Carbon.	Mercury.	Ammonia.	Sulphurous Acid.	Pressure in Atmospheres (14.6967 lb. per sq. in.).
	°	°	°	°	°	°	°	°	°	°	
1	100.00	34.96	78.30	56.32	60.18	76.52	46.25	357.35	−32.90	−10.01	1
2	120.60	55.87	96.82	78.03	82.59	101.37	69.25	397.32	−18.12	6.69	2
3	133.91	69.61	108.83	92.30	97.55	117.91	84.46	423.01	− 8.57	17.72	3
4	144.00	80.21	117.98	103.26	109.04	130.50	96.17	442.47	− 1.32	27.02	4
5	152.22	88.96	125.49	112.30	118.56	141.18	105.86	458.33	4.61	33.28	5
6	159.22	96.47	131.91	120.08	126.78	150.19	114.21	471.85	9.68	39.38	6
7	165.34	103.09	137.57	126.95	134.08	158.12	121.59	483.71	14.13	44.80	7
8	170.81	109.01	142.65	133.15	140.66		128.25	494.32	18.11	49.71	8
9	175.77	114.37	147.28	138.81	146.71		134.34	504.36	21.74	54.23	9
10	180.31	119.39	151.54	144.05	152.34		139.97	513.44	25.08	58.15	10

[To convert into Fahrenheit degrees multiply tabular values by 1.8 and add 32°].

[1] Tables 1 to 7 inclusive and Table 11 of the Appendix were computed with a Thomas calculating machine immediately after the appearance of the second volume of Regnault's work. The second edition of the present book, issued in 1866, already contained these tables, but in the present edition the numerical values of some columns differ somewhat from those given earlier because then the specific pressure corresponding to *one* atmosphere was assumed equal to 10334 kg., while in the present edition there is taken the more accurate value 10333 kg.

I have in my day undertaken the excessively laborious and time-absorbing computation of the tables in the hope that a comparison of the results of calculation might lead to certain common laws to which saturated vapors of different liquids are simultaneously subject. This expectation has not up to this time been realized, although the tables have been much used in the interval by others as well as myself.

Tables 8, 9, 10, and 12 are new ones which have been added, and further information concerning their production will be given later.

The law of change of pressure with temperature for the given collection of vapors can be easily shown graphically with the help of the foregoing auxiliary table, provided we do as Regnault did, lay off the temperature values as abscissas and the pressures as ordinates. We then get for each kind of vapor a special curve; the separate curves have similar courses and lie alongside of one another in the diagram; if the temperature is laid off from left to right, then the pressure curve for mercury will occupy the extreme right-hand position; the extreme left-hand curve will be that for ammonia; much farther to the left, in the neighborhood of the absolute zero, would lie the pressure curves corresponding to the saturated condition of oxygen, nitrogen, etc., for which curves at the present time only a few points have been determined. For example, von Wroblewski[1] gives for oxygen at atmospheric pressure (more exactly at 760 millimeters of mercury) [29.922 in.] the temperature of the saturated condition as −181.5° C. [−294.7° F.]; for nitrogen at the same pressure −193° [−315.4° F.]; for carbonic oxide gas (at 735 millimeters) [23.937 in.] the temperature is −190° C. [−310° F.]. v. Wroblewski gives also (ibid.), to be sure only for a narrow range of temperature, that is, for a short distance, the course of the pressure curves for the vapors mentioned.

For carbonic acid (compare Table 10 of the Appendix) Regnault[2] gives the temperature as −78.2° C. [108.76° F.] for atmospheric pressure (760 mm.).

For liquid atmospheric air the boiling point is −191° C. [311.8° F.].

The question of the liquefaction of Gases will be discussed more fully later.

---

[1] Transactions of the Imperial Academy of Sciences in Vienna, Vol. 91, 1885: "Über den Gebrauch des siedenden Sauerstoffs, Stickstoffs, Kohlenoxydes, sowie der atmosphärischen Luft als Kältemittel."

[2] Rel. II, 626.

§ 3. THE HEAT OF THE LIQUID, THE TOTAL HEAT AND THE LATENT HEAT.

The utilization of the principles of thermodynamics, for the investigation of the behavior of vapors, demands not only the experimental results discussed above, but also other results which have come from observations; these latter observations are of such a character that for at least one particular case we can decide upon the quantity of heat which is necessary to generate saturated vapor from its liquid.

Existing experimental results of this sort, at least those which we will use here, we likewise owe to Regnault; through him we know, for a series of different liquids, the quantity of heat which is necessary to convert at constant pressure p the unit of weight of liquid, possessing the initial temperature 0° C. [32° F.], into saturated vapor.

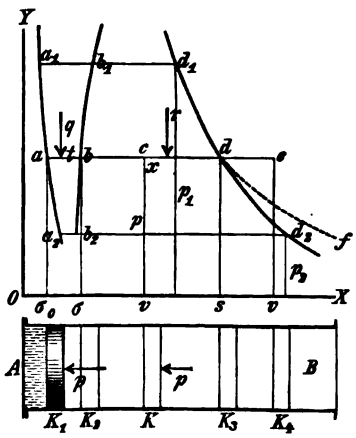

FIG. 1.

In Fig. 1 (repeated from p. 4) suppose a unit of weight of liquid of 0° C. [32° F.] temperature to be confined in cylinder AB, to the left of the piston position K_1; let the piston be acted upon from without by a pressure of p kg. [lb.] per unit of area. Point a gives the state of the liquid as regards the volume and the pressure, the former being indicated by the abscissa σ_0, and the latter by the ordinate p.

If heat is supplied to this liquid from without, then, before steam formation begins, it will be heated up to the temperature t which corresponds to the pressure p of the vapor to be generated; in so doing (Fig. 1) the volume of the liquid increases from σ_0 to σ, and point b, which lies upon the lower limit curve, designates the state of the liquid at this instant. I call the heat quantity neces-

sary for the change of state from a to b the heat of the liquid, and will designate it hereafter by q; this quantity is doubtless only a function of the temperature and can be represented by the expression

$$q=\int_0^t c\,dt; \quad \ldots\ldots\ldots \quad (11)$$

accordingly the ratio

$$c=\frac{dq}{dt} \quad \ldots\ldots\ldots \quad (12)$$

stands forth as the specific heat of the liquid at constant pressure p and likewise is a function of the temperature t. On account of the connection established between the temperature t and the pressure p in the preceding articles we can also regard both quantities q and c as functions of the pressure p. Now if the heat supply is continued from the point b on, hence from piston position K_2 (Fig. 1), steam formation will begin under the constant pressure p and will be continued to the point d, belonging to piston position K_3, till the last liquid element has passed into vapor; this vapor is dry saturated, the point d lying in the upper limit curve. The quantity of heat supplied, during the period of steam formation along the path bd, hence under constant pressure p and therefore also without change of temperature t, is called the latent heat, or (according to Clausius) the heat of evaporation; hereafter we will always designate it by r; it is likewise a function of the temperature t, or if we wish we may regard it as a function of the pressure p. The sum of the two heat quantities q and r we will designate by λ, i.e.,

$$\lambda=q+r, \quad \ldots\ldots\ldots \quad (13)$$

which is generally called (according to Regnault) the total heat of the steam; it gives the quantity of heat necessary to generate dry saturated steam, at constant pressure p and the corresponding temperature t, from liquid at 0° C. [32° F.].

Regnault,[1] from his experiments, derived the following

[1] Rel. I, 726.—Rel. II, 881.

empirical formulas for calculating the total heat of the vapors enumerated below:

Saturated Vapor of	Total Heat λ.
Water	$\lambda = 606.50 + 0.305\,t$
Ether	$= 94.00 + 0.45000\,t - 0.00055556\,t^2$
Acetone	$= 140.50 + 0.36644\,t - 0.000516\,t^2$
Chloroform	$= 67.00 + 0.1375\,t$
Chloride of carbon	$= 52.00 + 0.14625\,t - 0.000172\,t^2$
Bisulphide of carbon	$= 90.00 + 0.14601\,t - 0.0004123\,t^2$

(14)

Saturated Vapor of	Total Heat λ.
Water	$\lambda = 1091.7 + 0.305\ (t-32°)$
Ether	$= 169.2 + 0.45000\ (t-32°) - 0.00031364\ (t-32°)^2$
Acetone	$= 252.9 + 0.36644\ (t-32°) - 0.00028667\ (t-32°)^2$
Chloroform	$= 120.6 + 0.1375\ (t-32°)$
Chloride of carbon	$= 93.6 + 0.14625\ (t-32°) - 0.00009556\ (t-32°)^2$
Bisulphide of carbon	$= 162.0 + 0.14601\ (t-32°) - 0.00022906\ (t-32°)^2$

According to Regnault's[1] data the heat of the liquid q for these vapors is to be determined from the following formulas:

Saturated Vapor of	Heat of the Liquid q.
Water	$q = t + 0.00002\,t^2 + 0.0000003\,t^3$
Ether	$= 0.52901\,t + 0.0002959\,t^2$
Alcohol	$= 0.54754\,t + 0.0011218\,t^2 + 0.000002206\,t^3$
Acetone	$= 0.50643\,t + 0.0003965\,t^2$
Chloroform	$= 0.23235\,t + 0.0000507\,t^2$
Chloride of carbon	$= 0.19798\,t + 0.0000906\,t^2$
Bisulphide of carbon	$= 0.23523\,t + 0.0000815\,t^2$

(15)

Saturated Vapor of	Heat of the Liquid q.
Water	$q = t - 32° + 0.000011\ (t-32°)^2 + 0.0000001\ (t-32°)^3$
Ether	$= 0.52901\ (t-32°) + 0.0001644\ (t-32°)^2$
Alcohol	$= 0.54754\ (t-32°) + 0.0006232\ (t-32°)^2 + 0.000000681\ (t-32°)^3$
Acetone	$= 0.50643\ (t-32°) + 0.0002203\ (t-32°)^2$
Chloroform	$= 0.23235\ (t-32°) + 0.0000282\ (t-32°)^2$
Chloride of carbon	$= 0.19798\ (t-32°) + 0.0000503\ (t-32°)^2$
Bisulphide of carbon	$= 0.23523\ (t-32°) + 0.0000453\ (t-32°)^2$

[1] Rel. I, 746.—Rel. II, 262.

With the help of the preceding data and using equation (13) we find from $r=\lambda-q$ the following empirical formulas for the direct calculation of the latent heat (heat of evaporation):

Saturated Vapor of	Latent Heat.	
Water	$r=606.50-0.695\,t\ -0.00002\,t^2-0.0000003\,t^3$	
Ether	$=\ 94.00-0.07901\,t-0.0008514\,t^2$	
Acetone	$=140.50-0.13999\,t-0.0009125\,t^2$	(16)
Chloroform	$=\ 67.00-0.09485\,t-0.0000507\,t^2$	
Chloride of carbon	$=\ 52.00-0.05173\,t-0.0002626\,t^2$	
Bisulphide of carbon	$=\ 90.00-0.08922\,t-0.0004938\,t^2$	

Saturated Vapor of	Latent Heat.
Water	$r=1091.7-0.695\,(t-32°)\ -0.000011\ (t-32°)^2$ $-0.0000001\ (t-32°)^3$
Ether	$=\ 169.2-0.07901\,(t-32°)-0.00047804\,(t-32°)^2$
Acetone	$=\ 252.9-0.13999\,(t-32°)-0.00050697\,(t-32°)^2$
Chloroform	$=\ 120.6-0.09485\,(t-32°)-0.00002820\,(t-32°)^2$
Chloride of carbon	$=\ 93.6-0.05173\,(t-32°)-0.00014586\,(t-32°)^2$
Bisulphide of carbon	$=\ 162.0-0.08922\,(t-32°)-0.00027436\,(t-32°)^2$

According to the formulas here given the corresponding values have been computed for the vapors mentioned and inserted in the tables of the Appendix. Regnault did not set up an empirical formula for computing the total heat λ of alcohol, because this value varied with the temperature in a way that would have led to an empirical formula of very complicated form. Therefore in the table given for alcohol those values of λ were employed which were found by Regnault in a graphical way.

The kinds of vapor presented in the above tabulation are the only ones for which the heat quantities discussed can be computed from the given formulas, as is required by our subsequent investigations; for all other vapors, and especially for the vapors of carbonic acid, ammonia and sulphurous acid presented in the tables of the Appendix, there exist only a few observations.

Before making use in the following investigations of the just presented, empirical, formulas given by Regnault, it is necessary to make a few remarks concerning their reliability and range of application. Closer examination of Regnault's experimental methods, which are repeated in nearly every one of the

larger manuals of Physics, shows that the determination of the heat of the liquid q in particluar, and hence also of its specific heat c, took place under assumptions which were not completely realized.

In Fig. 1, p. 4, the curve a_1aa_2 represents the relation between the pressure and volume of the liquid at constant temperature 0° C. [32° F.]; that this curve approaches the axis of ordinates OY with increasing pressure, as indicated in the figure, is indubitable; on the other hand observations have shown that the decrements of volume of liquids, particularly of water, under constant temperature but with increasing pressure, are negligible in comparison with those which occur with the final steam formation. Consequently the curve a_1aa_2(Fig. 1) can be regarded as a v e r t i c a l l i n e; in other words, within the limits of pressure occurring in engineering the specific volume σ_0 of the liquid at 0° C. [32° F.] can be regarded as independent of the pressure.

The second curve b_1bb_2 (Fig. 1), the lower limit curve, gives the relation which exists between the pressure and volume of the liquid when its temperature corresponds exactly to the pressure at which the beginning of steam formation is possible. It may be safely assumed that this curve, as the figure shows, departs from the axis of ordinates as the pressure grows; we will come back to the question later, when investigating the equation of condition of vapors.

We may conclude with H i r n that within the ordinary pressure limits w a t e r, in particular, experiences an extraordinarily small increase of specific volume σ with growing pressure.

Let t be the temperature, σ the corresponding specific volume of w a t e r, and let σ_0 be its specific volume at 0° C. [32° F.], then H i r n's statements are reproduced by the following empirical formula:

$$\sigma = \sigma_0(1 + 0.00009\,t + 0.0000034\,t^2)$$

$$[\sigma = \sigma_0\{1 + 0.000005\,(t - 32°) + 0.00000105\,(t - 32°)^2\}].$$

From it we get for $\sigma_0 = 0.001$ [0.01602] at

$t = 100°$	$\sigma = 0.00104$	$t = 212°$	$\sigma = 0.0167$ cu. ft.
125	$= 0.00106$	$= 257$	$= 0.0170$
150	$= 0.00109$	$= 302$	$= 0.0175$
175	$= 0.00112$	$= 347$	$= 0.0179$
200	$= 0.00115$	$= 392$	$= 0.0184$

Up to $t=100°$ [212°] we can regard $\sigma=0.0010$ as constant. From here up to 200° [392°], or up to the corresponding steam pressure of 15.4 atmospheres, the increment of σ is so small that it is permissible, in general, to regard the increments of volume of liquid water as also negligible up to the given pressure limit, in comparison with the changes in volume experienced by their vapors when the pressures and temperatures change. This assumption is also made as regards the other liquids treated in the following investigations; it is therefore assumed that the volume σ of a unit of weight of liquid is a constant quantity whatever temperature the liquid may possess and whatever the pressure to which it may be subjected; or presenting it in another way, the two curves b_1bb_2 and a_1aa_2 (Fig. 1) are regarded as coinciding with the vertical line. We must at once add the remark, however, that this assumption is only permissible between certain temperature and pressure limits; in all the following investigations, therefore, it is tacitly assumed that these limits are not exceeded during the course of the cycles investigated.

In connection with the preceding statements we must not fail to refer to the fact that the manner in which Regnault computed from his experiments the heat of the liquid q and its specific heat c involved a tacit assumption of the invariability of the specific volume of the liquid. If this assumption is not granted, then we must doubt the accuracy of the empirical formulas given by Regnault for the heat of the liquid. He did not at all observe the quantity of heat required by a liquid in order to bring it at constant pressure from the temperature 0° [32°] to the corresponding temperature t of evaporation, nor did he observe the quantity of heat which must be withdrawn during a reversal of the operation; what Regnault did in his experiments with water was to let a certain quantity of water flow from a little steam boiler, under the steam pressure prevailing in the latter, into a calorimeter filled with cold water and to observe the rise of temperature which the liquid there, under atmospheric pressure, experienced. The occurrence taking place here is a complicated one, but can be pursued by means of our subsequent

presentations on the efflux of highly heated liquids; the result of these calculations is that Regnault's method of procedure is permissible, if we accept the invariability of the specific volume of water or, to speak more exactly, acknowledge that it varies but slightly.

Later experiments by Hirn, also by Pfaundler and Platter, indicate, however, that the specific heat of water grows more rapidly with the temperature than is the case in the above-mentioned statements by Regnault.

Nevertheless, in the present book we will hold fast to Regnault's statements.

§ 4. STEAM HEAT AND THE INNER AND OUTER LATENT HEAT.

The empirical formulas just adduced, which Regnault has given for calculating the total heat and the latent heat, are valid under the express proviso that the steam was generated under the constant pressure p and under just that pressure which corresponds, according to the law of tensions, to the assumed temperature.

During the passage from the liquid to the vapor condition work is produced, because of the increase of volume from σ to s (Fig. 2), which is represented by the area of the rectangle σbds. If we represent this work by L we get at once, for the present case,

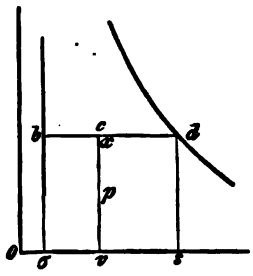

FIG. 2.

$$L = p(s - \sigma).$$

As the two magnitudes s and σ occur in the following formulas in the form of their difference, we will follow the practice of Clausius (as was done in deriving equation (2), p. 6) and make use of the simpler notation

$$u = s - \sigma,$$

representing the work in question by

$$L=pu.$$

In Fig. 2 the value u is measured by the distance bd, and may be regarded as a still to be determined function of the pressure p or, if we wish, of the temperature t.

As production of work is accompanied by a disappearance of heat, the unit of work corresponding to the heat quantity A,

$$AL=Apu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

represents the quantity of heat converted into work when a unit of weight of dry saturated steam is formed under constant pressure, or, conversely, represents the heat quantity generated when dry steam is condensed at constant pressure.

If the steam is formed at 0° C. [32° F.] temperature, the total heat is λ; therefore of the total heat supplied, only the heat quantity $(\lambda-Apu)$ remains behind in the steam. If this remnant is designated by J, we get the relation

$$J=\lambda-Apu. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

This heat I call the "steam heat"; it states how many more units of heat are contained in a unit of weight of saturated steam at a certain pressure than in the unit of weight of liquid *of 0° C. [32° F.] temperature.*

On the other hand, if the liquid at the beginning of the steam formation already possesses the corresponding temperature t, the heat quantity necessary to transform it, at constant pressure, into steam is measured by the latent heat r; accordingly the difference $(r-Apu)$ again represents the quantity of heat remaining behind in the steam, or consumed in the interior in forming the steam. Designating this difference by ρ we get the relation

$$\rho=r-Apu. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

I call the value ρ the "inner latent heat" and correspondingly call the value Apu the "external latent heat"; the two heat

quantities together give the total latent heat, or simply the latent heat. The inner latent heat ρ states how many more units of heat are contained in a unit of weight of saturated steam of given pressure than in the unit of weight of liquid *of the same temperature.*

Combining the two preceding equations (18) and (19) we get, with the help of equation (13), the relation

$$J = q + \rho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

I introduced the two quantities J and ρ into the theory of heat not only because all the formulas bearing on the behavior of saturated vapors took on a simpler form, but because these magnitudes had a more general significance than the quantities λ and r.

The steam heat J and the inner latent heat ρ are both independent of the manner in which the steam has been formed from the liquid, while in using the total heat λ and the latent heat r it must be expressly kept in view that they are only valid when evaporation or condensation, as the case may be, has taken place under constant pressure.

If we succeed in determining a single one of the three heat quantities J, ρ, and Apu, as a function of the pressure or of the temperature, then the two other values are also determined by the preceding formulas.

Thermodynamics gives us the means of establishing a relation for the calculation of the external latent heat Apu.

For this purpose let us suppose a unit of weight of a mixture of a vapor and liquid in which x kg. [lb.] of steam are mixed with $(1-x)$ kg. [lb.] of liquid, then the volume v, belonging to the point c (Fig. 2), is, according to equation (2), p. 6,

$$v = xu + \sigma.$$

If we supply the heat quantity dQ to this mixture, at constant temperature, the volume will increase by the amount

$$dv = u dx,$$

because in this case u is constant; in so doing the quantity dx of the liquid is converted at constant pressure into steam, so that

$$dQ = r dx,$$

and hence from combination with the preceding equation we get

$$dQ = \frac{r}{u} dv. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

For this quantity of heat we can find another expression from the fundamental equations given earlier.

From the second set of equations under III*a* (Vol. 1, p. 62), which is valid for all bodies, namely, according to

$$dQ = \frac{A}{\frac{\partial t}{\partial p}} [X dt + T dv],$$

we get

$$dQ = \frac{AT}{\frac{\partial t}{\partial p}} dv,$$

because here it is assumed that heat is imparted under constant temperature.

In the present case the temperature is only a function of the pressure p, hence the reciprocal value of the partial differential coefficient $\frac{\partial t}{\partial p}$ can be written in the form $\frac{dp}{dt}$ and we get

$$dQ = AT \frac{dp}{dt} dv.$$

The combination of this equation with equation (21) then brings us to the relation

$$\frac{r}{u} = AT \frac{dp}{dt}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

in which T represents the absolute temperature.

This equation constitutes the most important and prominent result thus far brought about by thermodynamics in its investigation of the behavior of saturated vapors.

The equation comes from Clapeyron (1834), but in his formula AT is replaced by the temperature function C which was designated above as the Carnot function (see Vol. I, Remarks, p. 130). On the basis of the then known values of the latent heat r of different vapors, and under the assumptions which at that time were made concerning the specific volume of vapors, Clapeyron calculated the value of the Carnot function for different temperatures.

After the true relation of the Carnot function to the temperature had been recognized and after the more exact experiments of Regnault on the latent heat were known, the procedure of Clausius was followed and the formula was reversed for a more exact determination of the values of u, a question to which we will return later. As the formula will occasion special investigations in other directions, we will, for ease of reference, designate it hereafter as Clapeyron's equation.

For the purpose of the investigations contained in this article, we get, by dividing both sides of equation (22) by Ap,

$$\frac{r}{Apu} = \frac{T}{p}\frac{dp}{dt}, \quad \ldots\ldots\ldots \quad (23)$$

and can then compute the value ϕ, where

$$\phi = \frac{Apu}{r}. \quad \ldots\ldots\ldots \quad (24)$$

For vapors for which r is known as a function of the temperature there can then be computed from the preceding formulas the values Apu, ρ, and J for every temperature.

The corresponding columns of Tables 1 to 9 of the Appendix were computed according to the preceding formulas and, beyond the headings of the several columns, need no further explanation. For the utilization of the tabular values, particularly for technical purposes, it is essential, however, that empirical formulas should be established for the heat quantities J, ρ, and Apu, so that these quantities can be rapidly calculated with sufficient accuracy.

As in this book we stick strictly to Regnault's results for the heat quantities λ and q, I do not wish to increase the num-

ber of empirical formulas any more than is absolutely necessary. I have only developed from the tabular values the formulas for the inner latent heat ρ, given in the following tabulation and reproducing these values with sufficient accuracy.

Saturated Vapors of	Inner Latent Heat.
Water	$\rho = 575.40 - 0.791\,t$
Ether	$= 85.54 - 0.10648\,t - 0.0007160\,t^2$
Acetone	$= 131.63 - 0.20184\,t - 0.0006280\,t^2$
Chloroform	$= 62.44 - 0.11282\,t - 0.0000140\,t^2$
Chloride of Carbon	$= 48.57 - 0.06844\,t - 0.0002080\,t^2$
Bisulphide of Carbon	$= 82.79 - 0.11446\,t - 0.0004020\,t^2$

(25)

Saturated Vapors of	Inner Latent Heat ρ.
Water	$\rho = 1035.72 - 0.791\,(t-32°)$
Ether	$= 153.97 - 0.10648\,(t-32°) - 0.000398\,(t-32°)^2$
Acetone	$= 236.93 - 0.20184\,(t-32°) - 0.000349\,(t-32°)^2$
Chloroform	$= 112.39 - 0.11282\,(t-32°) - 0.0000078\,(t-32°)^2$
Chloride of Carbon	$= 87.43 - 0.06844\,(t-32°) - 0.000116\,(t-32°)^2$
Bisulphide of Carbon	$= 149.02 - 0.11446\,(t-32°) - 0.000223\,(t-32°)^2$

In combination with the formulas given under (15) and (16) the preceding expressions will give us in the relations

$$Apu = r - \rho$$

and

$$J = q + \rho,$$

empirical formulas for these magnitudes.

It is worth noting that the vapor of water in particular has a very simple expression connecting ρ with the temperature; the great accuracy of the formula in question is evident from the equality of the differences in the values of Column 7, Table 1b of the Appendix.

The combination of equations (13) and (19) gives the external latent heat

$$Apu = \lambda - q - \rho, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

and we therefore get for the vapor of water, when we utilize the empirical formulas adduced in (16) and (25),

$$Apu = 31.10 + 1.096\,t - q \quad . \quad . \quad . \quad . \quad . \quad (26a)$$

$$[Apu = 55.98 + 1.096\,(t - 32°) - q].$$

In the two main Tables, 11 and 12 of the Appendix, for the vapor of water, the values of Apu and ρ are computed according to the preceding formulas instead of formulas (23) and (19). Moreover we get for the direct calculation of the external latent heat the following formulas:

Saturated Vapors of	External Latent Heat $Apu = r - \rho$.	
Water	$Apu = 31.10 + 0.096\,t - 0.00002\,t^2 - 0.0000003\,t^3$	
Ether	$= 7.46 + 0.02747\,t - 0.0001354\,t^2$	
Acetone	$= 8.87 + 0.06185\,t - 0.0002845\,t^2$	
Chloroform	$= 4.56 + 0.01797\,t - 0.0000367\,t^2$	(27)
Chloride of Carbon	$= 3.43 + 0.01671\,t - 0.0000546\,t^2$	
Bisulphide of Carbon	$= 7.21 + 0.02524\,t - 0.0000918\,t^2$	

Saturated Vapors of	External Latent Heat $Apu = r - \rho$.
Water	$Apu = 55.98 + 0.096\,(t - 32°) - 0.0000111\,(t - 32°)^2 - 0.000000093\,(t - 32°)^3$
Ether	$= 13.43 + 0.02747\,(t - 32°) - 0.0000752\,(t - 32°)^2$
Acetone	$= 15.97 + 0.06185\,(t - 32°) - 0.0001581\,(t - 32°)^2$
Chloroform	$= 8.21 + 0.01797\,(t - 32°) - 0.0000204\,(t - 32°)^2$
Chloride of Carbon	$= 5.81 + 0.01671\,(t - 32°) - 0.0000303\,(t - 32°)^2$
Bisulphide of Carbon	$= 12.98 + 0.02542\,(t - 32°) - 0.000051\,(t - 32°)^2$

In physical investigations there is often used for calculating the latent heat for the vapor of water an approximate formula which was proposed by Clausius, namely,

$$r = 607 - 0.708\,t \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

$$[r = 1092.6 - 0.708\,(t - 32°)].$$

Therefore we have

$$Apu = r - \rho = 31.6 + 0.083\,t \quad . \quad . \quad . \quad . \quad (28a)$$

$$[Apu = r - \rho = 56.88 + 0.083\,(t - 32°)].$$

But the differences of the values of columns 4 and 5 of Table 1b of the Appendix show that in exact calculations no use should be made of these formulas.[1]

§ 5. SPECIFIC WEIGHT OF SATURATED VAPORS.

In determining the specific weight, i.e., the weight of a cubic unit of vapor, the assumption formerly was that the connection between pressure, volume and temperature was the same as in gases; in other words, the equation of condition for the latter was employed in the form

$$ps = BT,$$

where s represented the specific volume of the dry saturated steam and B a constant belonging to the kind of vapor considered.

The specific weight γ is connected with the specific volume by the relation

$$\gamma s = 1$$

and hence we have, from the preceding formula,

$$\gamma = \frac{p}{BT}. \quad \ldots\ldots\ldots \quad (29)$$

If we here substitute for the vapor under consideration the values of the pressure p, determined by observation for the different temperatures, there will result the values of γ corresponding to the assumed pressures.

The computations made now show that the specific weight γ increases very nearly uniformly with the steam pressure, and this caused Navier to set up, for the vapor of water, the relation

$$\gamma = \alpha + \beta p, \quad \ldots\ldots\ldots \quad (30)$$

in which α and β are constant quantities whose values were determined. This relation is known in technical circles as

[1] The tables of the first edition of this book were calculated on the basis of Clausius' formula for the latent heat r and therefore differ from the tables of the present issue.

Navier's formula. Pambour made the most extensive use of it in his Theory of the Steam Engine, which for many years was regarded by engineers as an important treatise, and justly so from the physical standpoint of the time.

Of course Navier, Pambour, and others knew that relation (30) accurately reproduced the values of equation (29) only within certain pressure limits; but the accuracy was increased by calculating the constants α and β, for the vapor of water, both for low pressure and for high, a proceeding which is of no use at the present time because now much higher pressures are employed in steam engines than were then customary.

The erroneous part of these older methods of calculation consists in the assumption of the accuracy of equation (29), according to which saturated vapors obey the equation of condition of gases.

So far as the needs of technical investigations are concerned it belongs to the important results of thermodynamics that the doubts have been removed which have existed since Navier's time concerning the behavior of steam.

After the real significance of the Carnot function was recognized Clausius was the first to compute, from the following three equations, [equations (22), p. 29, and equation (1), p. 6:]

$$\frac{r}{u}=AT\frac{dp}{dt},\quad u=s-\sigma,\quad \text{and}\quad \gamma=\frac{1}{s}=\frac{1}{u+\sigma},$$

the specific weight of saturated vapor of water, and to reach the conclusion that with very small pressures the deviations from the results of the older method of calculation were insignificant, but that the deviations increased considerably with growing pressures. With the steam pressures used in engineering the differences are so great that we must finally cease making any use of the older procedure.

In Tables 1 to 10 of the Appendix the values of u are computed for different vapors; by adding the specific volume σ of the vapor in question the specific volume s is easily determined and its reciprocal value γ is the specific weight of the vapor considered at the given temperature.

In the two tables 11 and 12 given for the vapor of water the quantity u is determined from the relation

$$u=\frac{Apu}{Ap}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

If saturated vapor of water were really subject to the equation of condition of gases, we would have to (according to the statements on p. 114, Vol. I) put $B=46.954$ [$B=85.583$] and compute its specific weight according to equation (29).

In order to compare the results of the newer and the older method of computation, use may be made of the following tabulation; the experiments by Fairbairn and Tate,[1] moreover, sufficiently confirm the values obtained for γ by employing Clapeyron's equation (22).

1	2	3	4
	Specific Weight γ		
Pressure of saturated vapor of water in atmospheres.	According to Clapeyron's equation. See Table 11, Appendix.	According to the older method of computation. Equation (29).	ε
0.1	0.0687	0.0690	0.6211
0.5	0.3153	0.3102	0.6336
1	0.6059	0.5900	0.6401
2	1.1629	1.1182	0.6482
5	2.7500	2.5875	0.6624
10	5.2704	4.8546	0.6767
14	7.2283	6.5757	0.6852

We see from Cols. 2 and 3 how great the deviations are which occur with the higher pressures, i.e., with those used in engineering. Moreover Col. 4 gives the relative weight ε of the steam with respect to air of the same pressure and the same temperature. These values increase with the pressure, while, according to the older assumption, the value ε was taken as a constant and ordinarily taken as $\varepsilon=0.623$ for every pressure.

[1] Proc. of the Royal Soc., 1860. Phil. Mag., 4th Ser., Vol. XXI.—Civil-Engineer, 1860.—Zivilingenieur, Literatur- und Notizblatt, Vol. 6, p. 31.

In order to show how the different vapors behave at the ordinary temperature of boiling (at evaporation under atmospheric pressure) use may be made of the following tabulation. In calculating it, the values of the temperature at atmospheric pressure were taken from the table on p. 18, line 1, while the external latent heat Apu was computed from formulas (27), on p. 32, with the exception of the value for the vapor of alcohol, which was found by interpolation from Table 3*b*.

If we regard the equation of condition of gases as valid, then, because the specific volume σ of the liquid is very small, and can be neglected in many cases, we can write

$$Apu = ABT,$$

and then, according to equation (23),

$$r = AB\frac{T^2}{p}\frac{dp}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

Saturated Vapors of	Values of			Specific Weight $\gamma = \frac{1}{u+\sigma}$	Relative Weight ϵ Referred to Hydrogen.
	Apu	u	σ		
	for atmospheric pressure.				
Water	40.20	1.6495	0.0010	0.606	9.24
Ether	8.25	0.3385	0.0013	2.943	37.06
Alcohol	15.33	0.6291	0.0013	1.586	22.79
Acetone	11.45	0.4698	0.0012	2.123	28.59
Chloroform	5.51	0.2261	0.0006	4.411	60.10
Chloride of Carbon	4.39	0.1801	0.0006	5.534	79.10
Bisulphide of Carbon	8.18	0.3356	0.0008	2.973	38.82

For vapors for which no observations on the latent heat r exist, we can calculate such observations with the help of the curve of pressures; but the values obtained must be regarded as only rough approximations. For the vapor of water in particular, results are obtained which deviate extraordinarily from the experimental results of Regnault. Attention is here called to

the matter because, as still frequently happens in physical investigations, the assumption mentioned is regarded as permissible.

For technical investigations it is desirable to possess formulas from which the specific weight of saturated vapors can be computed rapidly and with sufficient accuracy, for the corresponding tables are not always available, and calculation in the way given above is very troublesome.

I have found for the vapor of water that within pressure limits ordinarily occurring in engineering the equation of the upper limit curve d_1dd_2, Fig. 1, p. 4, can be very simply represented by

$$ps^{\mu}=C. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

If the pressure is given in atmospheres (10333 kg. per sq. m. [2116.3 lb. per sq. ft.]) then we must put $\mu=1.0646$, and for the constant C of the right member we write the value 1.7049 [32.670].

If, in the preceding equation, we replace s by $1:\gamma$, we get for the direct calculation of the specific weight γ of the vapor of water

$$\gamma=\alpha p^{n}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

in which formula we must substitute $\alpha=0.6061$ [$\alpha=0.037837$] and $n=0.9393$, provided p is expressed in atmospheres. How accurately this formula reproduces the values of γ, which are given in Table 11 of the Appendix, is shown by the following tabulation:

p Atmosph.	γ		p Atmosph.	γ	
	Eq. (34).	Table 11 of Appendix.		Eq. (34).	Table 11 of Appendix.
0.5	0.316	0.315	6	3.262	3.263
1	0.606	0.606	8	4.274	4.274
2	1.162	1.163	10	5.270	5.270
3	1.701	1.702	12	6.255	6.254
4	2.229	2.230	14	7.229	7.228

If the steam pressure p is not given in atmospheres, but in kilograms per square centimeter (new atmos-

pheres) [pounds per square inch], then we must substitute in equation (34) $\alpha = 0.5875$ [0.00303] and must again make $n = 0.9393$. For the course of the upper limit curve we must put in equation (33) $\mu = 1.0646$ and $C = 1.7617$ [480.145]; however, it is sufficiently accurate to write $\mu = \frac{11}{10}$.

If the need should arise to possess such an auxiliary formula for the other vapors given in the tables of the Appendix, then the values of p and u there given can be employed to derive such a formula, which perhaps may prove to be of the same form.

§ 6. CLAPEYRON'S EQUATION.

If, in Fig. 3, we lay off the absolute temperature T as abscissa OT and the corresponding pressure p as ordinate, we get at a one point of the tension curve of the saturated vapor considered. If we draw a tangent at the point a of the curve, prolong it to its intersection c with the axis of ordinates, and pass horizontally over from a to the point b, then, on the one hand, the angle α of the tangent is given by

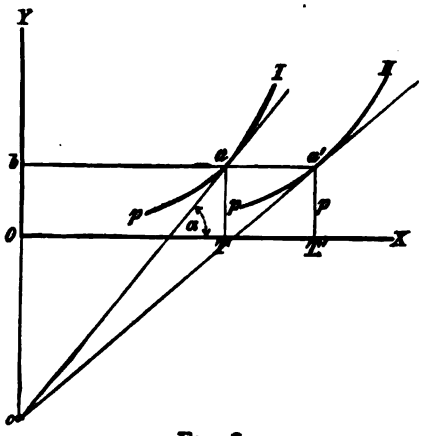

Fig. 3.

$$\tan \alpha = \frac{dp}{dt},$$

and on the other hand by

$$\tan \alpha = \frac{\overline{bc}}{\overline{ab}}.$$

As the distance $\overline{ab}$ measures the value T we get, from the combination of the two formulas,

$$\overline{bc} = T\frac{dp}{dt}.$$

Comparison with Clapeyron's equation (22), p. 29, namely, with

$$\frac{r}{u} = AT\frac{dp}{dt},$$

gives the result that the distance $\overline{bc}$ represents the value

$$\frac{r}{Au}.$$

From the relation $r=\rho+Apu$ we then get

$$\frac{\rho}{Au}=\frac{r}{Au}-p,$$

which quantity is represented by the distance $\overline{Oc}$ (Fig. 3) and can be computed from the equation

$$\frac{\rho}{Au}=T\frac{dp}{dt}-p. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

In this way the quantities considered have been determined for column 8 of the Tables 1*a* to 7*a* of the Appendix.

Differentiation of the last equation with respect to t, moreover, gives

$$\frac{d}{dt}\left(\frac{\rho}{u}\right)=AT\frac{d^2p}{dt^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

while from equation (35) directly follows[1]

$$\frac{\rho}{Apu}=T\frac{d}{dt}\left(\log_e\frac{p}{T}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

The two values $r:Au$ and $\rho:Au$ are functions of the pressure p, but now appear here, in Fig. 3, directly as pressures, and we can

---

[1] If, as was suggested on p. 36, we substitute in equation (37) simply $pu=BT$, we get

$$\frac{\rho}{A}=BT^2\frac{d}{dt}\left(\log_e\frac{p}{T}\right).$$

The expression on the right-hand side can be found in the classical investigations of Kirchhoff (Pogg. Ann., Vol. CIII, p. 202, and Vol. CIV, p. 612) and likewise in C. Neumann, "Vorlesungen über die mech. Theorie der Wärme," Leipsic, 1875, p. 188.—The expression represents nothing but the inner latent heat measured in units of work. The assumption $pu=BT$ like the preceding expression, cannot be entertained, however, so long as Regnault's experimental results on the latent heat r of the vapor of water inspire the confidence which has thus far been vouchsafed to them.

therefore designate $\rho : Au$ as the internal pressure and p as the external pressure.

The following is worthy of remark. Since u is approximately identical with the specific volume s of the saturated steam, the ratios $r:u$ and $\rho:u$ respectively represent the total latent heat and the inner latent heat per unit of volume of the vapor. According to the tables of the Appendix, like the pressure p itself, these values increase rapidly with growing temperature, and if we compare them for the different vapors with each other, at the same pressure, they will be found to differ but little.

For example, for the vapor of water (Table 1*a*), when $p=354.62$ mm. [13.9615 in.], we get $r:u=162.95$ [18.31], and for the vapor of alcohol, when $p=350.21$ mm. [13.788], we have $r:u=168.24$ [18.90].

Likewise, at equal pressures the values

$$\frac{\rho}{u} \text{ and } \frac{T}{p}\frac{dp}{dt}$$

prove to be approximately equal, as is evident from the above-given connection between these formulas.

If the relations given above were exactly correct we could regard the value $r:u$ as the same function of the pressure p for all vapors, and could put

$$\frac{r}{u}=f(p),$$

or, approximately, $r\gamma=f(p)$, where $f(p)$ would be simultaneously valid for all vapors.

For equal pressures the latent heats of the several vapors would be inversely proportional to their specific weights, or

> the generation of a cubic unit of steam under constant pressure would require the same heat quantity at this pressure with all kinds of vapors.

In most of the text-books on Physics attention is called to this proposition given by Despretz (1823).

If, as was done in Fig. 3, we draw the pressure curves for the different kinds of vapors, then at the points of equal pressure the corresponding tangents pass through the common point c, and substitution of the value $r:u=f(p)$ in Clapeyron's equation (22) gives

$$f(p)=AT\frac{dp}{dt},$$

or

$$\frac{dT}{T}=A\frac{dp}{f(p)}.$$

If we designate the integral of the right member by $F(p)$, which function is again, let us say, the same for all vapors, we get

$$\log_e T=F(p)+C.$$

If, for another vapor at the same pressure, the temperature were T', then there would follow for this vapor

$$\log_e T'=F(p)+C',$$

and by subtracting these equations

$$\log_e \frac{T}{T'}=C-C',$$

or, expressed in words, "with two kinds of vapors the ratio of the temperatures is a constant quantity when the corresponding pressures are alike." The pressure curves I, II, etc. (Fig. 3), corresponding to the different kinds of vapor will then stand in a very simple relation to each other; if the pressure curve were drawn for one kind of vapor, then the corresponding curve of a second vapor could at once be drawn, provided the given ratio were known for this second curve.

But more exact examination of Despretz' proposition shows that all the preceding, derived results are only rough approximations, for if we draw, according to Regnault's experiments, the pressure curves of the different vapors adjacent to each

other, as in Fig. 3, we see at once that the connection between the curves is by no means so simple as the one just presented.

The results of the calculation of $r:u$ for like pressures are contained in the following tabulation and likewise show great variation:

Vapors	Values of $\frac{r}{u}$ for Steam Pressures of		
	1	5	10 Atmospheres
Water	325.20	1374.9	2534.7
Ether	267.85	1094.5	2035.9
Alcohol	340.68	1407.8	2518.5
Acetone	275.44	1127.2	2012.5
Chloroform	269.97	1085.4	1917.2
Chloride of Carbon	257.63	1042.8	—
Bisulphide of Carbon	252.80	1033.2	1854.0
Mercury	282.13	1225.8	2250.3

The values in the same vertical column are by no means to be taken as equal, the variations being so marked that they certainly cannot be ascribed to errors of observation.

Another use has recently been found for Clapeyron's equation, particularly for equation (23) derived from it, namely, for

$$\frac{r}{Apu} = \frac{T}{p}\frac{dp}{dt};$$

with this help the effort has been made to derive the equation of the pressure curves of saturated vapors. We will here refer to a few of these efforts.

(*a*) Rankine's Formula.

If, according to Clausius, we allow ourselves to assume that, for the vapor of water in particular, the latent heat r may be assumed to be

$$r = \alpha - \beta T$$

and permit ourselves to make use of the formula for gases $pu = BT$, then the above equation gives

$$\frac{dp}{p} = \frac{(\alpha - \beta T)}{ABT^2} dT,$$

and from this we get

$$\log p = a - \frac{b}{T} - c \log T, \quad \ldots\ldots \quad (38)$$

the formula of Rankine, in which the constants a, b, and c are directly determined from the pressure experiments and not from the constants α, β, and B. The very method of deriving the formula shows that it can only be regarded as an empirical one.

But Guldberg[1] has shown that when the constants, calculated by him for the vapor of water, of ether, and of bisulphide of carbon are employed, this formula reproduces excellently Regnault's experimental results.

As no use will hereafter be made of this formula, the reader is referred for the value of the constants to Guldberg's aforementioned article.

(*b*) Unwin's Formula.

Unwin[2] found that the values of Col. 6 of my steam tables (Tables 1*a* to 7*a* of the Appendix) are reproduced with sufficient accuracy by the relation

$$\frac{T}{p}\frac{dp}{dt} = \left(\frac{\alpha}{T}\right)^n,$$

provided α and n are constants corresponding to a given vapor. The integration of this expression then gives for the equation of the pressure curve

$$n \log_e \frac{p_0}{p} = \left(\frac{\alpha}{T}\right)^n, \quad \ldots\ldots\ldots \quad (39)$$

in which p_0 represents a third constant. With the help of equation (23), p. 30, there also follows the relation

$$\frac{r}{Apu} = n \log_e \frac{p_0}{p}.$$

---

[1] Guldberg, "Uber die Zustandsgleichung der Körper," Zeitschr. des Vereins deutscher Ingenieure, Vol. 12, 1868, p. 676.

[2] W. O. Unwin, "The Relation of Pressure, Temperature, and Volume in Saturated Vapors," Philos. Magazine, Vol. 21, 1886, p. 299.

Unwin gives (elsewhere) the constants for several kinds of vapors and shows by the numerical results that his formula also very well reproduces Regnault's experimental results, and does it between wide temperature limits; at any rate Unwin's formula is also to be regarded as an empirical one; as regards the exponent n, it is to be taken equal to 1.25 for the vapor of water, 1.29 for alcohol, 1.153 for ether, 0.69 for mercury, and 0.77 for carbonic acid.

(c) Raoult Pictet's Formula.

If in Unwin's formula (equation 39) we make $n=1$ and suitably change the remaining constants we shall get

$$\log_e \frac{p_0}{p} = \frac{\alpha}{T}, \quad \ldots\ldots\ldots \quad (39a)$$

and this is the formula which Pictet[1] has given as valid for all vapors and presented as established theoretically; it was given by him in much simpler form and involved an additional assumption which will be mentioned later. But Szily[2] has called attention to the fact that during the derivation assumptions were made which, strictly speaking, cannot be regarded as permissible. Pictet in later works made use of his formula which led to lively discussions that will be touched upon later, and from the results of his computation drew far-reaching conclusions, so that we are justified in examining more fully the development of the formula.

If in Clapeyron's equation (22), p. 29, we take the generally permissible step of replacing the value u by the specific volume s of saturated steam, then

$$\frac{r}{s} = AT\frac{dp}{dt}.$$

Now for atmospheric air of the same temperature and of the same pressure we can calculate the specific volume v of the air

[1] Comptes rendus, Vol. XC, p. 1073, 1880.
[2] Journal de Physique, Vol. IX, 1880.

from $pv=BT$, and if the relative weight of the vapor, with respect to air, is designated by ϵ, we have $v=s\epsilon$, the combination of these formulas gives

$$\frac{dp}{p}=\frac{r\epsilon}{AB}\frac{dt}{T^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39b)$$

The equation can be integrated when $r\epsilon$ is known as a function of T; now r diminishes with increasing temperature while ϵ increases (see table, p. 35); therefore it seems possible that $r\epsilon$ may be treated as constant. If we accordingly make

$$\frac{r\epsilon}{AB}=\alpha,$$

the integration of equation (39*b*) gives equation (39*a*), the constant of integration being represented by $\log_e p_0$; on the other hand we also get, when p_1 represents pressure corresponding to T_1,

$$\log_e\frac{p_1}{p}=\frac{r\epsilon}{AB}\frac{(T_1-T)}{TT_1}. \quad . \quad . \quad . \quad . \quad . \quad (39c)$$

If we substitute what is at least approximately true for the vapor of water,

$$r=\delta-\beta T,$$

where δ and β are constants to be determined, we can also write

$$\log_e\frac{p_1}{p}=\frac{\epsilon}{AB}[r_1+\beta(T_1-T)]\frac{(T_1-T)}{TT_1}, \quad . \quad . \quad (39d)$$

and that is the form in which Pictet has given it. The above equations (39*a*) and (39*c*) prove themselves unserviceable, and the reason is that $r\epsilon$ can by no means be regarded as constant, and a closer examination shows that it is not constant. To be sure in setting up his formula he does not expressly assume this, but without it his formula cannot be at all brought into harmony with that of Clapeyron. Moreover, Pictet maintains that his formula is valid for all volatile liquids.

(*d*) Emil Herrmann's Formula.

If we examine the values given in Col. 9 of my steam tables (Tables 1*a* to 7*a* of the Appendix) it will be seen that these increase approximately with the temperature, so that we may write

$$\frac{Apu}{r}=\beta(T-\alpha),$$

where α and β are constant quantities which are easily determined from the tables, for each kind of vapor.

According to equation (23), p. 30, therefore,

$$\frac{T}{p}\frac{dp}{dt}=\frac{1}{\beta(T-\alpha)}.$$

The integration is readily effected, and we get for the equation of the pressure curve

$$\frac{p}{p_0}=\left(\frac{T-\alpha}{T}\right)^{\epsilon}, \quad \ldots\ldots\ldots \quad (40)$$

in which p_0 represents a third constant and the exponent ϵ of the right member is determined by

$$\epsilon=\frac{1}{\alpha\beta}.$$

Equation (40) is the relation given by Emil Herrmann.[1] He finds, not only for the vapors adduced in the Appendix of this book, but also for a series of other vapors, that the constant α is the same for all these vapors and that its value is $\alpha=85.8°$. Herrmann also gives numerical values for ϵ and p_0, and shows that for the vapor of water in particular the formula reproduces very satisfactorily the experimental results of Regnault between the wide temperature limits of $-32.84°$ to $+210°$ [$-27.11°$ to $+410°$]; he finds, especially for the vapor of water, $\epsilon=44.6$

[1] Emil Herrmann, "Kompendium der mechanischen Wärmetheorie." Berlin, 1879, p. 133.

and $\log_{10} p_0 = 7.94400$ [6.73917], provided p is expressed in millimeters [inches] of mercury.

Exactly this relation (40) has recently been given by Bertrand,[1] who doubtless did not know of Herrmann's work, but the former maintains that the exponent ϵ may be taken throughout as $\epsilon = 50$ for the vapors of water, ether, bisulphide of carbon, chloride of carbon, carbonic acid, ammonia, sulphurous acid, and sulphur, while Herrmann, as we know, says that the constant may be taken the same for all vapors. Shortly afterward Bertrand developed[2] an equation of the pressure curve of a different form; in this he started, as Rankine did, by assuming that the latent heat could be represented by $r = \alpha - \beta T$, but that it would be more correct to use in place of $pu = BT$ the expression

$$pu = B(T + T_0),$$

wnere T_0 represents a constant. The last-given relation has been frequently proposed by others, but it by no means agrees with the results of more exact calculations—always with the proviso that we stick closely to Regnault's experimental results.

Now the substitution of the given formulas in equation (23) leads Bertrand to an expression that can be integrated, but gives a very complicated and very inconvenient formula, which would hardly be used even if it were better established than is the case.

Similar unsuccessful efforts, like those indicated, to get the true form of the equation of the pressure curve of saturated vapors, with the help of Clapeyron's equation, have been made by others, but we will here omit further reference to them, touching the problem again later on.

So long as this relation cannot be found in a purely theoretical fashion it is advisable, for the purpose of more exact numerical computations, to hold fast to Regnault's formulas, which, on account of their great number of constants, after all best reproduce the results between the wide limits covered by the experiments.

[1] Comptes rendus, 1887, Vol. CIV, p. 1568.
[2] Ibid., Vol. CV, p. 389.

§ 7. RECENT INVESTIGATIONS BY BATTELLI ON THE BEHAVIOR OF THE VAPOR OF WATER.

The experimental results of Regnault on the pressure of saturated steam at different temperatures (see p. 9 and the tables in the Appendix), which we use exclusively in this treatise, agree almost exactly with the experiments of Magnus[1] within the same temperature limits, a proof of the wonderful accuracy with which these physicists made their determinations. Regnault's experiments extended from $-32°$ [$-25.6°$ F.] to the temperature $t=232°$ [449.6° F.], and those of Magnus from $-6.6°$ [20.12° F.] to 104.6° [220.28° F.]. Now there exist other recent experiments which extend to $t=365°$ [689° F.], a value of the temperature which plays an important part, as we will show later, in the theory of the steam. The first experimental series is due to Cailletet and Colardeau,[2] the second is due to Battelli; both series agree sufficiently well up to $t=200°$ [392° F.] with Regnault's results; from there on, as the following arrangement shows, they exhibit some discrepancies which are probably due to difficulties of observation. The second and third columns give the pressure of saturated steam in millimeters of mercury; p_C are the values according to Cailletet and Colardeau; p_B the values according to Battelli.

Temperature:	Pressure:	
$t°$ C.	p_C.	p_B.
200°	11628 mm.	11625 mm.
225	19076	19124
250	29792	29951
275	45144	48816
300	65512	67620
325	92416	94112
350	127300	126924
365	152380	149733

[1] Magnus, "Versuche über die Spannkräfte des Wasserdampfes," Poggendorff's Annalen, Vol. LXI, 1844, p. 225.

[2] Cailletet and Colardeau, "Nouvelle méthode de détermination du point critique. Application de cette méthode au cas particulier de l'eau, et à le recherche de la loi des tensions de la vapeur d'eau saturée. Annales de Chimie et de Physique, 1892, 16th Series, Vol. XXV, p. 519.

For the present purposes, however, those investigations of Battelli deserve fuller discussion, which relate to the determination of the specific volume of saturated and of superheated steam. Here we will first discuss the results for saturated steam.

With the help of the value r of the heat of vaporization, given by Regnault, and with the help of Clapeyron's equation, there was calculated the value of $u = s - \sigma$ in Table 1*b* of the Appendix, s representing here the specific volume of the saturated steam, and σ the specific volume of water at the same temperature and pressure. The specific weight γ of the steam is found from the reciprocal $1 : s$, and thus values are derived that are sufficiently confirmed by the experiments of Fairbairn and Tate (see p. 35). Other experimental results have not been presented until quite recently, although efforts have frequently been made to determine experimentally the specific weight γ; success in the case of saturated steam was probably wrecked by the fact that some condensation always takes place on the walls of the vessel.

Now we have the recent experiments of Battelli.[1]

The following tables give for saturated steam the complete experimental results. Col. 2 contains the steam pressure in millimeters of mercury (according to Battelli), and Col. 3 gives the observed specific volume in cubic meters per kilogram. Col. 4 gives the specific volume which is here designated by v instead of s and is computed from Tumlirz'[2] empirical formula

$$p(v+0.008402) = 3.4348\,T \quad . \quad . \quad . \quad . \quad . \quad (\alpha)$$
$$[p(v+0.13456) = 1.2034\,T],$$

[1] In the text above we made use of the following article: A. Battelli, "Sulle proprietà termiche dei vapori. Parte IV. Studio del vapor d'acqua," Memoria della reale Accademia delle scienze di Torino. Series 2, Vol. XLIII, 1893, pp. 63–98. A partial reproduction of Battelli is given in Annales de Chimie et de Physique, 1894, Series 7, Vol. III, p. 408. In these annals there is given in Series 6, Vol. 25, 1892, p. 38, the experimental method employed in the investigations of the behavior of ether vapors, and in the same year, Vol. 26, p. 394, the experiments with bisulphide of carbon can be found and the description of the experimental apparatus with the vapor of water.

[2] Tumlirz. "Die Zustandsgleichung des Wasserdampfes." Sitzungs-

in which p is expressed in millimeters [inches] of mercury, and $T=273°+t$ [$T=459.4°+t$]; Col. 5 gives the difference between the observed and computed values of v in percentages.

1. Temperature $t°$.	2. Pressure p mm.	3. Spec. Volume v observed, v cbm.	4. Spec. Volume v computed, from Tumlirz, v cbm.	5. Difference in Percentage of Observed Value.
−6.16°	2.80	327.246	327.322	0.02
+1.32	4.77	197.853	197.524	0.17
6.24	6.87	139.832	139.602	0.16
9.72	8.66	112.283	112.126	0.14
14.91	12.34	80.3114	80.128	0.23
21.05	18.07	55.7464	55.885	0.25
27.15	25.96	39.5342	39.705	0.43
57.01	129.14	8.73890	8.769	0.34
78.52	330.78	3.63241	3.642	0.25
99.60	749.12	1.69046	1.700	0.56
130.32	2060.1	0.661534	0.66405	0.38
144.21	3061.9	0.457233	0.45960	0.52
182.90	7971.4	0.187622	0.18805	0.23
202.21	12181.1	0.125372	0.12560	0.18
231.41	21272.1	0.072415	0.07305	0.87

The values for pressure and volume in cols. 2 and 3 refer to the state of equilibrium in the saturated condition; besides these values, Battelli also gives those which relate to the passage from the non-saturated to the saturated condition for the first moment of condensation. Here the values of the specific volume are larger throughout, and the values for the pressure somewhat smaller. Tumlirz also gives his formula (α) for superheated steam and, with 242 observation values, there are only two differences from Battelli's results which exceed 0.90%.

In order to facilitate comparison with the data given in the present treatise, the following tabulation has been computed. For the temperatures given in Col. 1 we have, in Col. 2, the pressure of saturated steam according to Regnault, and in Col. 3 the

berichte der kais. Akademie der Wissenschaften in Wien. Mathem.-naturw. Klasse, Vol. CVIII, Part IIa. Here Tumlirz gives the tables of Battelli very completely, as published in the papers of the Turin Academy, and he also gives the tables for superheated steam.

values of u, according to the relation $u = s - \sigma$, where $s = v$ has been determined from the preceding equation of Tumlirz, and σ by the data given by Hirn (p. 24). In Col. 4 the corresponding values of u, taken from Table 1b of the Appendix, are inserted.

1. Temperature t.	2. Pressure p mm.	3. u.	4. u. According to Table 1b.
0°	4.600	203.83	210.68
20	17.391	57.862	58.726
40	54.906	19.571	19.646
60	148.786	7.6780	7.6538
80	354.616	3.4097	3.3792
100	760.000	1.6764	1.6498
120	1491.280	0.8957	0.8753
140	2717.630	0.5125	0.4977
160	4651.620	0.3002	0.3001
180	7546.390	0.1967	0.1901
200	11688.960	0.1295	0.1257

Of course this table permits of no comparison between the values of Col. 4 and those directly deducible from Battelli's experiments; for this purpose the values of u must be determined by interpolation from the statements of the preceding table. But investigations in the direction indicated, whose results I will not present here, show irregularities which are probably due to the uncertainty of some observation values. For the purpose in hand, however, the preceding statements are sufficient, namely, to answer the question whether Battelli's experimental values ought to be the occasion for recalculating the tables of the Appendix for steam or not, and at the same time subject a whole series of my former special statements to correction. This question I believe I must answer in the negative.

The values of Cols. 3 and 4 show, to be sure, some not inconsiderable differences, particularly in the vicinity of temperatures of 100° C. [212° F.], but this may also be ascribed to the application of Tumlirz' formula; for example, we get directly from Battelli's statements (table, p. 50), by interpolation, $u = 1.6660$ for $t = 100°$, which would agree somewhat better with the corresponding value of Col. 4; at other temperatures, however,

greater differences appear. Our values of Col. 4, as explained, were calculated from Clapeyron's equation on the supposition that Regnault's formula for the latent heat, or heat of evaporation r (see p. 23), is perfectly reliable; now although doubt in this matter was expressed on p. 25, it is, to say the least, awkward to calculate the values of r backward from Clapeyron's equation, with the help of Battelli's values of u, and then to regard these values as the more reliable ones; it is just in this direction that my closer investigations of the tabulation on p. 50 give values for r which vary quite irregularly with the temperature t.

Even assuming that, by a reliable determination of the specific volume of dry saturated steam, we can arrive, in the way indicated, at a more exact determination of the vaporization heat r, nevertheless there will here come into play another fact when we take account of the results of physical research for utilization in technical investigations. In calculating the entropy of wet steams, the heat q of the liquid (p. 22), or the specific heat c of the water, is of equal importance with the vaporization heat r; it is just with respect to the reliability of Regnault's statements in this direction that well-grounded doubt has been expressed in the text above, and by others.

Under such circumstances it must seem thoroughly justifiable if, in the present book, we hold fast to the statements of the earlier edition with respect to the behavior of tne vapor of water. A recalculation of the steam tables for the purpose of technical computation need not be undertaken till Battelli's statements have been confirmed by new experiments and till new calorimetric observations have been recorded, for since Regnault's time such observations have not been instituted to any adequate extent.

During the investigations of non-saturated and superheated vapors there will be opportunity to return to Battelli's experimental results.

Here we may add that Tumlirz (ibid.)[1] concludes, from

[1] Compare also: Sitzungsberichte der Kais. Akademie der Wissenschaften in Wien, 1896, Vol. CV, Part II.

the comparison of the numerical values of Cols. 3 and 4 of the table on p. 51, that steam, generated at low temperatures from boiling water, must be, in part, dissociated in statu nascendi, but in so doing Tumlirz probably overestimates the degree of accuracy of both columns.

§ 8. HEAT EQUATIONS OF MIXTURES OF VAPOR AND LIQUID (WET VAPORS).

Let a unit of weight of a mixture of vapor and liquid be enclosed in a vessel; let the weight of the steam present be x (steam quality), and $(1-x)$ the liquid present; let t be the temperature, and p be the corresponding pressure.

The excess of heat present in the liquid over that in an equal weight of liquid at 0° C. [32° F.] temperature is

$$(1-x)q;$$

on the other hand for the steam quality x the excess of heat over that contained in x kilograms [pounds] of liquid at 0° C. [32° F.] is (see equation 18, p. 27)

$$xJ.$$

Accordingly the excess of the heat contained in the mixture over that contained in a unit of weight of liquid of 0° C. [32° F.] temperature is

$$(1-x)q+xJ,$$

or, if we consider equation (20), namely $J=q+\rho$, we get for the mixture's heat excess

$$q+x\rho.$$

Let us now suppose that this mixture experiences any change whatever; let the specific steam quantity at the beginning be x_1 and its temperature t_1, and let q_1 and ρ_1 be the corresponding values of q and ρ, then at the start the said heat excess is

$$q_1+x_1\rho_1.$$

During the change the heat contained in the mixture has been increased by the amount

$$q-q_1+x\rho-x_1\rho_1;$$

but this value is evidently identical with the change $U-U_1$ of the inner work, measured in units of heat; we therefore get the equation

$$A(U-U_1)=q-q_1+x\rho-x_1\rho_1,$$

and passing to the differential we get

$$AdU=dq+d(x\rho). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

This equation gives us the means of determining the change of inner work for any change of state whatever; here dq may be replaced by cdt, and the heat of the liquid q and the inner latent heat ρ are known from the formulas on p. 22 and p. 31, as functions of the temperature; of course this equation and the other formulas about to be developed will hold only so long as the steam remains saturated or so long as liquid is present with the steam in the mixture; in other words, so long as the pressure p can be regarded as a function of the temperature t only.

It is worth noting that the preceding formula can be derived in an elementary way without having recourse to the general formulas of the first section.

Further, let us assume that during the supposed change of state there has been performed an external work dL while the mass expands the amount dv; then the heat quantity dQ that must be supplied to the mixture is

$$dQ=A(dU+dL), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

which was given upon p. 26, Vol. I.

The two equations (41) and (42) in combination with the preceding now furnish the means of following the behavior of mixtures of vapor and liquid for all kinds of heat supply, changes of volume and changes of state, whether they are reversible or non-reversible.

For the case that the changes of state occur in reversible fashion, which case we will now examine more closely, we can, according to the presentation on p. 31, Vol. I, replace the value of the external work dL by pdv; the substitution of both this value and that of dU as given by equation (41) in equation (42) then furnishes

$$dQ = dq + d(x\rho) + Apdv. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

If we add to this the equation already given for the specific volume v of the mixture, namely,

$$v = xu + \sigma, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

then these equations, combined with Clapeyron's equation, can solve all problems that can be proposed with respect to reversible changes of state of mixtures of vapors and liquids, or, to speak more briefly, with espect to "wet vapors."

In order to solve certain problems it is advantageous to bring the preceding fundamental equation (43), from the start, into another form, and there are several such transformations.

First Transformation. If we substitute the value of x derived from equation (44) in equation (43), there results

$$dQ = dq + d\left(\frac{\rho}{u}(v-\sigma)\right) + Apdv. \quad . \quad . \quad . \quad . \quad (45)$$

If we effect the differentiation indicated, by treating the quantities q, ρ, and u as functions of p, there follows

$$dQ = \frac{d}{dp}\left(q - \sigma\frac{\rho}{u} + v\frac{\rho}{u}\right)dp + \left(\frac{\rho}{u} + Ap\right)dv,$$

or if we utilize in the second term of the right member the relation

$$r = \rho + Apu,$$

we shall get

$$dQ = \left[\frac{d}{dp}\left(q - \sigma\frac{\rho}{u}\right) + v\frac{d}{dp}\left(\frac{\rho}{u}\right)\right]dp + \frac{r}{u}dv. \quad . \quad . \quad . \quad (46)$$

If we compare this equation with the fundamental equation

$$dQ = A[Xdp + Ydv],$$

given in the first section, Vol. I, p. 32, there can be found for wet steam the two functions X and Y which can then be introduced into the general investigations and be regarded as functions of p and v. Accordingly,

$$AX = \frac{d}{dp}\left(q - \sigma\frac{\rho}{u}\right) + v\frac{d}{dp}\left(\frac{\rho}{u}\right), \quad \ldots\ldots \quad (47)$$

also

$$AY = \frac{r}{u}. \quad \ldots\ldots\ldots \quad (48)$$

The third function Z (Vol. I, p. 32) there employed bears to Y the relation

$$Y = Z + p.$$

Therefore we also have

$$AZ = \frac{\rho}{u}. \quad \ldots\ldots\ldots \quad (49)$$

Considering Clapeyron's equation (22), p. 29, there also follows

$$Y = T\frac{dp}{dt} \quad \text{and} \quad Z = T\frac{dp}{dt} - p.$$

We recognize from the preceding that with wet steam the two functions Y and Z are only functions of the pressure p, as was also the case with gases, for which

$$Y = \frac{\kappa p}{\kappa - 1} \quad \text{and} \quad Z = \frac{p}{\kappa - 1}$$

were found (Vol. I, pp. 124 and 131). On the other hand the quantity X, which for gases was

$$X = \frac{v}{\kappa - 1}$$

and only a function of the volume, becomes, according to equation (47), with wet steams, a function of the volume and of the pressure.

If there were need of determining numerically the differential coefficients occurring in equation (47), then the following relations could be employed:

$$\frac{dq}{dp}=\frac{c}{\frac{dp}{dt}},$$

where c represents the specific heat of the liquid; and, utilizing equation (36), p. 39, we should have

$$\frac{d}{dp}\left(\frac{\rho}{u}\right)=AT\frac{d}{dt}\left(\log_e\frac{dp}{dt}\right).$$

The preceding transformation of the fundamental equation (43) is only presented in order to bring out the significance of the functions X, Y, and Z.

Second Transformation. If the differentiation indicated in equation (45) is effected by considering q, ρ, and u as functions of the temperature t, there follows

$$dQ=\left[c-\sigma\frac{d}{dt}\left(\frac{\rho}{u}\right)\right]dt+d\left(v\frac{\rho}{u}\right)+Apdv.$$

If we simplify by substituting

$$\omega=c-\sigma\frac{d}{dt}\left(\frac{\rho}{u}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

where ω is for the present to be regarded as a function of the temperature, we get

$$dQ=\omega dt+d\left(v\frac{\rho}{u}\right)+Apdv. \quad . \quad . \quad . \quad . \quad . \quad (51)$$

Comparison with the fundamental equation

$$dQ=AdU+Apdv$$

gives, for the change dU of the inner work, the formula

$$AdU = \omega dt + d\left(v\frac{\rho}{u}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

If we introduce the relation (35), p. 39, into the second term of the right member of equation (51) we get, after some easily followed transformations,

$$dQ = \omega dt + ATd\left(v\frac{dp}{dt}\right), \quad . \quad . \quad . \quad . \quad . \quad (51a)$$

a form of equation used by E. Herrmann.[1]

As early as the second edition of this book (1866), p. 349, I pointed out that the quantity ω for the vapor of water, according to equation (50), is a constant and with great accuracy we can take $\omega = 1$, between the wide temperature limits 0° C. [32° F.] and 200° C. [392° F.].

E. Herrmann has extended the investigation to the rest of the vapors contained in my tables,[2] and only with alcohol and acetone did he find greater variations; with other vapors the constant character of ω clearly appears; he finds as mean values:

for ether	$\omega = 0.532$	chloride of carbon	$\omega = 0.201$
chloroform	0.232	bisulphide of carbon	0.235.

Third Transformation. If we differentiate equation (44), which was given above for the specific volume of the mixture, we get, because σ can be regarded as constant,

$$dv = d(xu),$$

therefore

$$Apdv = Apd(xu),$$

and from this, according to a well-known principle in differential calculus,

$$Apdv = Ad(pux) - Axudp.$$

---

[1] Kompendium der mechanischen Wärmetheorie. Berlin, 1879, p. 123.

[2] Ibid., p. 139.

If we substitute this value in equation (43) we get

$$dQ = dq + d(x\rho) + Ad(xpu) - Axudp.$$

The second and third terms of the right-hand member of this equation can be reduced to one term if we substitute the latent heat r for $\rho + Apu$; both terms can then be replaced by $d(xr)$. For the last term, moreover, there is obtained from equation (22), p. 29,

$$Axudp = \frac{xr}{T}dt;$$

hence, and because $dq = cdt$,

$$dQ = cdt + d(xr) - \frac{xr}{T}dt. \quad \ldots\ldots\ldots \quad (53)$$

In this form the equation was first given by Clausius; there also follows from it

$$dQ = dq + Td\left(\frac{xr}{T}\right).$$

If we introduce into the formulas a temperature function τ, for which the relation

$$\tau = \int_0^t \frac{dq}{T} \quad \ldots\ldots\ldots \quad (54)$$

holds, we finally get

$$dQ = Td\left(\tau + \frac{xr}{T}\right), \quad \ldots\ldots \quad (55)$$

and this formula is of special value in certain investigations.

With the help of the empirical formulas (15) for q, p. 22, the temperature function τ, which will be shown later to have a special significance, can easily be determined and its numerical value found for every temperature.

Clausius considered it permissible in the case of the vapor of water to substitute a constant mean value for the specific heat c (Clausius made $c = 1.013$), and this gives

$$\tau = c \log_e \frac{T}{273}. \quad \ldots\ldots\ldots \quad (54a)$$

It is more exact for water to use the relation

$$\tau = 2.4318893 \log_{10} \frac{273+t}{273} - 0.0002057\, t + 0.00000045\, t^2 \quad . \quad (54b)$$

$$[\tau = 2.4318893 \log_{10} \frac{459.4+t}{491.4} - 0.0001143(t-32°) + 0.000000139\,(t-32°)^2],$$

according to which formula a series of values for τ have been computed (p. 62).

Fourth Transformation. If we effect in equation (53) the differentiation indicated in the second term, we get

$$dQ = cdt + rdx + xdr - x\frac{r}{T}dt,$$

and if in the right member $xcdt$ is both added and subtracted, there follows, after some rearrangement of the terms,

$$dQ = (1-x)cdt + rdx + x\left(c + \frac{dr}{dt} - \frac{r}{T}\right)dt.$$

Let us designate the quantity in the parenthesis of the third term by h, so that

$$h = c + \frac{dr}{dt} - \frac{r}{T}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

then we will get

$$dQ = (1-x)cdt + rdx + xhdt. \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

In this form the equation is also first found with Clausius; the first term $(1-x)cdt$ of the right member represents that part of the supplied heat quantity dQ which is expended in raising the temperature of the liquid $(1-x)$ present an amount dt; at the same time the liquid element dx is converted into steam and requires for this purpose the heat quantity rdx, which constitutes the second term; finally $xhdt$ represents that part of dQ which enters the steam part of the mixture; the unit of weight of steam requires the heat quantity hdt and consequently, as Clausius has already pointed out, the factor h plays the part of a kind of specific heat; hdt is the heat quantity which must be imparted to the unit of weight of steam when its mass expands through dv in order that

the steam, after being heated through dt, may again be in the saturated condition.

If we substitute in equation (56) the total heat λ in place of r by making use of the relation $r=\lambda-q$ (see p. 21) and of $dq=cdt$, then we get the following expression:

$$hT=T\frac{d\lambda}{dt}-\lambda+q, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56a)$$

which can also serve for the computation of the function h.

§ 9. THE ENTROPY OF WET VAPORS AND THE TRANSFORMATION OF THEIR PRESSURE CURVES.

During the derivation of the fundamental formulas in the first section, Vol. I, the expression

$$\frac{dQ}{AS}$$

appeared as a complete differential, and later it was found that the function S was simply identical with the absolute temperature T. The integral

$$P=\int\frac{dQ}{AT}$$

was designated as heat weight (entropy, according to Clausius) and was specially determined for gases on p. 135, Vol. I.

This value can easily be calculated for wet vapors, the two forms (51a) and (55) of the heat equation being best suited for this purpose. The last-mentioned equation gives directly

$$P=P_0+\frac{1}{A}\left(\tau+\frac{xr}{T}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

and the other

$$P'=P_0'+\frac{\omega}{A}\log_e T+v\frac{dp}{dt}, \quad . \quad . \quad . \quad . \quad . \quad (59)$$

where P_0 and P_0' are to be regarded as integration constants.

Of these two identical equations that one is chosen for the solution of particular problems which evidently leads most quickly to the solution; in the following discussion we will for the present take equation (58) as a basis.

Suppose the unit of weight of wet steam of volume v_1 and pressure p_1 to be given, and let it expand according to the curve *acb* (Fig. 4*a*) to the volume v_2 and to the pressure p_2, then for

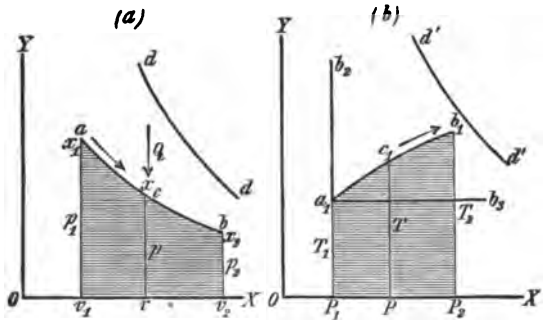

FIG. 4.

every intermediate state v and p the corresponding temperature t is at once given by this pressure p, and the equation $v = xu + \sigma$ enables us to calculate the steam quality x for the instant. Conversely, the steam quality x may be given beforehand for every point of the curve *acb* and the determination of the corresponding specific volume v be required.

Auxiliary Table for

1. Pressure in Atmospheres $\frac{p}{(1\,\text{atm} = 10333\,\text{kg})}$	2. T	3. $\frac{dp}{dt}$ kg per sq. m.	4. $\frac{1}{A}\log_e T$	5. $\frac{r}{A}$	6. $\frac{r}{AT}$
0.5	354.71	207.41	2489.43	111.397	656.725
1	373.00	369.67	2510.75	132.954	609.856
2	393.60	653.86	2533.54	156.100	562.173
3	406.91	909.22	2547.64	170.478	533.871
4	417.00	1146.57	2558.03	181.103	513.587
5	425.22	1371.00	2566.30	189.583	497.755
6	432.22	1585.43	2573.23	196.698	484.725
7	438.34	1791.32	2579.19	202.846	473.667
8	443.81	1990.00	2584.45	208.277	464.030
9	448.77	2183.25	2589.16	213.153	455.482
10	453.31	2370.92	2593.43	217.571	447.820
11	457.50	2553.97	2597.33	221.612	440.875
12	461.41	2732.60	2600.94	225.364	434.502
13	465.08	2907.43	2604.30	228.858	428.613
14	468.53	3078.75	2607.43	232.123	423.160

If we calculate the value P according to equation (58), for several points of the curve, and lay it off in Fig. 4*b* as abscissa and the corresponding absolute temperature T as ordinate, we will get in the curve $a_1c_1b_1$ the transformation of the pressure curve *acb*; in so doing the constant P_0 can be chosen quite arbitrarily and may at pleasure be taken $P_0=0$. Here the hatched area bounded by curve *acb* in Fig. 4*a* represents the outer work produced during expansion, while the hatched area in Fig. 4*b* represents the heat quantity Q which must be supplied to the vapor from the outside during the assumed expansion of the vapor, and this heat is measured in units of work.

In so doing it is of course assumed that during the whole change of state the steam remains wet and consequently the expansion curve *acb* does not overstep the upper limit curve (see p. 62) indicated by *dd* in Fig. 4*a*. To assist such representations the accompanying auxiliary table was calculated specially for the vapor of water, which table contains not only the temperature functions occurring in the preceding formulas, but also others, calculated for different pressures, that will arise in subsequent investigations. [For English units see Table 13, Appendix.]

Saturated Steam.

7. $\frac{1}{A}\left(\tau+\frac{r}{T}\right)$	8. $\int_0^t hdt$	9. $\frac{1}{A}\frac{dq}{dp}$	10. $\frac{1}{A}\frac{d}{dp}\left(\frac{\rho}{u}\right)$	11. $\frac{d}{dt}\left(\frac{\rho}{u}\right)$	Pressure in Atmospheres p
768.122	−126.747	2.0631	11.976	5.859	0.5
742.810	−148.470	1.1618	11.088	9.668	1
718.273	−170.639	0.6600	10.194	15.722	2
704.349	−183.778	0.4763	9.675	20.749	3
694.690	−193.163	0.3788	9.299	25.149	4
687.338	−200.457	0.3175	9.014	29.160	5
681.423	−206.394	0.2752	8.774	32.808	6
676.513	−211.481	0.2440	8.557	36.155	7
672.307	−215.862	0.2201	8.409	39.471	8
668.635	−219.726	0.2009	8.264	42.557	9
665.391	−223.178	0.1853	8.143	45.533	10
662.487	−226.292	0.1723	8.004	48.217	11
659.866	−229.134	0.1613	7.896	50.892	12
657.471	−231.752	0.1518	7.794	53.450	13
655.283	−234.165	0.1435	7.699	55.910	14

As a special part of the investigation let us first discuss the transformation of the two limit curves.

In Fig. 5*a* let the state of a unit of weight of wet steam at the point c be given by the volume v and the pressure p, and let the steam quality x be known from $v = xu + \sigma$.

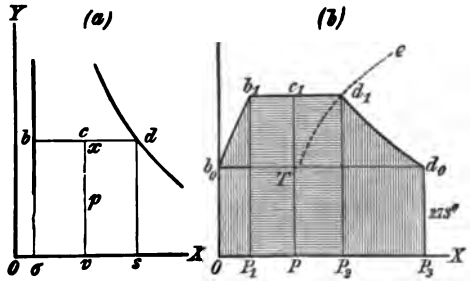

FIG. 5.

Then the transformation c_1 of this point in Fig. 5*b* is determined when we calculate the value P according to equation (58) and lay it off as abscissa OP, in doing which the constant P_0 can be placed equal to zero; hence the abscissa is

$$P = \frac{1}{A}\left(\tau + \frac{xr}{T}\right)$$

and T is the ordinate. For the unit of weight of liquid of the same temperature we have $x = 0$ and for dry saturated steam $x = 1$; for the transformations b_1 and d_1 of the two points b and d (Fig. 4*a*) the abscissas are

$$P_1 = \frac{\tau}{A} \quad \text{and} \quad P_2 = \frac{1}{A}\left(\tau + \frac{r}{T}\right),$$

which magnitudes can be taken from the preceding auxiliary table for steam, corresponding to the different pressures. The horizontal $b_1 d_1$ is the transformation of the horizontal pressure curve bd, and the rectangle lying (horizontally hatched) below $b_1 d_1$ therefore represents, according to the principles enunciated, the heat quantity necessary for steam formation under constant pressure, i.e.,

represents the latent heat r, but measured in units of work. The area of this rectangle can be computed, by utilizing the preceding expressions, from

$$(P_2-P_1)T=\frac{r}{A}.$$

In order to transform the lower limit curve bb (Fig. 5a) lay off the temperature values T as ordinates and the corresponding values of $\tau:A$ (see Col. 5 of the preceding table) as abscissas; for the temperature $t=0$ we also have $\tau=0$ and therefore $\overline{Ob_0}=273$ [491.4] (Fig. 5b); hence the curve b_0b_1 is the transformation of the pressure curve when the liquid is heated from 0° to $t°$, and the area lying (vertically hatched in Fig. 5b) under this curve b_0b_1 consequently represents the heat of the liquid q, measured in work.

If we wish to transform, in like manner, the upper limit curve dd, we lay off the values

$$\frac{1}{A}\left(\tau+\frac{r}{T}\right)$$

as abscissas (Col. 7 of the above table) and the corresponding temperatures T as ordinates; let the curve d_1d_0 (Fig. 5b) represent the transformation. If the point d_0 is to correspond to the temperature 0° C. [32° F.], i.e., to $T=273°$ [491.4°], then we find for water, for example, the abscissa $\overline{OP_3}=941.958$ [1716.93], according to the preceding formula, because $\tau=0$ and $r=606.5$ [1091.7].

The area (vertically hatched in Fig. 5b) lying under the curve d_1d_0 now represents the heat quantity, measured in units of work, which must be imparted to the dry saturated steam if the latter is to expand from the pressure p, and corresponding temperature t, according to the limit curve, till its temperature has fallen to 0° C. [32° F.] and to the corresponding pressure.

Conversely, if the steam is to be compressed according to the limit curve, a withdrawal of heat must take place; if heat were not imparted in the first case a part of the steam

would condense, while in the other case the steam would be superheated.

Now, for the vapor of water, the transformation of the upper limit curve has in fact the course indicated in Fig. 5*b*, and therefore this kind of vapor partially condenses during adiabatic expansion. Rankine and Clausius first called attention to this particular behavior of water, but each of them reached the result mentioned in a somewhat different way. In its day, the discovery was rightly regarded as a highly important result of the first steps in thermodynamics and effected a decided change in the views which then prevailed in technical circles concerning the behavior of steam during its expansion in the cylinder of the steam engine.

Pambour took for a basis of his theory of the steam engine the hypothesis that the steam in the steam cylinder is dry saturated and expands in such a way as to remain dry saturated during the expansion and fall of pressure, or, in the words of the above-given presentation, the steam expands according to the upper limit curve. At that time, and for a long time afterwards, even in the earlier edition of this book, there was tacitly connected with the thought of the expansion of steam in steam engines the assumption that the cylinder walls had little or no influence on the expanding steam; in other words, that adiabatic expansion might be assumed. The significance of the discovery mentioned, particularly its technical importance, was supposed to lie principally in the recognition that dry saturated steam partially condensed in the steam cylinder during adiabatic expansion and that Pambour's assumption had thus become untenable. But the new views concerning the changes of state of the steam in the cylinder of the steam engine, which we will discuss more fully later on, give a subordinate importance to the discovery mentioned, so far at least as this special technical problem is concerned.

We have already mentioned that expansion of the steam according to the upper limit curve *dd* (Fig. 5*a*) makes a supply of heat necessary when the transformation d_1d_0 of the limit curve has the course indicated in Fig. 5*b*, i.e., when there is connected

with increasing temperature T a decrease of the abscissa

$$P=\frac{1}{A}\left(\tau+\frac{r}{T}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

as occurs, in fact, in Col. 7 of the above-given table for steam.

If, on the other hand, the transformation of the pressure curve has a course like that of the dotted curve d_1e in Fig. 5*b*, then the vapor considered will exhibit the opposite behavior, the value P of equation (60) will increase with growing temperature and accordingly heat must be supplied to the steam during its expansion as per limit curve.

Hirn[1] first called attention to the fact that the vapor of ether really behaved in this exceptional fashion; with respect to this behavior, therefore, we must place the kinds of vapor in two groups; those of one group range themselves on the side of the vapor of water, the other group on the side of the vapor of ether. Immediately after Regnault's experimental results were made known, I examined all the rest of his vapors in the direction indicated.[2]

For this purpose it is only necessary to investigate whether, for the vapor considered, the value P found from equation (60) increases or decreases with growing temperature, or whether the differential is positive or negative; the former is the case with ether, the latter with water.

If we consider the meaning of τ according to equation (54), p. 59, differentiation gives

$$ATdP=\left[c+\frac{dr}{dt}-\frac{r}{T}\right]dt. \quad . \quad . \quad . \quad . \quad . \quad (61)$$

[1] Hirn, "Confirmation expérimentale de la seconde proposition de la théorie mécanique de la chaleur et des équations qui en découlent." Cosmos, Year XII, Vol. 22, p. 413.

[2] Compare the articles by the author in the Vierteljahresschrift der naturf. Gesellschaft in Zurich, Year 1863, p. 68: "Das Verhalten verschiedener Dämpfe bei der Expansion und Compression" and "Tabelle für gesättigte Aetherdämpfe." See also Combes, "Exposé des principes de la théorie mécanique de la chaleur." Bulletin de la Société d'Encouragement pour l'Industrie Nationale, Paris, 1863–1864.

The expression in the brackets, according to equation (56), p. 60, is nothing else than the temperature function h, introduced by Clausius; its sign decides the question at once, and it was this very fact that the negative sign occurs with the vapor of water which led Clausius to his discovery in his first article (1850).

The sign of h is also decided by equation (56a); if we there utilize the empirical formulas for λ and q, given under (14) and (15), there follows for

vapor of water:

$$hT = -523.23 + t + 0.00002\,t^2 + 0.0000003\,t^3$$
$$[hT = -941.814 + (t-32°) + 0.0000111\,(t-32°)^2 + 0.000000093(t-32°)^3],$$

vapor of ether:

$$hT = +28.85 + 0.2257\,t - 0.0002596\,t^2,$$
$$[hT = +51.93 + 0.2257\,(t-32°) - 0.0001442\,(t-32°)^2],$$

vapor of acetone:

$$hT = -40.473 + 0.2247\,t - 0.000119\,t^2,$$
$$[hT = -72.851 + 0.2247\,(t-32°) - 0.0000661\,(t-32°)^2],$$

vapor of chloroform:

$$hT = -29.462 + 0.2323\,t - 0.0000507\,t^2,$$
$$[hT = -53.032 + 0.2323\,(t-32°) - 0.0000282\,(t-32°)^2],$$

vapor of chloride of carbon:

$$hT = -12.087 + 0.1041\,t - 0.000081\,t^2,$$
$$[hT = -21.757 + 0.1041\,(t-32°) - 0.000045\,(t-32°)^2],$$

vapor of bisulphide of carbon:

$$hT = -50.139 + 0.0101\,t - 0.0003308\,t^2.$$
$$[hT = -90.250 + 0.0101\,(t-32°) - 0.000184\,(t-32°)^2].$$

For the ordinarily occurring temperatures the equation is at once decided by the sign of the first term of the right member of these expressions; this term represents the value of hT for $t=0$;

if we designate the value of the temperature function for $t=0$ by the subscript 0, there follows, from equation (56a), because $q=0$,

$$(hT)_0 = 273\left(\frac{d\lambda}{dt}\right)_0 - \lambda_0$$

$$\left[(hT)_0 = 491.4\left(\frac{d\lambda}{dt}\right)_0 - \lambda_0\right].$$

For a vapor whose total heat is represented by the empirical formula

$$\lambda = \alpha + \beta t + \delta t^2,$$

where α, β, and δ are constant factors, there follows therefore

$$(hT)_0 = 273\beta - \alpha$$

$$[(hT)_0 = 491.4\beta - \alpha].$$

For example, R e g n a u l t gives for the vapor of benzine

$$\lambda = 109.00 + 0.24429\, t - 0.0001315\, t^2$$

$$[\lambda = 196.20 + 0.24429\, (t-32°) - 0.0000731\, (t-32°)^2];$$

for such a vapor, therefore, calculation gives $(hT)_0 = -42.309$ $[-76.156]$. Benzine vapor consequently ranges itself on the side of water vapor.

For alcohol vapor, as already mentioned, R e g n a u l t gives an empirical formula only for the liquid heat q (equations 15, p. 22). But then from the values given in Table 3b of the Appendix we get

$t=0°$	40°	80°
$\tau+\frac{r}{T}=3.1413$	3.1172	3.0447

As the latter values diminish with growing temperature this vapor also behaves like the vapor of water.

Of the kinds of vapor which thus far permit investigation, only the vapor of ether exhibits the exceptional behavior mentioned.

If we replace the bracketed quantity of the right member of equation (61) by h, and remember that, according to the definition

of the entropy P, the left side of this equation represents nothing but dQ there follows

$$dQ = hdt,$$

which formula also directly ensues from the heat equation (57), given by Clausius. For if the change of state takes place along the limit curve we simply substitute $x=1$ in the quoted equation, which immediately leads to the preceding formula whose integration determines the heat quantity Q which must be imparted to dry saturated steam when it expands according to the limit curve.

The auxiliary table on p. 63 gives, in Col. 8, for the vapor of water the values of

$$\int_0^t hdt,$$

which appear negative for the reasons mentioned.

Example. Let dry saturated steam of 5 atmospheres pressure expand along the limit curve till the pressure has fallen to 1 atmosphere, then if t_1 represents the initial and t_2 the final temperature, we shall have

$$Q = \int_{t_1}^{t_2} hdt = \int_0^{t_2} hdt - \int_0^{t_1} hdt,$$

and therefore, according to the table on p. 63,

$$Q = -148.470 - (-200.457) = +51.987 \text{ cal.}$$
$$[Q = +93.577 \text{ B.t.u.}],$$

which amount represents the heat quantity which in this case must be supplied to the steam from without.

The change of inner work, measured in units of heat, is found by integrating equation (41), p. 54:

$$AdU = dq + d(x\rho),$$

and if we substitute $x=1$ and utilize the corresponding statement of the main Table 11 of the Appendix, the integration becomes

$$A(U_2 - U_1) = q_2 - q_1 + \rho_2 - \rho_1 = -11.935 \text{ Cal.} [-21.483 \text{ B.t.u.}];$$

consequently we have here a diminution of inner work. The outer work L, produced by expansion, is determined by

$$Q = A(U_2 - U_1) + AL,$$

from which follows

$$AL = 63.922 \text{ Cal.} \quad \text{and} \quad L = 27102.9 \text{ mkg.}$$
$$[AL = 115.06 \text{ B.t.u.} \quad \text{and} \quad L = 88922 \text{ ft-lb.}].$$

Therefore there is consumed in outer work not only the whole of the heat supplied, but also a part of the inner work.

§ 10. THE ISOTHERMAL AND THE ISODYNAMIC CURVE OF WET VAPORS.

(*a*) Isothermal Curve. The change of pressure p with volume v takes place according to the isothermal curve, when the temperature t is kept constant during the changes of state. Now because with wet vapor the pressure depends only on the temperature, we have also the pressure constant during the assumed change of state, and therefore the isothermal curve is a straight line running parallel to the axis of abscissas.

We have already fully discussed the matter of evaporation and condensation under constant pressure in §§ 3 and 4, so that the question of isothermal changes of state is settled by those discussions. That the transformation of the isothermal curve is likewise a straight line parallel to the axis of abscissas was explained when considering Fig. 5, p. 64.

(*b*) Isodynamic Curve. If the inner work U (intrinsic energy) is kept constant, while the pressure and volume of wet vapors change, then we designate the corresponding pressure curve as the "isodynamic curve." If U is constant, i.e., $dU = 0$, it follows from equation (41), p. 54, namely, from

$$AdU = dq + d(x\rho),$$

by integration that

$$q + x\rho = q_1 + x_1\rho_1, \quad \ldots\ldots\ldots \quad (62)$$

if we regard as given the temperature t_1 and the steam quality x_1, for the initial condition.

This equation in combination with the two formulas

$$v = xu + \sigma \quad \text{and} \quad v_1 = x_1u_1 + \sigma \quad \ldots\ldots \quad (63)$$

solves the proposed question as to the course of the isodynamic curve; of course the equation of the curve just obtained is not in the usual form, i.e., the pressure p is not expressed as a function of the volume v, but the equations secured enable us to compute the coordinates of the several points, and their lay-off then shows the course of the curve.

If there is given, for the initial condition, the steam quality x_1, likewise the temperature t_1 and hence the pressure p_1, we shall also know the values q_1, ρ_1, and u_1, and can compute the right member of equation (62), and with its aid get from the second of equations (63) the specific volume v_1 for the initial condition.

For any other value of the temperature t or of the pressure p we can compute, from equation (62), the steam quality x, and from the first of equations (63) the volume v. The combination of the equations gives, moreover, for the direct computation of v, the formula

$$q+\frac{\rho}{u}(v-\sigma)=q_1+\frac{\rho_1}{u_1}(v_1-\sigma). \quad . \quad . \quad . \quad . \quad . \quad (64)$$

Example. Let a cylinder contain one kilogram [pound] of water and steam, of pressure $p_1=5$ atmospheres, and of it let $x=0.80$ be in the form of vapor, then, according to Table 11 of the Appendix,

$$q_1=153.741, \quad \rho_1=454.99_1 \quad u_1=0.3626$$
$$[q_1=276.734, \quad \rho_1=818.982_1 \quad u_1=5.8085].$$

On the other hand, for the pressure of one atmosphere, we get

$$q=100.500, \quad \rho=496.300, \quad u=1.6495$$
$$[q=180.9, \quad \rho=893.34 \quad u=26.423].$$

Hence we find from equation (62)

$$x=0.8407,$$

and according equations (63)

$$v_1=0.2911 \quad \text{and} \quad v=1.3877 \text{ cbm.}$$
$$[v_1=4.0223 \quad \text{and} \quad v=22.229 \text{ cu. ft.}].$$

The ratio of expansion is therefore

$$\frac{v}{v_1}=4.766.$$

Accordingly, at the end of the expansion from 5 to 1 atmospheres, we have more steam on hand than at the beginning, because $x > x_1$ results, and therefore the expansion involves a partial evaporation.

If we allow ourselves to make the assumption that the isodynamic curve is subject to the law

$$pv^\nu = p_1 v_1^\nu,$$

where the exponent ν is a constant quantity, then, in the present example, we find

$$\nu = \frac{\log p_1 - \log p}{\log v - \log v_1} = 1.031.$$

Hence the exponent ν differs but little from unity, a result which is also obtained when other pressure limits are assumed. The iosdynamic curve of wet vapor has therefore approximately the course of an equilateral hyperbola; wet vapors approach gases in this respect.

If we subtract $x_1\rho$ from both sides of equation (62) we get from it

$$x - x_1 = \frac{q_1 - q + x_1(\rho_1 - \rho)}{\rho} \quad \ldots\ldots \quad (65)$$

If we consider the formulas for q, given under (15), p. 22, and likewise the formulas for ρ, according to (25), p. 31, it is seen that the right side of equation (65) becomes positive for all vapors mentioned above (also for vapor of ether) when the temperature diminishes, i.e., when expansion takes place and hence $x > x_1$.

With wet vapors, therefore, expansion, according to the isodynamic curve, involves partial evaporation, and compression involves a condensation of the steam. If the expansion is continued long enough it is possible that $x = 1$ will result, and dry saturated steam ensue; at this point the isodynamic curve passes through the limit curve; with a further continuation of the expansion the steam passes into a superheated condition in which the isodynamic curve takes a different course from that in wet field.

For the point of intersection of the limit curve, $x = 1$ must be substituted in equation (62); the formula then becomes

$$q + \rho = q_1 + x_1\rho_1,$$

and furnishes the means of computing the temperature, and hence the pressure and volume of the point of intersection.

Example. If the point of intersection is to lie exactly at one atmosphere of pressure, and if the initial pressure is 5 atmospheres, then the steam quality x_1, necessary at the beginning, will be found to be $x_1 = 0.965$, from the preceding formula and the utilization of the numerical values given in the preceding example.

As regards the heat quantity which must be imparted to wet vapors during isodynamic change of state, we have the fundamental equation

$$dQ = A(dU + pdv),$$

and because $dU = 0$,

$$dQ = Apdv,$$

according to which the area, bounded by the isodynamic curve, represents the heat quantity Q measured in units of work; it is therefore unnecessary, for the graphical determination of this heat quantity, to first transform the curve.

If, in the preceding equation, we designate the work pdv by dL, then by utilizing the first of equations (63) we get

$$dL = pd(xu),$$

from which results

$$dL = d(pux) - xu\frac{dp}{dt}dt,$$

or, with the use of Clapeyron's equation (22), p. 29,

$$dL = d(pux) - \frac{xr}{AT}dt,$$

a formula which is still general for every kind of expansion.

If, for the present case, we substitute in the second term of the right member the value of x which comes from equation (62), we get

$$dL = d(pux) - \frac{r}{A\rho T}(q_1 + x_1\rho_1 - q)dt.$$

The first term of the right side can be integrated at once, but the second term can only be integrated when the temperature functions

$$\frac{r}{A\rho T} \quad \text{and} \quad \frac{rq}{A\rho T}$$

are replaced by simpler empirical formulas. When the work L is produced the heat quantity becomes simply $Q=AL$.

§ 11. THE ADIABATIC CURVE OF WET VAPORS.

During adiabatic change of state no heat is imparted or withdrawn during the entire course and therefore $dQ=0$, and if we here draw on equation (55), p. 59, we get

$$Td\left(\tau+\frac{xr}{T}\right)=0,$$

or, providing the quantities belonging to the initial condition with the subscript 1,

$$\tau+\frac{xr}{T}=\tau_1+\frac{x_1r_1}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

Accordingly the change of entropy P given by equation (58), p. 61, is a constant quantity; hence the transformation of the adiabatic curve ab (Fig. 6a) is a straight line $a'b'$ lying parallel to the axis of ordinates, as Fig. 6b shows.

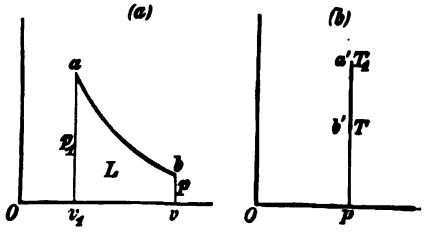

FIG. 6.

Equation (66) in combination with equations

$$v_1=x_1u_1+\sigma \quad \text{and} \quad v=xu+\sigma \quad . \quad . \quad . \quad . \quad (67)$$

now gives us the means of fixing the course of the adiabatics of wet vapors. If the pressure and volume p_1 and v_1 are known for the initial condition a, the first of equations (67) determines the steam quality x_1, or, conversely, we determine from the latter the specific volume v_1.

For any other pressure p we then get for equation (66) the steam quality x, and from the second of equations (67) the specific volume v belonging to the point b (Fig. 6a). In this way a series of points on the adiabatic curves can be established and its course determined.

The work L, produced during adiabatic expansion, is found from the fundamental equation

$$dQ = A(dU + dL)$$

when we make $dQ = 0$ and employ for dU equation (41), p. 54; then

$$AdL = -dq - d(x\rho),$$

and from this, by integration,

$$AL = q_1 - q + x_1\rho_1 - x\rho. \quad . \quad . \quad . \quad . \quad . \quad (68)$$

Accordingly, when the specific steam quality x is determined for the final pressure p by equation (66), the work can be calculated.

Example. Let a kilogram [pound] of dry saturated steam expand from the initial pressure of 4 atmospheres down to the final pressure of 1 atmosphere, then, according to the table on p. 62, and also Table 11 of the Appendix,

$$\frac{r_1}{A} = 181.103, \quad \frac{r_1}{AT_1} = 513.587, \quad u_1 = 0.4474$$

$$\left[\frac{r_1}{A} = 330.10, \quad \frac{r_1}{AT_1} = 936.12, \quad u_1 = 7.167\right]$$

and

$$\frac{r}{A} = 132.954, \quad \frac{r}{AT} = 609.856, \quad u_1 = 1.6495$$

$$\left[\frac{r}{A} = 242.34, \quad \frac{r}{AT} = 1111.77, \quad u_1 = 26.423\right].$$

We therefore get from equation (66), for $x_1 = 1$, when we divide both sides by A, the value

$$x = 0.9211,$$

and then

$$v_1 = 0.4484, \quad v = 1.5203, \quad \frac{v}{v_1} = 3.390.$$
$$[v_1 = 7.183, \quad v = 24.354].$$

Since $x < x_1$, adiabatic expansion here involves partial condensation.

Furthermore, for the present case, according to Table 11,

$$q_1 = 145.310, \quad q = 100.500, \quad \rho_1 = 461.496, \quad \rho = 496.300$$
$$[q_1 = 261.558, \quad q = 180.900, \quad \rho_1 = 830.693, \quad \rho = 893.340],$$

and hence, from equation (68),

$$AL = 49.064 \text{ cal. } [88.495 \text{ B.t.u.}]$$

or

$$L = 20845.5 \text{ mkg. } [68391.5 \text{ ft-lb.}].$$

For the solution of the questions treated in the foregoing we might also have made use of equation (51*a*), p. 58. If we there substitute $dQ = 0$, there follows

$$\omega dt + ATd\left(v\frac{dp}{dt}\right) = 0,$$

and from this results the equation of the adiabatic curve, provided we assume, as was there indicated, the value ω constant,

$$v\left(\frac{dp}{dt}\right) - v_1\left(\frac{dp}{dt}\right)_1 = \frac{\omega}{A}\log_e\frac{T_1}{T}, \quad \ldots \quad (69)$$

from which we can determine v directly for an assumed pressure p; for the vapor of water we must specially substitute $\omega = 1$. The table in the Appendix and also the table on p. 62 give the value of the differential coefficients occurring in this formula.

In like manner a second expression can be developed for the produced or consumed work L.

According to equation (52), p. 58,

$$AdU = \omega dt + d\left(v\frac{\rho}{u}\right), \quad \ldots \quad (70)$$

which equation is a general one. But with adiabatic change of state

$$AdL = -AdU,$$

hence we also have

$$AdL = -\omega dt - d\left(v\frac{\rho}{u}\right),$$

and accordingly we have in the place of equation (68)

$$AL = \omega(T_1 - T) + v_1\frac{\rho_1}{u_1} - v\frac{\rho}{u}. \quad \ldots \quad (71)$$

The work can therefore be computed directly, as soon as the initial and final volumes and the initial and final pressures are given.

For practical calculations, however, it is more advantageous to make use of equations (66) to (68).

It may happen, for instance, that the adiabatic crosses the limit curve. This may occur during the adiabatic expansion of the vapor of ether and during the adiabatic compression of the vapor of water and of the remaining vapors adduced in this treatise; but then the formulas lose their validity. If such a crossing of the limit curve takes place, it is manifested by equation (66) furnishing for the steam quality the impossible value $x > 1$. Therefore, in order to better control this matter, the quantity x should always be computed beforehand and we may thus use to better advantage equations (66) to (68). Equation (66) enables us to recognize that with the adiabatic change of state of wet vapor there is always connected a change of steam quality x. If we assume for the start that the steam is dry saturated, that is assume $x_1 = 1$, then a condensation of steam is connected with the expansion of the vapor of water; while, in the converse case of adiabatic compression, $x > 1$ occurs, a sign that in so doing the steam is superheated. All the other vapors mentioned in this book behave in like manner, with the sole exception of the vapor of ether, which, initially supposed to be dry saturated, acts in just the opposite way; namely, it is superheated by expansion and is condensed by compression, a phenomenon which was fully

discussed on p. 67. Those considerations, to be sure, referred only to the behavior of d r y saturated vapor; but a closer consideration of equation (66) shows that the vapor of water (and the other vapors before grouped with it, in this sense) may sometimes condense during adiabatic c o m p r e s s i o n, when it is not dry at the beginning but is w e t, and that it is here only a question of the initial value x_1 of the steam quality; conversely an evaporation occurs with expansion. Thus there follows from equation (66)

$$(x-x_1)\frac{r}{T}=\tau_1-\tau-x_1\left(\frac{r}{T}-\frac{r_1}{T_1}\right). \quad . \quad . \quad . \quad . \quad (72)$$

If we think of expansion, and therefore assume $t<t_1$, we have for $x_1=1$, in the case of v a p o r o f w a t e r, a negative value for the right member of this equation, and accordingly $x<x_1$, i.e., the steam present diminishes.

On the other hand if we assume that at the beginning o n l y water is present, so that $x_1=0$, then the right-hand member becomes positive, hence $x>x_1$; i.e., steam formation occurs.

Finally we recognize that, between the two initial conditions assumed and the presupposed temperature limits, there may exist an initial condition for which the steam quality x at the end of the expansion will be the same as at the beginning (but not during the course of the expansion); according to equation (72) this case occurs when

$$x_1=\frac{\tau_1-\tau}{\frac{r}{T}-\frac{r_1}{T_1}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

For example, if the vapor of water is given and expanded adiabatically from 4 atmospheres to 1 atmosphere, then the preceding formula gives

$$x_1=0.501,$$

when we utilize the corresponding value of Cols. 5 and 6 in the table on p. 62. We get in this way for other pressure limits other values of x_1 which differ but little from 0.50, so that we can at least say i n g e n e r a l that, during the adiabatic expansion of

vapor of water, there occurs a condensation of steam, provided, at the start, there is, weight for weight, more steam than water present; on the other hand if initially the weight of water is in excess, then evaporation is connected with the expansion.

At the first glance it would appear as if there were here a very simple law; but it is very easily shown that this is not the case. If we assume that the steam quality x_1, of equation (73), is really a constant quantity k for any pressure limits whatsoever, we find that when we pass to the differential of equation (73),

$$kd\left(\frac{r}{T}\right) = -d\tau,$$

or when we execute the differentiation and utilize the value for $d\tau$ resulting from equation (54), we get

$$k\left(\frac{dr}{dt} - \frac{r}{T}\right) = -c,$$

where c means the specific heat of the liquid. If we utilize here Clausius' temperature function h from equation (56), p. 60, there follows

$$k = \frac{c}{c-h}.$$

Thus, for example, for water and the temperature values

	$t=0°$	100°	200° C.
	[$t=32°$	212°	392° F.]
we get			
	$c=$ 1	1.013	1.044
	$h=-1.9166$	-1.1333	-0.6766

from which we respectively find

$$k = \quad 0.343 \qquad 0.472 \qquad 0.607,$$

so that the value k is by no means a constant quantity.

The adiabatic curve of wet vapors plays an important part in certain technical problems which we will discuss later; it there-

fore seems highly desirable to represent the equation of this curve in the form $p=f(v)$ even if it is only an approximate formula. The first effort of this sort is due to Rankine[1]; he concludes, from the result of numerical calculations, that the adiabatic curve of the vapor of water is given by the approximate formula

$$pv^{\mu}=p_1v_1^{\mu}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

where p_1 is the initial pressure, v_1 the initial volume of saturated steam, and μ is a constant quantity for which Rankine gives the value 1.111. Rankine does not say whether, in applying this formula, we must assume at the beginning of the expansion pure steam without admixture of water or not. But this question is not unimportant, as I will show.

Later Grashof[2] returned to this question and concluded, from the results of numerical calculations, that $\mu=1.140$ should be used; in these calculations, however, it was expressly assumed that, while the steam was saturated at the beginning of expansion, no water was mixed with it. On account of the importance of the matter, I will go into the question more fully and in the same way as Rankine and Grashof, but will determine the value μ of equation (74) with the help of my exact tables given here, once under the assumption that no water is mixed with the saturated steam at the beginning, and then under the assumption that we are dealing with a mixture of water and vapor.

Suppose that we have a certain initial condition (p_1v_1) and that the volume v has been calculated for an assumed terminal pressure p in the manner presented on p. 76; then we could determine for equation (74) the exponent μ according to the formula

$$\mu=\frac{\log p_1-\log p}{\log v-\log v_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

For instance, in the example on p. 76, dry saturated steam was assumed of the pressure $p_1=4$ atmospheres and expansion to the pressure of 1 atmosphere was supposed to occur; we found

[1] Rankine, Manual of Applied Mechanics.—A Manual of the Steam Engine, p. 385.

[2] Grashof, Vorwort in "Völker's Indicator." Berlin, 1863. Zeitschrift des Vereins deutscher Ingenieure, Vol. VIII, p. 151.

there $v_1 = 0.4484$ and $v = 1.5203$, and, accordingly, the preceding equation gives for this one case

$$\mu = 1.1354.$$

In like manner, for other initial and final values, we can determine the value μ and see whether or not this value really remains constant. The following tabulation gives the results of such calculations for the initial pressures of 8, 4, 2, and 1 atmospheres and final pressures of 4, 2, 1, and 0.5 atmospheres; throughout, the assumption is made that the steam is dry saturated at the beginning; there are tabulated the specific quality x at the end of the expansion, the expansion ratio ε, i.e., the relation $v : v_1$, and finally the corresponding values of μ.

Initial Pressure p_1 in Atmospheres	Final Pressure p in Atmospheres.			
	0.5	1	2	4
8	$x =$ 0.8541 $\varepsilon =$11.577 $\mu =$ 1.1321	0.8844 6.236 1.1360	0.9182 3.375 1.1396	0.9564 1.834 1.1431
4	$x =$ 0.8882 $\varepsilon =$ 6.282 $\mu =$ 1.1315	0.9211 3.390 1.1354	0.9580 1.837 1.1399	
2	$x =$ 0.9241 $\varepsilon =$ 3.409 $\mu =$ 1.1304	0.9598 1.842 1.1344		
1	$x =$ 0.9614 $\varepsilon =$ 1.848 $\mu =$ 1.1291			

A glance at this tabulation shows that for a given initial pressure the value μ diminishes decidedly the smaller the final pressure p, and that, for a given final pressure, μ becomes greater the larger the initial pressure is assumed to be; nevertheless the variations are so insignificant that within the pressure limits assumed in the table, which include nearly all the cases occurring in steam engines, we may assume a constant mean value for μ, and this value is

$$\mu = 1.135,$$

and therefore one closely agreeing with that given by Grashof. But the results of the computation are only valid for dry saturated steam, therefore only true for the assumption that the steam quality at the beginning is $x_1 = 1$. In order to recognize the influence of the value x_1 on the exponent μ we add the following tabulation of the results of computations made under the assumptions that $x_1 = 0.90$, 0.80, or 0.70; the table contains only the values of μ.

Initial Pressure p_1 in Atmospheres	Initial Steam Quality x_1	Final Pressure in Atmospheres		
		0.5	1	2
4	$x_1 = 0.90$	$\mu = 1.1236$	1.1268	1.1302
	0.80	$= 1.1140$	1.1164	1.1190
	0.70	$= 1.1019$	1.1036	1.1047
2	$x_1 = 0.90$	$\mu = 1.1231$	1.1264	
	0.80	$= 1.1141$	1.1166	
	0.70	$= 1.1028$	1.1044	
1	$x_1 = 0.90$	$\mu = 1.1223$		
	0.80	$= 1.1138$		
	0.70	$= 1.1032$		

This grouping shows that the exponent μ of the empirical formula (74) varies with the initial steam quality x_1, but that it is again nearly constant for equal values of the initial steam quality. If we take the mean of these values of μ which belong to equal values of x_1, we get, for the initial steam quality,

$x_1 = 1$	$\mu = 1.135$
0.90	1.125
0.80	1.115
0.70	1.103

and these values can readily be concentrated in the empirical formula

$$\mu = 1.035 + 0.100\, x_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

which formula, of course, is only permissible for the present limits 0.7 to 1 here assumed for x_1; most of the cases occurring in engineering will be covered by this relation.

In all cases in which use is made of the approximate formula $pv^{\mu}=p_1v_1^{\mu}$ we can compute the expansion work L by formula

$$L=\int_{v_1}^{v} pdv,$$

and with this approximate formula as a base we get, as may readily be seen,

$$\left.\begin{aligned} L&=\frac{p_1v_1}{\mu-1}\left[1-\left(\frac{v_1}{v}\right)^{\mu-1}\right] \\ \text{or} \quad L&=\frac{p_1v_1}{\mu-1}\left[1-\left(\frac{p}{p_1}\right)^{\frac{\mu-1}{\mu}}\right], \\ \text{or also} \quad L&=\frac{1}{\mu-1}[p_1v_1-pv], \end{aligned}\right\} \quad \ldots\ldots \quad (77)$$

formulas, which, within the designated limits, give numerical results which agree sufficiently well with those furnished with the exact equation (68), p. 76.

The preceding equations agree with those which we found before (Vol. I, p. 140) for the adiabatic expansion of gases, with the difference that there we substituted the value $\kappa=1.410$ for the exponent μ.

One might be inclined to infer from this that the preceding formulas and the relation (74) represent the true law of adiabatic change of state of wet vapors. A closer examination of the fundamental equations (66) to (68), p. 75 and p. 76, easily convinces one that the equations just established can only be regarded as approximate formulas, and that they can only be applied for pressure limits which do not greatly transcend those of the two tabulations on p. 82 and p. 83. If the limits of the pressures are far apart, values are obtained for the exponent μ which differ not inconsiderably from those of the two tabulations on p. 82 and p. 83, values which no longer correspond to the empirical formula (76). Still greater discrepancies occur if we take the steam quality x_1 less than 0.5 at the beginning, and the approximate computation be-

comes wholly untenable for the case in which $x_1 = 0$. Hence, if use is hereafter made of the approximate formulas (74) and (77), it will only be within the indicated limits of the initial and final pressure.

Charpentier[1] enunciates, to be sure without proof, the proposition that with dry saturated steam, i.e., for $x_1 = 1$ we must determine the steam quality x at the end of the adiabatic expansion according to the formula

$$x = \frac{Apu}{Ap_1u_1}, \quad \ldots \ldots \ldots \quad (78)$$

but the theoretically deduced formula (66), p. 75, shows that x must be computed from the relation

$$x\frac{r}{T} = \tau_1 - \tau + \frac{r_1}{T_1}.$$

Charpentier's formula for the pressure limits assumed in the tabulation on p. 83 gives values for the quality x which differ but little from those of the table which was computed from the last of the preceding formulas; for the special case in which the steam is dry in the beginning, however, just as good an agreement is obtained when the steam quality at the end is determined by the formula

$$x = \frac{\rho_1}{\rho}.$$

A closer examination of this assumption and of Charpentier's statement shows that the resulting connection between the several temperature functions entering into the formulas is not confirmed so long as we hold fast to Regnault's experimental results on liquid and total heat.

The preceding investigations concerning the course of the adiabatic curve of wet vapors bore particularly on the question of expansion, under the assumption that the steam was saturated

[1] "Sur la détente adiabatique de la vapeur d'eau." Comptes rendus, 1884, XCVIII, pp. 85–87, 425–428.

at the beginning, or that comparatively little liquid was mixed with it.

The question as to the behavior of wet steam during adiabatic compression, when the weight of the liquid present exceeds that of the vapor, is one of technical importance. In this case condensation of the steam is connected with compression, and it is of such a character that if the compression is continued long enough there will be only liquid present without any steam.

Let p_1 be the pressure at the beginning and x_1 the steam quality; suppose the compression of the unit of weight of mixture to be continued till finally only liquid is on hand, then σ will be the final volume and $v_1 = x_1 u_1 + \sigma$ will be the initial volume; the ratio ε of these two values may be called the compression ratio; accordingly

$$\varepsilon = \frac{x_1 u_1 + \sigma}{\sigma}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

Moreover, because at the end of compression we must have $x = 0$, equation (66), p. 75, gives us

$$\tau = \tau_1 + \frac{x_1 r_1}{T_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

from this the final temperature t and the final pressure p, at which the whole mass is liquefied, are determined, and then follows, from equation (68), the work of compression L from the formula

$$AL = q_1 - q + x_1 \rho_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

Example 1. Let the initial pressure be 0.2 atmosphere or $p_1 = 2066.6$ kg. [433.26 lb.]; corresponding to this pressure we get the values

$$\tau_1 = 0.2005, \quad \frac{r_1}{T_1} = 1.6924, \quad u_1 = 7.5428 \quad [u_1 = 120.83],$$
$$q_1 = 60.589 \quad \text{and} \quad \rho_1 = 527.584$$
$$[q_1 = 109.06 \quad \text{and} \quad \rho_1 = 949.65].$$

Consequently we get for the two cases

$x_1 = 0.10$	or	$x_1 = 0.01$,
$\varepsilon = 755.4$		$\varepsilon = 76.4$,
$\tau = 0.3697$		$\tau = 0.2175$.

The values correspond to the pressures

$$p=2.000, \qquad p=0.256 \text{ atmospheres,}$$

existing for complete liquefaction, and according to equation (81) we find respectively

$$AL=8.07 \qquad AL=0.28 \text{ cal.}$$
$$[AL=14.526 \qquad AL=0.504 \text{ B.t.u.}].$$

Example 2. Let the initial pressure be 1 atmosphere or $p=10333$ kg. [2116.3 lb.]. Corresponding to this pressure we have the values

$$\tau_1=0.3136, \quad \frac{r_1}{T_1}=1.4383, \quad u_1=1.6494,$$
$$q_1=100.50, \quad \rho_1=496.30.$$
$$\left[\tau_1=0.3136, \quad \frac{r_1}{T_1}=1.4383, \quad u_1=26.422,\right.$$
$$\left. q_1=180.9, \quad \rho_1=893.34.\right]$$

Therefore during compression to the point of liquefaction we get, for the two cases, the following results:

$$x_1=0.10 \quad \text{or} \quad x_1=0.01$$
$$\epsilon=165.9 \qquad \epsilon=17.5$$
$$\tau=0.4575 \qquad \tau=0.3280,$$

and with sufficient accuracy we have for these values the terminal pressures

$$p=5.5 \quad \text{or} \quad p=1.2 \text{ atmospheres.}$$

The work L is found from equation (81) to be, respectively,

$$AL=7.34 \quad \text{to} \quad AL=0.28 \text{ cal.}$$
$$[AL=13.212 \quad \text{to} \quad AL=0.504 \text{ B.t.u.}].$$

If for the first of the two cases, i.e., for the initial value $x_1=0.10$, compression is carried only from 1 to 3 atmospheres, then, according to equation (66), the steam quality at the end is $x=0.0439$ and the corresponding volume $v=0.0206$ [0.3300], while at the initial pressure $p_1=1$ atmosphere the volume amounts to $v_1=0.1659$ [2.670]; if we here assume that the compression curve follows the law $pv^\mu=p_1v_1^\mu$, then $\mu=0.527$.

The two preceding problems correspond in certain cases to what happens during compression in the cylinder of a steam engine, according as the engine is condensing or non-condensing. It has not yet been proved how much water is, in these cases, mixed with the steam at the beginning of compression. If we make the possible assumption that the water

present is considerable, then the pressure curve rises less rapidly than with vapor containing little water. We have therefore a means of inferring, from the course of the compression curve of an indicator diagram, something concerning the water carried by the steam which, confined in the steam cylinder, is compressed there and finally is mixed with the fresh steam just entering. Decisive experiments and results of observations on this question unfortunately do not exist, but we will take it up again later.

APPLICATIONS.

I. Reversible Changes of State of Wet Vapors.

§ 12. HEATING OR COOLING OF WET VAPORS AT CONSTANT VOLUME.

If a unit of weight of a mixture of liquid and vapor has the pressure p_1 and the steam quality x_1, then its volume is given by

$$v_1 = x_1 u_1 + \sigma;$$

now if, in consequence of heat being imparted or withdrawn, the pressure changes to the value p, and if x is the steam quality at the end, then, because a change of volume does not occur, we also have

$$v_1 = xu + \sigma.$$

By equating these two equations we get, for the steam quality x at the end,

$$x = \frac{x_1 u_1}{u}, \quad \ldots\ldots\ldots \quad (1)$$

and it may be calculated from the assumed terminal pressure p, because this also determines the value u. Since, in the present case, $dv = 0$, there follows from equation (43), p. 55,

$$dQ = dq + d(x\rho),$$

and from this the heat to be imparted is

$$Q = q - q_1 + x\rho - x_1\rho_1,$$

or, with the help of equation (1), we have

$$Q=q-q_1+x_1u_1\left(\frac{\rho}{u}-\frac{\rho_1}{u_1}\right). \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We get from equation (58), p. 61, the corresponding increase of entropy:

$$P-P_1=\frac{1}{A}\left(\tau-\tau_1+\frac{xr}{T}-\frac{x_1r_1}{T_1}\right), \quad . \quad . \quad . \quad . \quad (3)$$

in which formula the value x, from equation (1), may also be substituted. If we lay off the last computed quantity as abscissa and the corresponding temperature T as ordinate for different values of p, we get the transformation of the present pressure curve. The pressure curve itself is here a straight line parallel to the axis of ordinates.

If, in the present case, we suppose heat to be imparted, i.e., suppose temperature and pressure to increase, then at a certain instant all the liquid present will be completely evaporated so that $x=1$ will obtain. If we designate the pressure for this state by p_2 we shall get from equation (1)

$$u_2=x_1u_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

and the quantity of heat Q_2, which must be supplied up to this point, will become

$$Q_2=q_2-q_1+x_1u_1\left(\frac{\rho_2}{u_2}-\frac{\rho_1}{u_1}\right). \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

A continued heat supply will still further increase the pressure and temperature, but the preceding formulas can no longer be applied to the occurrences that will now take place, because from here on the steam passes into the superheated condition.

Example. Suppose a vessel contains a mixture of G kg. [lb.] of water and its vapor; let 0.763 G kg. [lb.] be steam and let the rest, 0.237 G, be present in the liquid condition; let the pressure amount to 1.5 atmospheres and let the pressure sink to 0.1 atmosphere while heat is withdrawn at constant volume.

Here, according to Table 11 of the Appendix, we find for the pressure of 1.5 atmospheres

$$u_1 = 1.1259, \quad q_1 = 112.408, \quad \frac{\rho_1}{u_1} = 432.56$$

$$\left[u_1 = 18.036, \quad q_1 = 202.334, \quad \frac{\rho_1}{u_1} = 48.606\right],$$

and corresponding to the pressure of 0.1 atmosphere

$$u = 14.552, \quad q = 46.282, \quad \frac{\rho}{u} = 37.029$$

$$\left[u = 233.10, \quad q = 83.307, \quad \frac{\rho}{u} = 4.161\right].$$

For an initial steam quality $x_1 = 0.763$ we get, according to equation (1), for the fraction of steam present at the end,

$$x = 0.0590,$$

so that finally the total steam on hand is $Gx = 0.059\,G$ and the quantity of water present is $G(1-x) = 0.941\,G$. The quantity of heat which must here be withdrawn per unit of weight is found from equation (2), and the foregoing numerical results are found to be 405.86 cal. [730.548 B.t.u.], and therefore for G kg. [lb.] of the mixture

$$Q = 405.86\,G \quad [Q = 730.548\,G].$$

If the cooling had been effected by allowing cold water to stream past the outer walls of the vessel, so that it was thereby heated from t_0 to t_0', and if G_0 kg. [lb.] of cooling water had thus been employed, then this water would have absorbed the heat quantity

$$Q = G_0(q_0 - q_0');$$

let us assume that in so doing the cooling water is warmed from $t_0 = 15°$ C. [59° F.] to $t_0' = 35°$ C. [95° F.], then Table 1b of the Appendix will give for the assumed temperature $q_0 = 15.005$ [27.009] and $q_0' = 35.037$ [63.0666], and we then get

$$Q = 20.03\,G_0 \quad [Q = 36.054\,G_0];$$

hence there follows, by equating the two determined values of Q for the quantity of cooling water,

$$G_0 = \frac{405.86}{20.03} G = 20.26\,G.$$

With the same initial wet steam of 1½ atmospheres pressure and steam quality $x_1 = 0.763$ kg. [lb.] let us now make the following experiment. Let heat be imparted to this mixture at constant volume till the last ele-

ment of water has been converted into steam and the corresponding pressure p_2, to be determined, has been obtained.

Here we have, according to equation (1*a*),

$$u_2 = 0.8589 \ [13.759],$$

and to this value there corresponds, according to Table 11 of the Appendix, a pressure $p_2 = 2$ atmospheres and a temperature $t_2 = 120.60°$ C. [249.08° F.]. Furthermore the table gives

$$q_2 = 121.417, \quad \frac{\rho_2}{u_2} = 558.86$$

$$\left[q_2 = 218.551, \quad \frac{\rho_2}{u_2} = 62.798\right];$$

hence, according to equation (2*a*), the necessary heat quantity for G kg. [lb.] of the mixture is

$$Q_2 = 117.53\, G \text{ cal.}$$
$$[Q_2 = 211.554\, G \text{ B.t.u.}].$$

In order to investigate the change of state of wet steam at constant volume we might also have used general equation (51*a*), p. 58, i.e.,

$$dQ = \omega dt + ATd\left(v\frac{dp}{dt}\right).$$

If we indicate the heating under constant volume by providing dQ with the subscript v, then, because $v = v_1$ is a constant, we also have, according to the preceding formula,

$$\frac{dQ_v}{dt} = \omega + AvT\frac{d^2p}{dt^2}, \quad \ldots\ldots\ldots \quad (4)$$

or, with the help of equation (36), p. 39,

$$\frac{dQ_v}{dt} = \omega + v\frac{d}{dt}\left(\frac{\rho}{u}\right).$$

This quantity represents nothing but the **specific heat of wet steam at constant volume**, and if we designate it by c_v, which value is a constant with gases, we will get

$$c_v = \omega + v\frac{d}{dt}\left(\frac{\rho}{u}\right); \quad \ldots\ldots\ldots \quad (4a)$$

hence c_v here appears as a function of the pressure and of the volume. If we substitute $v=xu+\sigma$, and consider the relation (50), p. 57, there will also follow

$$c_v=c+xu\frac{d}{dt}\left(\frac{\rho}{u}\right), \quad \ldots\ldots\ldots \quad (4b)$$

where c is the specific heat of water. According to equation (36), p. 39, we can also write

$$c_v=c+AxuT\frac{d^2p}{dt^2},$$

or, utilizing Clapeyron's equation (22), p. 29, after a few easily followed transformations,

$$c_v=c+x\left[\frac{dr}{dt}-\frac{r}{T}-\frac{r}{u}\frac{du}{dt}\right]. \quad \ldots\ldots\ldots \quad (4c)$$

For dry saturated steam we make $x=1$; but in so doing we must only consider withdrawal of heat at constant volume, for with heat supply the steam would pass into the superheated condition and, in this case, the equations would lose their validity. Equations (4*b*) and (4*c*) give for dry steam

$$c_v=c+u\frac{d}{dt}\left(\frac{\rho}{u}\right), \quad \ldots\ldots\ldots \quad (4d)$$

or

$$c_v=c+\frac{dr}{dt}-\frac{r}{T}-\frac{r}{u}\frac{du}{dt},$$

and if, with the last equation, we utilize the Clausius temperature function h, as per equation (56), we get

$$c=h-\frac{r}{u}\frac{du}{dt}, \quad \ldots\ldots\ldots \quad (4e)$$

an equation which was also found in another way by J. J. Müller,[1] only he substituted, approximately, the quantity u for

[1] J. J. Müller, "Über die spezifische Wärme der gesättigten Dämpfe," Poggendorff's Annalen, Jubelband, 1874, p. 227.

the specific volume s of dry steam. For calculating the specific heat of steam with the help of the auxiliary table, p. 63, Col. 11, it is most convenient to use equation (4*a*) or (4*d*).

We thus obtain for dry saturated steam of:

$p =$	0.5	1	5	10	14	atmospheres
$c_v =$	19.583	16.957	11.603	9.638	8.732	

The values decrease with growing pressure, i.e., with increasing temperature, and approach, as equation 4*d* shows, the value c and a limit value which probably lies in the neighborhood of 400° C. [752° F.], as J. J. Müller has already observed.

With the help of equation (4*b*) there could be computed for different pressures the steam quality x, or from equation (4*a*) there could be calculated the specific volume v of wet steam under the hypothesis that c_v is constant for all points of the contemplated curve.

The formulas to which the proposed problem has led furnish the means of discussing a question that is of technical importance in running steam boilers.[1]

In this connection let us investigate the law according to which the steam pressure in a boiler increases with the time, when the heating is continued beyond the instant at which the valve in the steam supply pipe is closed.

At the moment of shutting off the steam let G be the weight of steam and water in the boiler, Gx_1 the weight of the steam present, and let the temperature be t_1 and the pressure p_1; then with continued heating of the boiler during the time ϑ the steam quantity has risen to Gx, the temperature to t, and the pressure to p; the heat quantity q, introduced into the boiler at this time, is, according to equation (2),

$$Q = G\left[q - q_1 + x_1 u_1\left(\frac{\rho}{u} - \frac{\rho_1}{u_1}\right)\right],$$

because here we have a case of heat imparted under constant volume.

[1] Compare the author's article "Neue Tabelle fur gesättigte Wasserdampfe." Schweiserische polytechnische Zeitschrift. Jahrg. 1863, Vol. VIII.

Let us in addition designate the heat quantity entering the boiler every minute by Q_m, then the time (in minutes) during which the boiler pressure rises from p_1 to p is found from

$$\vartheta = \frac{G}{Q_m}\left[q - q_1 + x_1 u_1 \left(\frac{\rho}{u} - \frac{\rho_1}{u_1}\right)\right], \quad . \quad . \quad . \quad . \quad (5)$$

and this answers the question proposed; but the preceding equation can be brought into simpler and sufficiently exact form for practical cases. In ordinary steam boilers the weight of the confined steam is so small, relatively to the water present, that the term affected by x_1, in equation (5), can be omitted; moreover for the pressure limits occurring in steam boilers the heat q of the liquid can be calculated by the formula $q = ct$, and in so doing we can substitute for the specific heat c of water the mean value $c = 1.0224$; under these assumptions we get, with sufficient accuracy,

$$\vartheta = \frac{Gc(t - t_1)}{Q_m}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

and from this we see that the time ϑ within which the boiler pressure increases from p_1 to p, after the steam discharge is interrupted, is, 1. proportional to the rise of temperature $(t - t_1)$; 2. proportional to the total weight G of steam and water, or, because the steam weight can be neglected, proportional to the weight of water present; finally, 3. this time is inversely proportional to the heat quantity Q_m which enters the boiler in a unit of time. In a series of special investigations Fairbairn (1853) [1] was the first to show that equal periods of pressure rise corresponded to nearly equal intervals of temperature, with one and the same boiler containing the same amount on each occasion; but now my formulas explain the procedure completely and point out the great influence possessed by the weight of water present and by the size of the heating surface (as shown by the value Q_m). Thus it is easily explained that in locomotive boilers which have a great heating surface and a small cubic capacity a much more rapid

[1] Compare V. Burg, "Über die Wirksamkeit der Sicherheitsventile." Sitzungsberichte der k. k. Akademie der Wissenschaften zu Wien, Vol. XLV, p. 312.

rise of steam pressure occurs when steam discharge is interrupted than in the boilers of stationary engines; for we can easily draw the additional conclusion from the preceding formulas that the time needed for a particular rise of temperature is nearly directly proportional to the volume of the steam boiler and inversely proportional to the size of its heating surface; moreover, for a particular boiler, the values of p in Col. 3, Table 1*a*, give approximately the law according to which the steam pressure increases in equal intervals of time.

The temperature increment dt demands, according to equation (5*a*), the time

$$d\vartheta = \frac{Gc}{Q_m} dt. \quad \ldots \ldots \ldots \quad (6)$$

We can call the value $\frac{dp}{d\vartheta}$ the velocity with which the steam pressure grows in a shut-off steam boiler subjected to continued heating; if we designate the velocity by w we get for it

$$w = \frac{dp}{d\vartheta} = \frac{Q_m}{Gc} \frac{dp}{dt}; \quad \ldots \ldots \ldots \quad (7)$$

it is therefore proportional to the differential coefficient of the pressure p with respect to the temperature.

The values in Col. 4 of Table 1*a*, and also those in Col. 3 of the auxiliary table on p. 62, accordingly furnish a measure for this velocity, and it is evident that it grows rapidly with the pressure.

If, in calculating the velocity w, we employ the more exact formula, we find

$$w = \frac{Q_m}{Gc} \frac{dp}{dt} \left[\frac{c}{c + x_1 u_1 \frac{d}{dt}\left(\frac{\rho}{u}\right)} \right] \ldots \ldots \quad (7a)$$

The expression in the brackets differs so little from unity that we may, without hesitation, employ the simpler equation (7) when calculating the velocity of pressure increase in steam boilers.

During the normal running of a steam boiler plant the velocity w of the pressure change should be small, and the irregularities in the stoking, i.e., variations in the value of the heat quantity Q_m,

entering the boiler in a unit of time, should have small influence on w; that is the case, according to equation (7), when the weight of water and steam in the boiler is very great and the differential coefficient $\frac{dp}{dt}$ is small. Accordingly it is far easier to run a large low-pressure boiler than a small high-pressure boiler, particularly when the latter, like a locomotive boiler, is provided with a large heating surface.

Example. Suppose an ordinary, cylindrical, steam boiler has 18 sq. m. [193.75 sq. ft.] heating surface and a capacity of 11 cbm. [388.5 cu. ft.] (the boiler corresponds to an engine of about 15 horse powers) and suppose it generates, when running normally, 25 kg. [5.12 lb.] of steam per hour per square meter [square foot] of heating surface at 5 atmospheres pressure, i.e., at a temperature $t_1 = 152.22°$ C. [274.3° F.]; suppose, moreover, that 0.6 of the boiler is filled with water and that the latter is fed at 15° C. [59° F.] temperature; then the quantity of steam generated per minute, when it is regularly supplying steam, is

$$\frac{18 \times 25}{60} = 7.5 \text{ kg.}$$

$$\left[\frac{193.75 \times 5.12}{60} = 16.533 \text{ lb.}\right].$$

If the steam is generated under the constant pressure of 5 atmospheres, from water at 0° [32°] temperature, then one kilogram [pound] demands the total heat $\lambda = q + r = q + \rho + Apu = 652.93$ (Table 11) [1175.274]; but as the feed water already possesses the temperature of $t_0 = 15°$ [59°] we must subtract from λ the corresponding value $q_0 = 15.005$ [27.009]; we get $\lambda - q_0 = 637.93$ [1148.265], and for the heat quantity Q_m, which enters the boiler per minute during the normal running of the engine, $Q_m = 7.5 \times 637.93 = 4784$ cal. [18985 B.t.u.]. The volume of water in the boiler amounts to 0.6×11, and the steam volume to 0.4×11 cbm. [155.4 cu. ft.]; since one cubic meter of water weighs 1000 kg. and one cubic meter [cubic foot] of steam of 5 atmospheres pressure (Table 11), weighs 2.75 kg. [0.17167 lb.], we find that the weight of water amounts to 6600 kg. [14550 lb.] and the steam weight to 12.1 kg. [26.68 lb.], so that the total weight $G = 6612.1$ kg. [14577.2 lb.] and the steam quality is

$$x_1 = \frac{12.10}{6612.1} = 0.00183.$$

Now let the steam supply be interrupted, but the imparting of heat to the boiler go on unchanged; in what time will the pressure then grow to 10 atmospheres?

According to Table 11, we have for the initial pressure of 5 atmospheres the values $q_1 = 153.74$ [276.732]; $u_1 = 0.3626$ [5.805]; $\rho_1 : u_1 = 1254.7$ [140.987], and for the final pressure of 10 atmospheres the table gives the values $q = 182.72$ [328.90], $\rho : u = 2293.0$ [257.66], and therefore equation (5) gives the time

$$\theta = 41.00 \text{ minutes.}$$

The approximate formula (5*a*), when the mean $c = 1.0224$ is employed and the corresponding temperatures are introduced, gives

$$\theta = 1.413\ (t - t_1) = 39.69 \text{ minutes,}$$

which is sufficiently accurate.

For the same example equation (7) gives, for the velocity of the increase of pressure,

$$w = 0.708 \frac{dp}{dt},$$

and we therefore get for the initial and the final pressure, respectively,

$w = 0.094$ for 5 [$w = 0.052$ atm. for 1° F.],
0.162 for 10 atmospheres [$w = 0.090$ atm. for 1° F.],

when we take the differential coefficients from the auxiliary table, p. 62, first dividing them by 10333 in order to get w in atmospheres.

These values give for this boiler, and for the corresponding pressures, the increment of pressure per minute, measured in atmospheres, under the supposition that, from the moment mentioned, the increment of pressure is proportional to the time.

The experiments of Fairbairn, above mentioned, which confirm the correctness of the formulas given here, were made on a locomotive boiler and led Fairbairn to the relation

$$\theta = \frac{1}{4.44}(t - t_1),$$

where the temperatures are to the Fahrenheit scale.

If we compute according to Celsius, then the time for Fairbairn's boiler is

$$\theta = 0.405(t - t_1),$$

and the velocity of increase of pressure is

$$w = 2.466 \frac{dp}{dt};$$

therefore the time θ is less and the velocity w essentially greater than the stationary boiler assumed above, which is explicable from what has been said.

The significance of the factor 0.405 of the formula was not recognized by Fairbairn; consequently when he answers a question as to the time of explosion of a boiler, on the basis of the numerical value 0.405, the result will only be valid for the boiler with which the experiments were conducted. The above-given formulas, however, solve the problem for every proposed case.

§ 13. THE CURVE OF CONSTANT STEAM WEIGHT AND ZERO CURVE OF WET STEAM.

(a) Curve of Constant Steam Weight.

If we start from a certain initial state (p_1, v_1, x_1) of the unit of weight of wet steam, then the question arises as to the course of the pressure curve *EE* (Fig. 7) when the change of state is of such a character that the steam quality x_1 remains constant. Here the specific volume for the initial state is given by

$$v_1 = x_1 u_1 + \sigma,$$

and under the assumption made the volume for any other pressure is

$$v = x_1 u + \sigma,$$

so that this at once gives us the means of following the course of the curve of constant steam weight. For $x_1 = 1$ this curve passes into the limit curve *DD*; if, as before, we designate by s the specific volume of dry steam corresponding to the pressure p, we have

$$s = u + \sigma.$$

From a combination of the last two equations, eliminating u, we get

$$v - \sigma = x_1(s - \sigma). \quad . \quad . \quad (8)$$

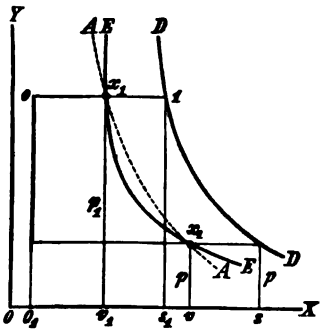

FIG. 7.

If the limit curve *DD* is represented accordingly, we can easily get the new curve for every other value of $x_1 < 1$. If we lay

off $OO_1 = \sigma$, then the abscissas for the same pressure, estimated from O_1, bear a constant ratio to each other; from the figure we have simply

$$x_1 = \frac{\overline{O_1 v}}{\overline{O_1 s}} = \frac{\overline{O_1 v_1}}{\overline{O_1 s_1}}. \quad \ldots\ldots\ldots \quad (8a)$$

By differentiation of equation (8) and multiplying both sides by p we get

$$pdv = x_1 pds;$$

then integration gives

$$L = x_1 L_1, \quad \ldots\ldots\ldots \quad (9)$$

where L_1 represents the work produced during expansion according to the limit curve, and L is the work produced for the same pressure limits when the changes of state occur according to the curve of constant steam weight.

The heat quantity Q_1, which is to be supplied during expansion, according to the limit curve DD, is found from equation (43), p. 55, to be

$$Q_1 = q - q_1 + \rho - \rho_1 + AL_1;$$

on the other hand, utilizing equation (9) we get for the curve EE

$$Q = q - q_1 + x_1(\rho - \rho_1 + AL_1),$$

from which follows by combination

$$Q = (1 - x_1)(q - q_1) + x_1 Q_1. \quad \ldots\ldots \quad (10)$$

The heat quantity Q_1, corresponding to the limit curve, may be found from the presentation on p. 70 from the formula

$$Q_1 = \int_{t_1}^{t} h dt.$$

Example. If the initial pressure is 5 atmospheres, and if the expansion is continued till the pressure has fallen to 1 atmosphere, then from the last formula we get for the limit curve (see example, p. 70)

$$Q_1 = 51.987 \text{ Cal. } [93.5766 \text{ B.t.u.}];$$

because $q_1 = 153.741$ [276.734] and $q = 100.500$ [180.9] we get, from equation (10), for the curve of constant steam weight x_1 for the same limit pressures,

$$Q = 105.288\, x_1 - 53.241 = 105.228\, (x_1 - 0.506).$$

From this we get a positive value for Q when $x_1 > 0.506$ and a negative value when $x_1 < 0.506$. In the first case heat must be supplied, in the second withdrawn, in order that at the end the steam quality may be the same as at the beginning. For the initial steam quality $x_1 = 0.506$ we get for the assumed limit pressures $Q = 0$, which is a proof that the curve of constant steam weight is intersected by an adiabatic curve AA (dotted in the figure) at the two points x_1 (Fig. 7).

(b) Zero Curve of Wet Vapors.[1]

The quantity of heat corresponding to an infinitesimal change of state of a unit of weight of wet steam, according to equation (57), p. 60, is

$$dQ = (1-x)cdt + xhdt + rdx. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Moreover, from what has been said above, we know that the Clausius temperature function

$$h = c + \frac{dr}{dt} - \frac{r}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11a)$$

is negative for vapors within certain limits, particularly for the vapor of water; it is negative in all practically occurring cases. Consequently we can conceive of a pressure curve in which the steam quality x varies in such a way that the first two terms of the right member of equation (11) are equal to zero, so that the relation

$$(1-x)c + xh = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12a)$$

obtains.

From this we derive

$$x = \frac{c}{c-h}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

[1] The designation zero curve is due to J. Weyrauch, who first called attention to the remarkable properties of this curve in his article "Zur Theorie der Dämpfe." Zeitschrift des Vereins deutscher Ingenieure, Vol. XX, 1876, p. 193.

and the corresponding specific volume

$$v=\frac{cu}{c-h}+\sigma, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

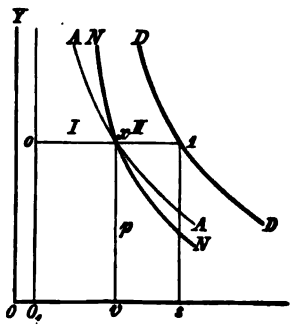

Fig. 8.

and this is the equation sought for the zero curve. As the quantities c, h, and u are given as functions of the temperature and hence of the pressure, we can compute v for every pressure and can therefore picture the curve.

In Fig. 8 let DD again represent the pressure curve and NN the zero curve.

From equation (13) follows

$$v-\sigma=\frac{cu}{c-h}.$$

For the limit curve at the same pressure

$$s-\sigma=u,$$

and from this, by division,

$$\frac{v-\sigma}{s-\sigma}=\frac{c}{c-h}.$$

If, in Fig. 8, we lay off $OO_1=\sigma$, then

$$\frac{\overline{O_1v}}{\overline{O_1s}}=\frac{c}{c-h}.$$

As the expression on the right can be computed for every temperature, we can easily construct the zero curve backwards from the limit curve.

From the intersection of the zero curve with the limit curve we have $x=1$; accordingly we get from equation (12), for this point, $h=0$. From the empirical formulas given on p. 68 for the calculation of h we can, if need be, determine the corresponding temperature t, and hence the pressure at the point of intersection.

For vapor of water in particular we get

	$p=$ 0.1	0.5	1	2	4	8	14 atm.
1.	$t=46.21°$	81.71°	100.00°	120.60°	144.00°	170.81°	195.53°
[1.]	[$t=115.18$	179.08	212.00	249.08	291.20	339.46	383.95]
2.	$c=1.0038$	1.0039	1.0130	1.0179	1.0244	1.0331	1.0420
3.	$-h=1.4942$	1.2439	1.1333	1.0209	0.9063	0.7894	0.6930
4.	$x=\frac{c}{c-h}=0.4018$	0.4479	0.4720	0.4992	0.5306	0.5669	0.6005
5.	x from (14) $=0.3999$	0.4475	0.4720	0.4996	0.5309	0.5669	0.6000

If we lay off the temperature t as abscissa and the value x (according to line 4) as ordinate, then for the vapor of water, as Weyrauch has pointed out, the points lie in nearly a straight line and we can therefore substitute

$$x=\frac{c}{c-h}=\alpha+\beta t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

indeed, in this case of the vapor of water

$$x=\frac{c}{c-h}=0.338+0.00134\,t\,[=0.338+0.000744\,(t'-32)]. \quad (14a)$$

Comparison of the values in lines 4 and 5 of the preceding tabulation shows an excellent agreement between the pressure limits 0.5 to 14 atmospheres, limits which are rarely overstepped with steam.

With the help of equation (13) we then get for the equation of the zero curve

$$v=(\alpha+\beta t)u+\sigma.$$

For the intersection of the zero curve with the limit curve we have $x=1$ and therefore $h=0$, and accordingly we get at the intersection from equation (14a) the temperature 494° C. [921.2° F.] in the case of water, provided the empirical formula used may be regarded as still reliable for the far-off temperature. Differentiation of equation (14) gives

$$dx=\beta dt.$$

For expansion according to the zero curve we get, from equation (11), with the help of (12a),

$$dQ=rdx=\beta r dt, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

which equation can be integrated since r is known as a function of t; during expansion it is necessary to withdraw heat, during compression to impart it; in the first case the steam quality diminishes, in the other it increases.

From the fundamental equation

$$dQ = dq + d(x\rho) + AdL$$

there can easily be computed the work L, if we substitute dQ from equation (16) and x from equation (14).

The zero curve has peculiar relations to the adiabatic and to the curve of constant steam weight.

From equation (11) follows, for the adiabatic curve, if we there substitute $dQ = 0$,

$$dx = -\frac{c-h}{r}\left[\frac{c}{c-h} - x\right]dt. \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The zero curve NN now divides the area within the limit curve DD into two parts I and II, Fig. 8, p. 102; for all points in space I, to the left of the zero curve, we have, for like pressure,

$$x < \frac{c}{c-h},$$

and at the right of it, in space II,

$$x > \frac{c}{c-h};$$

for the passage through the zero curve the compared values are equal.

The values $(c-h)$, r, and c are positive; if we assume adiabatic expansion, and AA in Fig. 8 the adiabatic considered, then dt is negative and, according to the preceding equation (17), the value of dx is positive in space I, while it is negative in space II; from this it follows that adiabatic expansion in space I involves evaporation, on the other hand in space II it involves condensation. If, as in Fig. 8, the adiabatic AA cuts the zero curve, then $dx = 0$ at the point of intersection, and therefore a change of sign takes place

in x; before this point evaporation occurs with expansion, after it condensation; the opposite takes place, of course, if we assume adiabatic compression.

For the curve of constant steam weight equation (11) gives

$$dQ=(c-h)\left[\frac{c}{c-h}-x_1\right]dt, \quad . \quad . \quad . \quad . \quad . \quad (18)$$

when we there make $x=x_1=$constant.

For all points in space I (for like pressure) the quantity in the brackets of the preceding equation is positive, in space II, on the other hand, it is negative. If we suppose expansion according to the curve of constant steam weight, then dt is negative; from this it follows that dQ is negative in space I and positive in space II.

Therefore during expansion, according to the curve of constant steam weight, h e a t w i t h d r a w a l is necessary in space I and h e a t s u p p l y in space II.

If the curve of constant steam weight in its course cuts the zero curve, then at the zero point itself $dQ=0$; ahead of this point heat must be withdrawn: after it heat must be imparted. During c o m p r e s s i o n, according to the same curve, the conditions are just reversed.

The circumstance that $dx=0$ at the intersection of an adiabatic with a zero curve, and also that $dQ=0$ at the intersection of a zero curve with the curve of constant steam weight, led W e y r a u c h to give the name zero curve to the curve here discussed.

The transformation of the zero curve can also be very easily effected. If we substitute, in the formula

$$AP=\tau+\frac{xr}{T}$$

for the heat weight P of wet steam, the value x from equation (12), there follows

$$AP=\tau+\frac{c}{c-h}\frac{r}{T}.$$

Now if we lay off P as abscissa and T as ordinate we get the transformation desired; in like manner the curves of constant

steam weight can be transformed, for here we have only to substitute $x=x_1$ in the last formula but one. If these transformations cut the zero curve, then, at the point of intersection, the element of the first curve will be vertical and will constitute one element in the transformation of an adiabatic curve, which latter transformation, according to an earlier presentation, is a vertical straight line.

§ 14. INVESTIGATION OF DIFFERENT HYPOTHESES CONCERNING THE EXPANSION OF STEAM IN THE CYLINDER OF THE STEAM ENGINE.

The course of the expansion curve in the indicator diagram of the steam engine has often given rise to discussion; the question is of great importance in judging the performance of existing engines and in designing new ones. The fact that no one has thus far succeeded in so representing, analytically, the course of the expansion curve that an agreement of calculations with the observations of indicator diagrams occurs, can easily be explained. In the first place there is no doubt that an exchange of heat takes place between the expanding steam and the cylinder walls, and in the second place it can be regarded as certain that the expansion curve, strictly speaking, is n o equilibrium-pressure curve.

As regards the exchange of heat mentioned, we will return to it when specially discussing the steam engine; at this place we will only emphasize the fact that certain observations (which will be discussed more fully later on), indicate that, in general, the walls extract heat from the steam during the first part of expansion, and impart heat to it toward the end of expansion. But t h i s occurrence need constitute no hindrance to expressing the course of the expansion curve analytically; it is different, however, as regards the other influence just mentioned and to which I first called attention.[1]

The steam piston moves (in the crank train) with variable velocity; at the ends of the stroke it stands still for a moment and

[1] "Kalorimetrische Untersuchung der Dampfmaschinen." Zivilingenieur, Vol. XXVII, 1881, p. 395.

has its greatest velocity near the middle of the stroke. During admission the steam acquires a stormy motion which is not quieted till towards the end of the piston stroke; therefore it will generally happen that at the end of the admission and in the first part of the expansion the steam will not act with the full equilibrium-pressure on the piston, and therefore for these positions the indicator diagram will represent the piston pressure somewhat too small, smaller than it would be if the piston moved slowly and uniformly forward. Doubtless, moreover, this stormy motion promotes this exchange of heat between steam and cylinder walls. If complete quiet is attained by the end of the piston stroke, then no loss of work is connected with the whole occurrence, because an increase of pressure takes place toward the end of the piston stroke. If we compare the course of the expansion curve, during slow and uniform piston motion, with that which obtains in crank engines, then a peculiar shifting of the expansion curve ought to occur which should be more marked, the faster the engine runs; in the first part the curve will be somewhat depressed, in the second part somewhat raised. Concerning the amount of this shifting with the speeed of the running of the engine, nothing certain can, to be sure, be said at this time. The ordinary utilization of the indicator diagram for the determination of the indicated work of a steam engine is but slightly affected by said circumstances. But the question is a different one if we wish to utilize the pressure data of the indicator diagram to work backwards and draw conclusions as to the instantaneous constitution of the steam in the cylinder; here, at any rate, the data of the diagram at the beginning of expansion cannot be regarded as perfectly reliable even if we disregard the fact that the gradual closing of the steam port at the end of admission exerts an influence on the magnitude and law of change of the steam pressure in the cylinder.

The circumstances are much more favorable for the compression curve of an indicator diagram; here, doubtless, there is always present an equilibrium-pressure curve, i.e., a reversible one; it is only necessary to compare an indicator diagram of a cold-vapor engine with that of a hot-vapor engine in order to at once recognize that the compression curve of the former has a

far more regular course than the expansion curve of the latter; if, as a matter of experiment, one could in a steam engine, at the end of the piston stroke, again compress the steam which had just expanded, and force it back into the boiler along its former path of admission, it will hardly be maintained that compression and expansion curve would cover each other even when the influence of the walls during the forward and return stroke of the piston was disregarded.

By the preceding remarks it was intended to point out in advance the reasons explaining the fact that the different hypotheses hitherto set up for the course of expansion curves all depart more or less from the course of really observed curves.

First Hypothesis. Polytropic Curve.[1]

Let us suppose a unit of weight of wet steam to expand from the volume v_1 to the volume v, the pressure changing at the same time from p_1 to p and let the equation of the expansion curve be

$$pv^n = p_1 v_1^n, \quad \ldots\ldots\ldots \quad (19)$$

then the work of expansion L, when the exponent n is a constant, is found from the formula [2]

$$L = \frac{1}{n-1}[p_1 v_1 - pv]. \quad \ldots\ldots\ldots \quad (20)$$

We have specially for the vapor of water, to which we just now confine our attention, the formula

$$ps^{n_1} = p_1 s_1^{n_1} \quad \ldots\ldots\ldots \quad (21)$$

for the relation between the pressure p and the volume s, i.e., we have the equation of the upper limit curve, a relation which has already been given on p. 37 as an approximate formula, but one proved to be sufficiently exact within the pressure limits occurring in practice; here we must take the exponent $n_1 = 1.0646$. There-

[1] See Vol. 1, p. 151. [2] See Vol. 1, p. 154.

fore, within said practical limits, this limit curve is to be regarded as a polytropic curve.

If we assume, in addition, that while the steam in the cylinder is wet, the steam weight present is always greater than the weight of the water present so that $x > 0.5$, then we can neglect the minute quantity σ in the formulas

$$v = xu + \sigma \quad \text{and} \quad s = u + \sigma,$$

and get in place of the two equations (19) and (21)

$$p(xu)^n = p_1(x_1u_1)^n \quad \text{and} \quad pu^{n_1} = p_1u_1^{n_1}. \quad . \quad . \quad (21a)$$

If we eliminate u and u_1 from the two equations and at the same time consider equation (19), we get

$$\frac{x}{x_1} = \left(\frac{p_1}{p}\right)^{\frac{n_1-n}{n_1n}} \quad \text{or} \quad \frac{x}{x_1} = \left(\frac{v}{v_1}\right)^{\frac{n_1-n}{n_1}}. \quad . \quad . \quad . \quad (22)$$

Accordingly, starting from a certain initial condition (p_1, v_1) we can compute, for any other value of p or v, the steam quality x, provided its initial value x_1 is given.

From the second of the preceding equations we recognize at once whether evaporation or condensation accompanies the expansion taking place according to the polytropic curve contemplated. Thus, if $n < n_1$, i.e., $n < 1.0646$, then we get $x > x_1$, i.e., the steam quality grows, a part of the water present is evaporated; conversely condensation occurs during expansion for $n > n_1$.

It is now easy to settle whether, in the proposed case, expansion involves a supply or withdrawal of heat.

From equation (20) follows that, because $v = xu$,

$$AdL = -\frac{Ad(pux)}{n-1},$$

and from the fundamental equation

$$dQ = dq + d(x\rho) + AdL$$

we get directly

$$dQ = dq + d\left[x\left(\rho - \frac{Apu}{n-1}\right)\right], \quad . \quad . \quad . \quad . \quad . \quad (23)$$

integrating which gives

$$Q = q - q_1 + x\left(\rho - \frac{Apu}{n-1}\right) - x_1\left(\rho_1 - \frac{Ap_1u_1}{n-1}\right), \quad . \quad . \quad (24)$$

in which equation we substitute the value of x from one of equations (22). According as Q is positive or negative, heat must be imparted or withdrawn.

If we are not dealing with the unit of weight, so that G is the weight of the expanding mixture of steam and liquid, it is simply necessary to multiply the right member of equation (24) by G; if, in so doing, we consider the relations

$$Gx_1u_1 = V_1 \text{ and } Gxu = V, \quad . \quad . \quad . \quad . \quad (25)$$

there follows

$$Q = G(q - q_1) + V\left(\frac{\rho}{u} - \frac{Ap}{n-1}\right) - V_1\left(\frac{\rho_1}{u_1} - \frac{Ap_1}{n-1}\right). \quad . \quad . \quad (26)$$

In this formula V_1 represents the cylinder volume of the steam at the beginning of expansion and V the cylinder volume at any other position of the piston, that is, includes the volume of the clearance space in both cases.

In practically utilizing this equation, however, a special difficulty is encountered with respect to the determination of the weight G of the confined steam quantity. This weight G is not identical with the weight of the mixture per piston stroke supplied by the steam boiler, but also includes the weight of the mass that has remained over in the hurtful space from the preceding stroke, and the determination of this weight cannot be effected with certainty at the present time, because no means have been found thus far for determining the quantity of water which is present with the steam in the hurtful space.

Of course, the preceding equation (26) is valid only under the express assumption that the expansion curve can be regarded as a

polytropic curve, and that the exponent n is really the same for all parts of the curve of the indicator diagram. To test this matter we ascertain from the indicator diagram the volume V and the corresponding pressure p for a series of points. If for one of the intervals V' and V'' represent the initial and final volumes and p' and p'' the corresponding pressures, then, according to equation (19) we find

$$n=\frac{\log p'-\log p''}{\log V''-\log V'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

If this quantity is found to be the same, or nearly the same, for all intervals, then the assumption that the curve of expansion is a polytropic curve is a permissible one.

Only a few investigations in the direction indicated have been published. Leloutre[1] holds that the assumption of a constant value n for the expansion curve is permissible, but concludes, from experiments, that n is different for different engines, according to the height of the admission pressure and the ratio of expansion.

Lüders[2] finds from indicator diagrams furnished by Bissinger[3] that the value n is, throughout, smaller than 1, but diminishes greatly with the expansion (varying in the special case mentioned from $n=0.903$ to $n=0.535$); a like result is given by my own investigations,[4] utilizing the experimental results found by Hallauer from a Corliss engine (here the value of n varied from $n=0.900$ to $n=0.436$).

For each of the two cases of special investigation referred to, the expansion curve did not prove to be a polytropic curve, but because in all cases $n<1.0646$ we must conclude, from equation (22), that evaporation of the water present took place during the whole course of expansion.

---

[1] Leloutre, "Recherches expérimentales" (Bulletin de la société industrielle du Nord de la France, 1874).

[2] Lüders, "Zur Theorie des Indikatordiagrammes." Zivilingenieur, 1881, Vol. XXVII, p. 225.

[3] Bissinger, "Versuche über den Nutzen des Dampfmantels." Zivilingenieur, 1874, Vol. XX, p. 339.

[4] "Kalorimetrische Untersuchungen," etc. Zivilingenieur, 1881, Vol. XXVII, p. 405.

It is possible that in certain cases the investigation of other indicator diagrams may show polytropic expansion.

Second Hypothesis. Pambour.

In his formerly highly esteemed treatise on the steam engine, Pambour starts from the assumption that the steam in the cylinder of the steam engine is dry saturated, and that it also remains dry during the expansion and the pressure diminution connected therewith; in other words, he assumes that the steam neither condenses nor passes over into the superheated condition. According to our way of expressing the matter, Pambour assumed that the steam expanded according to the limit curve.

From what has been said above concerning the vapor of water within the pressure limits occurring in practice, this limit curve is also a polytropic curve, only we must assume for it $n=n_1=1.0646$; the formulas (22) to (26), developed above, therefore apply here; equation (22) only gives $x=x_1$, while according to Pambour's view $x=x_1=1$ for the whole course of expansion.

Moreover, it is evident that the substitution of $n=n_1$ in equation (22) leads directly to the curve of constant steam weight, and that therefore such a curve is of the same species as the limit curve.

Accordingly, for the curve of constant steam weight we have, because $x=x_1$,

$$p_1 v_1^{n_1} = p v^{n_1}, \quad \ldots\ldots\ldots \quad (19a)$$

and according to equation (20) the work produced per unit of weight is

$$L = \frac{x_1}{n_1 - 1}[p_1 u_1 - p u] \quad \ldots\ldots\ldots \quad (20a)$$

and the heat to be supplied is

$$Q = q - q_1 + x_1\left[\left(\rho - \frac{Apu}{n_1-1}\right) - \left(\rho_1 - \frac{Ap_1u_1}{n_1-1}\right)\right] \ldots \quad (24a)$$

By means of these formulas the propositions are extended which were found above for the curve of constant steam weight, but in so doing we must not overlook the fact that the formulas given here are approximate formulas which are valid only so long as it is permissible to neglect the specific volume σ of the water relatively to the volume $x_1 u$ of the steam contained in the unit of weight.

If we substitute $x_1 = 1$ in the preceding equations we get the quantities mentioned for expansion according to the limit curve; here, for the purpose of greater accuracy, we replace u_1 and u by v_1 and v respectively; the work therefore is

$$L = \frac{1}{n_1 - 1}[p_1 v_1 - pv], \quad \ldots\ldots\ldots \quad (20b)$$

and the heat to be imparted

$$Q = (q + \rho) + (q_1 - \rho_1) + AL. \quad \ldots\ldots\ldots \quad (24b)$$

Pambour, of course, could not see that the here assumed expansion required heat to be supplied; he calculated the work of the steam by another formula.

Thus, according to Navier, the specific weight of dry saturated steam was formerly determined from the formula

$$\gamma = \beta(a + p),$$

where β and a are certain constant quantities which may be taken different for the cases of high pressure and low pressure. From the relation $v\gamma = 1$ Pambour found the equation of the expansion curve (limit curve) to be

$$(a + p)v = (a + p_1)v_1, \quad \ldots\ldots\ldots \quad (28)$$

and hence this curve should be an equilateral hyperbola in which the horizontal asymptote lies at a distance a below the axis of abscissas, and the vertical asymptote coincides with the axis of ordinates.

Utilizing equation (28), the work of expansion, according to Pambour, becomes

$$L=\int_{v_1}^{v} pdv=(a+p_1)v_1 \log_e \frac{v}{v_1}-a(v-v_1), \quad . \quad . \quad . \quad (29)$$

while this work is more correctly found, from a combination of equations (22) and (19), to be

$$L=\frac{p_1v_1}{n_1-1}\left[1-\left(\frac{v}{v_1}\right)^{n_1-1}\right].$$

Engineers, very properly, no longer make any use of Pambour's formulas.

Third Hypothesis. Poncelet.

Poncelet assumes that the curve of expansion is an equilateral hyperbola whose asymptotes coincide with the axes of coordinates; he simply makes

$$pv=p_1v_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Accordingly the curve is a polytropic one with the exponent $n=1$ The work of expansion is then found from

$$L=\int_{v_1}^{v} pdv=p_1v_1 \log_e \frac{v}{v_1}, \quad . \quad . \quad . \quad . \quad . \quad (31)$$

or from

$$L=p_1v_1 \log_e \frac{p}{p_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31a)$$

These are the formulas which, at the present time, are almost universally used in steam-engine calculations, and there is a certain justification in so doing because, as a matter of fact, the expansion curve of the indicator diagram has a course which never deviates much to the one side or the other from the equilateral

hyperbola. It therefore seems proper to test this curve also in a thermodynamic way.

Provided the steam is wet at the beginning, equation (30) must be written in the form

$$p(xu+\sigma)=p_1(x_1u_1+\sigma), \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

or, if we again neglect the value σ because it is so small, we get

$$pxu=p_1x_1u_1,$$

or

$$x=\frac{Ap_1u_1}{Apu}x_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32a)$$

In the case of the vapor of water the value Apu diminishes with the pressure, hence we have $x>x_1$.[1] Poncelet's hypothesis leads to the proposition that evaporation is connected with expansion; therefore if the steam was dry at the beginning, or became dry during the course of the expansion, then it would be superheated at the end of the expansion.

During this kind of expansion it is evident that heat must be imparted to the expanding steam, i.e., heat must be given off to the cylinder walls. The quantity of heat Q, absorbed by the unit of weight of mixture, is determined by equation

$$Q=q-q_1+x\rho-x_1\rho_1+AL, \quad . \quad . \quad . \quad . \quad . \quad (33)$$

where x is determined by equation (32a) and L by equation (31). The utilization of these equations of course demands the knowledge of the steam quality x_1 at the beginning of expansion, a question which we will discuss again when considering steam engines.

[1] Upon p. 85 there is given an expression similar to equation (32a), but the fraction in the right member is there inverted and, according to Charpentier it represents the ratio of the steam qualities during adiabatic expansion.

§ 15. CONTINUATION OF THE INVESTIGATION OF DIFFERENT CURVES OF EXPANSION.

Fourth Hypothesis. Adiabatic Curve.

If we could neglect as insignificant the influence of the cylinder walls on the expanding steam, then the expansion curve would coincide with the adiabatic line.

We have already shown on p. 81 that for wet steam the course of the adiabatic curve is represented with sufficient accuracy by the equation

$$pv^n = p_1 v_1^n,$$

where the exponent n is given by equation (76), p. 83, namely, by

$$n = 1.035 + 0.1\, x_1,$$

where x_1 is the initial steam quality and is confined between 0.7 and 1.

Therefore, under the assumptions made, the curve again appears as a polytropic one, only here we have $n > n_1$ between the given limits of x_1, and accordingly equation (22) is written in the form

$$\frac{x}{x_1} = \left(\frac{v_1}{v}\right)^{\frac{n-n_1}{n_1}} = \left(\frac{p}{p_1}\right)^{\frac{n-n_1}{nn_1}}.$$

It follows from this that with growing volume the steam quality diminishes and condensation therefore accompanies the expansion, as was already pointed out above in our fuller discussion of the adiabatic expansion of wet vapors.

The work of expansion of the unit of weight of wet steam should then be calculated by the formula

$$L = \frac{p_1 v_1}{n-1}\left[1 - \left(\frac{v_1}{v}\right)^{n-1}\right].$$

The measurement of indicator diagrams has shown that, in general, the real expansion curve sinks less rapidly than the adiabatic curve, even when we assume much moisture at the outset of

expansion; we are therefore compelled to conclude that the cylinder walls give off heat to the expanding steam, if we wish to hold fast to the assumption that the expansion curve be regarded as an equilibrium-pressure curve; but if we concede the possibility that we have here to do with the non-reversible change of pressure, as was suggested in the introductory remarks to § 14, on p. 107, and that consequently the expansion curve is depressed at the beginning and raised toward the end, then it is by no means necessary to explain the occurrences solely by heat exchange between cylinder walls and steam. In reality both actions probably occur simultaneously.

Fifth Hypothesis. Expansion when Heat Supply is Proportional to Change of Temperature.

If the quantity of steam and liquid confined in the cylinder is G kg. [lb.] the heat quantity dQ, absorbed during an infinitesimal change of state is, according to equation (55), p. 59,

$$dQ = GTd\left(\tau + \frac{xr}{T}\right).$$

If we suppose this mass to be in contact with a body having the weight G' and whose temperature at every instant is equal to that of the mass of steam and whose specific heat has the constant value c, then we also have

$$dQ = -cG'dT. \quad \ldots\ldots\ldots \quad (34)$$

After equating these two expressions and substituting, for the sake of simplicity,

$$\frac{G'}{G} = \mu, \quad \ldots\ldots\ldots \quad (35)$$

we get

$$-\mu c dT = Td\left(\tau + \frac{xr}{T}\right),$$

or

$$d\left(\mu c \log_e T + \tau + \frac{xr}{T}\right) = 0,$$

and from this, by integration,

$$\mu c \log_e T + \tau + \frac{xr}{T} = \mu c \log_e T_1 + \tau_1 + \frac{x_1 r_1}{T_1}, \quad . \; . \quad (36)$$

while the combination of equations (34) and (35) gives the transferred heat quantity

$$Q = \mu c G(T_1 - T). \quad . \; . \; . \; . \; . \; . \; . \quad (37)$$

For the special assumption that the body G', which delivers heat, is water, the factor c will represent the variable specific heat of water, and equation (34) can be written in the form $dQ = -G'dq$. Equation (36), on account of the meaning given to τ, takes the form

$$(\mu+1)\tau + \frac{xr}{T} = (\mu+1)\tau_1 + \frac{x_1 r_1}{T_1}, \; . \; . \; . \; . \quad (36a)$$

and equation (37) becomes

$$Q = \mu G(q_1 - q). \; . \; . \; . \; . \; . \; . \; . \quad (37a)$$

Under the supposition that the ratio μ is given, we can compute for any pressure p the steam quality x, from equation (36) or (36*a*), and can compute from equation (37) or (37*a*) the heat quantity which thus far has been imparted to the expanding steam. Equation (36) or (36*a*) represents a whole host of expansion curves of the same species, according to the value taken for μ; for $\mu = 0$ the adiabatic curve appears.

The work of expansion, measured in units of heat, is determined by the formula

$$AL = Q - G[q - q_1 + x\rho - x_1\rho_1], \quad . \; . \; . \; . \quad (38)$$

where Q is to be substituted from equation (37) or (37*a*).

Now in order to ascertain whether or not the here discussed curve of expansion is one which occurs in steam engines, we must measure the volume V and the pressure p for its different points and determine the value x from the relation

$$V = G(xu + \sigma),$$

and substitute it in equation (36) or (36*a*). If the ratio μ should really prove to be constant, or nearly constant, for the different points, this would show that the here assumed course of the curve of expansion is the one before us.

Emil Herrmann[1] believes the present hypothesis may be regarded as confirmed on the basis of a closer investigation of the indicator diagrams of two different engines. Hirn also, who considered the question earlier, comes to the same conclusion. My own investigations,[2] on the basis of the course of an expansion curve found by Hallauer with a Corliss engine, do not confirm this, but rather indicate that the quantity μ, occurring in the preceding formulas, experiences a very marked increase with increasing expansion.

According to all the observations thus far presented the course of the expansion curve is not reproduced with sufficient accuracy for all cases by any one of the five hypotheses just discussed. In the one case one of the hypotheses, and in the other case another, brings about a better agreement. A perfectly clear rendering of the course of the curve is hardly to be expected from the analytical treatment; that the curve will have a different course in the case of a cylinder with a steam jacket than in the case of one without a steam jacket seems at once self-evident. Moreover the following circumstances will be of influence: the height of the initial pressure, the degree of expansion, the moisture in the steam at the end of admission, the velocity with which the crank turns, the kind and mode of action of the valve gear, etc.

For the purpose of designing new engines, Poncelet's assumption will be taken as a basis for a long time to come, if for no other reason than that it leads to the simplest formulas for the work of expansion and compression; on the other hand, in purely theoretical investigations, as in the subsequent ones here given, adiabatic changes of state will be assumed, because Thermodynamics demands such changes for the perfect (Carnot) cycle of steam engines.

[1] Kompendium der mechanischen Wärmetheorie. Berlin, 1879, p. 214.
[2] Zivilingenieur, Vol. XXVII, p. 405.

II. Non-reversible Change of State of Wet Vapors.

§ 16. EXPANSION OF WET VAPORS UNDER DIFFERENT CIRCUMSTANCES.

In § 15, p. 74, of the first volume, the non-reversible operation was treated in a general fashion, and then, in § 23, p. 128, Vol. I, the behavior of gases in this respect was subjected to investigation. The propositions there established can easily be transferred to the case presented here.

In Fig. 9 let a unit of weight of wet steam be confined in a cylinder by a piston K; let pressure, volume, and temperature be given by p_1, v_1, and T_1, and let x_1 be the steam quality. Now if the pressure acting from without against the piston is suddenly reduced from p_1 to p_1', then the equilibrium is disturbed, and the piston is pushed back by the steam mass and performs work; the law to which the external pressure is thus subjected is determined by the curve *acb*, the working-pressure curve.

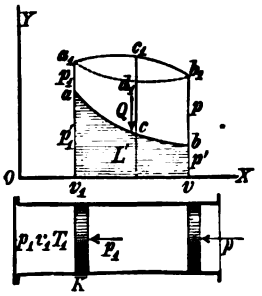

FIG. 9.

If after the expansion from v_1 to v the piston (supposed to be without weight) is suddenly held fast, and if p' is the external pressure at the instant, then the stormily moved mass will gradually pass into a state of equilibrium, the internal pressure will grow from p' to the equilibrium-pressure p, and this pressure will correspond to the equilibrium-temperature t; the steam quality will settle itself to a particular value x, which satisfies the equation

$$v = xu + \sigma, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

while in the initial state of equilibrium

$$v_1 = x_1 u_1 + \sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

obtained.

Points a_1 and b_1 lie in the equilibrium-pressure curve $a_1c_1b_1$ (Fig. 9).

The introduction of the whole procedure occurs here by "release," and indeed this takes place by the sudden diminution of the initial external pressure p_1 to the value p_1'.

The external work L' thus performed is given by the area (hatched in the figure) lying below the working pressure curve *acb*, and is determined by

$$L' = \int_{v_1}^{v} p' dv, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where p' must be regarded as given by a function of v. The change of inner work $U - U_1$ is determined from the equilibrium-pressure curve

$$A(U - U_1) = (q + x\rho) - (q_1 + x_1\rho_1),$$

and hence there follows, for the heat quantity Q' which is to be supplied during this non-reversible operation,

$$Q' = (q + x\rho) - (q_1 + x_1\rho_1) + AL', \quad . \quad . \quad . \quad . \quad (4)$$

or, passing to the differential,

$$dQ' = dq + d(x\rho) + Ap'dv. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The preceding formulas now give the solution of the problems presented, but in so doing it is not possible to determine the temperature and the steam quality existing immediately at the end of the occurrence.

The preceding equations were developed under the assumption that both the curves are given, the working-pressure curve *acb* and the equilibrium-pressure curve $a_1c_1b_1$, and also that the external work L' and the supplied heat Q' are to be determined. If the two curves coincide, that is if $p' = p$, along the whole path of the change of state, then the r e v e r s i b l e operation is before us, which was fully treated in the article above, and can only be considered as a special case of the present general one.

The formulas given here can easily be brought into other forms with the help of the relations developed before for the reversible operation; the general presentation in Vol. I, § 15, p. 74, suggests the necessary procedure; there was also fully discussed the transformation of non-reversible changes of state. We must add that the equations given here, of course, lose their validity when the steam becomes superheated during the change of state.

First Special Case. Non-reversible Isothermal Expansion.

At constant equilibrium-temperature $t=t_1$, the equilibrium-pressure curve $a_1c_1b_1$ is a horizontal straight line, because the pressure $p=p_1$ becomes constant.

From equations (1) and (2) can then be found

$$v-v_1=(x-x_1)u_1, \quad \ldots\ldots\ldots \quad (6)$$

and equation (4) gives

$$Q'=(x-x_1)\rho_1+A\int_{v_1}^{v} p'dv. \quad \ldots\ldots \quad (7)$$

According to the manner in which the external pressure p' varies with the volume v, we shall here have an infinite number of special cases.

Now selecting only one case, let us assume that the external pressure p' is kept constant during expansion; then the working pressure curve is the horizontal *acb*.

The external work L can then be found by

$$L'=Ap'(v-v_1)=Ap'u_1(x-x_1), \quad \ldots\ldots \quad (8)$$

and the heat to be supplied from

$$Q'=(x-x_1)(\rho_1+Ap'u_1)=(v-v_1)\left(\frac{\rho_1}{u_1}+Ap'\right). \quad \ldots \quad (9)$$

If, in a special case, $p'=0$, for example if wet steam expands from the space of volume v_1 into a vacuum having the volume $v-v_1$, then we have

$$L'=0 \quad \text{and} \quad Q'=(x-x_1)\rho_1=(v-v_1)\frac{\rho_1}{u_1},$$

but here, according to equation (6), the condition

$$v-v_1\leq(1-x_1)u_1$$

must be satisfied in order that no superheating whatever may obtain. Finally, if equilibrium-pressure p_1 is identical with the external pressure p', then the reversible cycle will be before us; equations (8) and (9) then give the formulas

$$L'=Ap_1u_1(x-x_1) \quad \text{and} \quad Q'=(x-x_1)r_1,$$

which were established above when investigating steam formation under constant pressure along a reversible path.

Second Special Case. Non-reversible Isodynamic Expansion.

In the case presented, the equilibrium pressure-curve is the isodynamic curve in which $U=U_1$ holds. Hence there follows, from the just established equations,

$$q+x\rho=q_1+x_1\rho_1, \quad \ldots\ldots \quad (10)$$

and, according to equation (4),

$$Q'=AL'=A\int_{v_1}^{v}p'dv.$$

Here again there exist an infinite number of special cases according to the variability of the external pressure p'.

For example, if p' is constant, there follows

$$Q'=AL'=Ap'(v-v_1)=Ap'(xu-x_1u_1), \quad \ldots \quad (11)$$

which equation as well as equation (10) remains valid only so long as $x \leq 1$, or so long as

$$v - v_1 \leq u - x_1 u_1$$

obtains.

Third Special Case. Non-reversible Adiabatic Expansion.

In the present case, because heat must be neither supplied nor withdrawn, we have $Q' = 0$; and therefore, according to equation (4),

$$A \int_{v_1}^{v} p' dv = (q_1 + x_1 \rho_1) - (q + x\rho). \quad . \quad . \quad . \quad . \quad (12)$$

Among the infinite number of special cases which can arise, according to the law of change of the external pressure p', we will again pick out the special one in which p' is constant. Here we find

$$L' = p'(v - v_1) = p'(xu - x_1 u_1), \quad . \quad . \quad . \quad . \quad (13)$$

and hence, according to equation (12),

$$q + x(\rho + Ap'u) = q_1 + (x_1 \rho_1 + Ap'u_1). \quad . \quad . \quad . \quad (14)$$

When, for the initial condition, there are given p_1, x_1, and also the constant external pressure p', we can determine, from the preceding equation, for any assumed equilibrium-pressure $p < p_1$, the corresponding steam quality x, and then, according to equations (1) and (2), p. 120, the volume v and the initial volume v_1, and from these can be laid off the adiabatic curve (equilibrium curve) presented here.

As this equilibrium-pressure p constantly diminishes, while the external pressure p' remains constant, we have before us a certain expansion ratio $v : v_1$ for which $p = p'$.

If we regard p' as given, then for the terminal value $p = p'$ in equation (14) there is also known $q' = q$, $\rho' = \rho$, $u' = u$; consequently the corresponding value $x' = x$ is obtained from

$$q' + x' r' = q_1 + x_1 \left(\rho_1 + Ap_1 u_1 \frac{p'}{p_1} \right), \quad . \quad . \quad . \quad . \quad (14a)$$

and then the corresponding value $v' = x'u' + \sigma$ and the corresponding external work

$$L' = p'(v' - v_1) = p'(x'u' - x_1 u_1). \quad . \quad . \quad . \quad . \quad (13a)$$

Example 1. Suppose a unit of weight of wet steam to expand adiabatically from the initial pressure of five atmospheres, overcoming, in so doing, a constant external pressure of one atmosphere and to continue this till the equilibrium-pressure at the end is equal to two atmospheres; then, according to Table 11 of the Appendix,

$$\begin{array}{llll} \text{for } p_1 = 5 \times 10333, & q_1 = 153.741, & \rho_1 = 454.994, & u_1 = 0.3626 \\ [\text{for } p_1 = 5 \times 2116.31, & q_1 = 276{,}734, & \rho_1 = 818.989, & u_1 = 5.7951], \\ \text{for } p = 2 \times 10333, & q = 121.417, & \rho = 480.005, & u = 0.8589 \\ [\text{for } p = 2 \times 2116.31, & q = 218.551, & \rho = 864.009, & u = 13.7586], \end{array}$$

and equation (14) gives, because $p' = 1 \times 10333$ [1×2116.31],

$$x = 0.0645 + 0.9259\, x_1.$$

The inital volume is

$$v_1 = x_1 u_1 + \sigma = 0.3626\, x_1 + 0.0010$$
$$[v_1 = 5.7951\, x_1 + 0.016].$$

The final volume is

$$v = xu + \sigma = 0.7953\, x_1 + 0.0564$$
$$[v = 12.7391\, x_1 + 0.9034],$$

and the external work performed, according to equation (13), is

$$L' = 4471.1\, x_1 + 572.4 \text{ mkg.}$$
$$[L' = 14696\, x_1 + 1878.1 \text{ ft-lb.}]$$

If the steam is dry saturated at the beginning so that $x_1 = 1$, there follows, from the preceding formulas,

$$\begin{array}{llll} x = 0.9904, & v_1 = 0.3636, & v = 0.8517, & L' = 5043.5 \\ [x = 0.9904, & v_1 = 5.8245, & v = 13.6433, & L' = 16547]. \end{array}$$

As $x < x_1$ here, the assumed change of state involves condensation; on the other hand evaporation would have taken place if $x > x_1$ had obtained or $x_1 < 0.870$ had been assumed.

Example 2. Suppose a unit of weight of wet steam to expand adiabatically from an initial pressure of five atmospheres, overcoming, in so doing, a constant external pressure of one atmosphere, and let this continue until the equilibrium-pressure at the end has become equal to the external pressure, then, with the help of the tables of the Appendix, we

determine from equation (14*a*) the steam quality x' at the end, by the formula

$$x' = 0.8645\, x_1 + 0.0992,$$

and the external work L', according to equation (13*a*), by

$$L' = 10988.1\, x_1 + 1690.5 \text{ mkg.}$$
$$[L' = 36051\, x_1 + 5546.3 \text{ ft-lb.}].$$

If the steam is dry saturated at the beginning, we get, from the preceding formulas,

$$x' = 0.9637 \quad \text{and} \quad L' = 12678.5$$
$$[x' = 0.9637 \quad \text{and} \quad L' = 41597],$$

consequently there will again be condensation; hence the steam must have been superheated at the beginning if it is just dry saturated at the end, in other words, if we are to have $x' = 1$ at the end; for in this case the preceding formula gives for $x' = 1$ the result $x_1 = 1.042$, therefore $x_1 > 1$, which impossible condition simply indicates superheating.

Equations (13) and (14) given above are also naturally valid under the assumption that no steam is present at the beginning, but only liquid, provided the latter is under a pressure that is equal to the steam pressure corresponding to its temperature; in this case $x_1 = 0$ and therefore, according to equation (13), the external work is

$$L' = p' x u, \quad \ldots\ldots\ldots \quad (13b)$$

where x is determined according to equation (14), from the relation

$$x(\rho + A p' u) = q_1 - q. \quad \ldots\ldots\ldots \quad (14b)$$

If expansion here continues till the equilibrium-pressure at the end becomes identical with the constant external pressure, then the steam quality x' at the end can be computed according to equation (14*a*):

$$x' = \frac{q_1 - q'}{r'}, \quad \ldots\ldots\ldots \quad (14c)$$

and the external work from equation (13*a*):

$$L' = (q_1 - q')\frac{p' u'}{r'} \ldots\ldots\ldots \quad (13c)$$

The last equations correspond to the occurrence which is ordinarily designated in the older technical papers as "spontaneous evaporation"; there, however, they found a theoretical explanation which the thermodynamics of the present day does not regard as correct and which does not at all correspond to the preceding formulas.

Example. Suppose a unit of weight of water to be under a pressure of 10 atmospheres, and to have a temperature $t_1 = 180.31°$ C. [356.56° F.], which corresponds to the steam pressure of 10 atmospheres (Table 11 of the Appendix); now let the external pressure be suddenly brought to one atmosphere and suppose it to be maintained constant during the change of state (evaporation) then following. Here we have $q_1 = 182.719$ cal. [328.894 B.t.u.] and $q' = 100.500$ cal. [180.9 B.t.u.], corresponding to the pressure of one atmosphere, and $r' = \rho' + Ap'u' = 536.50$ [965.7], and hence we find the steam quality x' from equation (14c) to be

$$x' = 0.1717,$$

and the external work, according to equation (13c),

$$L' = 2926 \text{ mkg.}$$
$$[L' = 9600 \text{ ft-lb.}]$$

The volume $v_1 = \sigma = 0.0010$ [0.01602] increases to $v = x'u' + \sigma = 0.2832$ cbm. [4.537 cu. ft.].

This case occurs when a steam boiler explodes under the pressure of 10 atmospheres without supposing any work to be expended in destroying the boiler. Disregarding the steam present in the boiler, for here the external atmospheric pressure is overcome, there will remain behind the quantity of water

$$(1 - x')\, G = 0.8283\, G,$$

provided G kg. [lb.] of water were originally present.

In conclusion we will point out that equations (13) and (14), given above, also include the case of expansion of wet steam into a vacuum; here $p' = 0$, consequently $L' = 0$ and so, according to equation (14),

$$x = \frac{q_1 + x_1\rho_1 - q}{\rho}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

from which the steam quality x is determined when the pressure p, at the end of the spreading of the steam in the vacuum, is known.

The initial volume $v_1 = x_1 u_1 + \sigma$ may be assumed as known; now if the final volume $v = xu + \sigma$ is given by the capacity $v - v_1$ of the empty space in which the expansion takes place, combination with equation (15) will give

$$\frac{\rho}{u} = \frac{q_1 + x_1 \rho_1 - q}{v - \sigma},$$

and in addition the corresponding terminal pressure p, and finally, by equation (15), the steam quality x; or, conversely, if the terminal pressure p, after the spreading and after the passage into the equilibrium condition, is given, there can be found the volume at the end $v = xu + \sigma$ and hence the volume of the vacuum $v - v_1$.[1]

Example. Suppose a unit of weight of wet steam to expand adiabatically from five atmospheres into a vacuum and finally, after equalization of pressure, to take a pressure of one atmosphere.

Here, according to Table 11 of the Appendix,

$$q_1 = 153.741, \quad \rho_1 = 454.994, \quad \rho = 496.300, \quad \text{and} \quad q = 100.500$$
$$[q_1 = 276.734, \quad \rho_1 = 818.989, \quad \rho = 893.340, \quad \text{and} \quad q = 180.90],$$

and hence, according to equation (15),

$$x = 0.1073 + 0.9168\, x_1, \quad \ldots\ldots\ldots \quad (15a)$$

and because $u = 1.6495$ [26.4232] the volume at the end becomes

$$v = xu + \sigma = 0.1780 + 1.5123\, x_1$$
$$[v = 2.8512 + 24.2248\, x_1],$$

while the initial volume amounts to

$$v_1 = x_1 u_1 + \sigma = 0.0010 + 0.3626\, x_1$$
$$[v_1 = 0.01602 + 5.7951\, x_1];$$

therefore the capacity of the initially empty space is

$$v - v_1 = 0.1770 + 1.1497\, x_1$$
$$[v - v_1 = 2.8352 + 18.4297\, x_1].$$

---

[1] Compare with the author's article "Beiträge zur Theorie der Dämpfe" in Poggendorff's Annalen, 1860.

If only dry saturated steam is on hand at the beginning so that $x_1 = 1$, the preceding formula will give for x, the value

$$x = 1.0241,$$

which points to the fact that dry saturated steam passes into the condition of superheated steam when the former expands into a vacuum; consequently for this case the given formulas lose their validity.

If, in the proposed example, the steam is just dry saturated after it has finished spreading in the vacuum so that $x = 1$, then, according to equation (15*a*), this presupposes the steam quality $x_1 = 0.9737$ at the beginning. In this case, according to the preceding formulas, the initial volume is $v_1 = 0.3541$ [5.6723], the final volume is $v = 1.6505$ [26.4392], the volume in the initially empty space is $v - v_1 = 1.2964$ cbm. [20.7669].

§ 17. MIXTURE OF WET VAPORS OF THE SAME KIND.

Let the two spaces V_1 and V_2 (Fig. 10) contain steam of the same kind; in space V_1 let there be confined G_1 kg. [lb.] of pressure p_1 and the steam quality x_1; in space V_2, separated by the partition ab from the former space, let there be enclosed G_2 kg. [lb.] of steam having the pressure p_2 and the steam quality x_2.

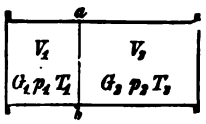

FIG. 10.

Then the following relations exist:

$$V_1 = G_1(x_1 u_1 + \sigma) \quad \text{and} \quad V_2 = G_2(x_2 u_2 + \sigma). \quad . \quad . \quad (16)$$

The heat contents J_1 in the first space, i.e., the excess of its heat over that in G kg. [lb.] of water of the temperature 0° C. [32° F.], amounts to

$$J_1 = G_1(q_1 + x_1 \rho_1), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and the contents in the second space likewise amount to

$$J_2 = G_2(q_2 + x_2 \rho_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17a)$$

If we now suppose the partition ab to be removed, a mixture occurs, and after equilibrium is established and heat has neither been supplied nor withdrawn, there will result a mean pressure p and a steam quality x which can be easily calculated.

After mixture the volume will be

$$V=(G_1+G_2)(xu+\sigma),$$

and the heat contents

$$J=(G_1+G_2)(q+x\rho).$$

But, according to the assumptions made,

$$V=V_1+V_2$$

and

$$J=J_1+J_2,$$

so that we get with the utilization of equations (16) and (17)

$$(G_1+G_2)xu=G_1x_1u_1+G_2x_2u_2$$

and

$$(G_1+G_2)(q+x\rho)=G_1(q_1+x_1\rho_1)+G_2(q_2+x_2\rho_2),$$

or

$$xu=\frac{G_1x_1u_1+G_2x_2u_2}{G_1+G_2} \quad \ldots\ldots \quad (18)$$

and

$$q+x\rho=\frac{G_1q_1+G_2q_2}{G_1+G_2}+\frac{G_1x_1\rho_1+G_2x_2\rho_2}{G_1+G_2} \quad \ldots \quad (19)$$

These equations can be extended to the case when more than two spaces are brought into communication; here u, q, and ρ are known functions of the temperature t, and therefore the temperature, after the mixture, can be determined from these equations, and hence the pressure p belonging to this temperature and the steam quality x at the end of the mixture. Of course it is here assumed that $x>1$ will not occur in this operation; in such a case the steam will be superheated at the end of the mixture and the occurrence will have to be judged by other formulas which will be given later.

If the values of the right member of equations (18) and (19)

are known, the mixing temperature t can of course only be found by the method of trial. If the two spaces at the beginning contain only liquid without steam, so that $x_1=0$ and also $x_2=0$, then equation (18) will give $x=0$ and accordingly, after combination, there will only be liquid on hand without steam; from equation (19) there follows

$$q=\frac{G_1q_1+G_2q_2}{G_1+G_2},$$

a well-known formula that has long been employed for determining the mixing temperature of liquids, only it is customary to substitute, as an approximation, the temperature in place of the heat of the liquid.

On the other hand, if dry saturated steam were present in both spaces so that $x_1=1$ and $x_2=1$, then the two equations (18) and (19) would give

$$xu=\frac{G_1u_1+G_2u_2}{G_1+G_2}, \quad \ldots\ldots (18a)$$

$$q+x\rho=\frac{G_1(q_1+\rho_1)+G_2(q_2+\rho_2)}{G_1+G_2}. \quad \ldots\ldots (19a)$$

If, in so doing, we also presuppose that saturated vapors behave like gases, then the pressure p after the mixture will be found from the equation (Vol. I, p. 176, equation (20a))

$$p=\frac{V_1p_1+V_2p_2}{V_1+V_2}. \quad \ldots\ldots\ldots (20)$$

Example. In space V_1 there are G_1 kg. [lb.] of dry saturated steam of 10 atmospheres pressure, while in space V_2 there are $G_2=0.3836\ G_1$ kg. [lb.] of dry steam of 1 atmosphere pressure.

Utilizing the values of Table 11 of the Appendix belonging to the two pressures we get, from equations (18a) and (19a),

$$xu=0.5926$$
$$[xu=9.49281],$$
$$q+x\rho=610.31$$
$$[q+x\rho=1098.558].$$

From these two expressions can be found the pressure p of 3 atmospheres belonging to the values u, q, and ρ, and also the steam quality $x = 1.012$.

The steam is accordingly slightly superheated after the mixture, but the values differ so little from 1 that we can assume the steam as approximately dry and the pressure found is 3 atmospheres.

If we had found the pressure p of the steam, under the suppositions underlying equation (20), we should have had

$$V_1 = (u_1 + \sigma)G_1 = 0.1897\, G_1 \quad \text{and} \quad V_2 = (u_2 + \sigma)G_2 = 0.6331\, G_1$$

$$[V_1 = 3.0388\, G_1 \quad \text{and} \quad V_2 = 10.1416\, G_1],$$

$$p = 3.075 \text{ atmospheres},$$

a result differing but little from the formerly found value.

Equation (20) has often been used for determining the mixing pressure of dry saturated vapors, but without proof of its reliability; the problem is encountered when investigating more closely the Woolf and compound steam engines; it arises when the receiver-space is put into communication with one or the other of the steam cylinders and the consequent sudden changes of pressure are subjected to calculation.

If we investigate in the manner of the preceding example the problem for other values of the pressures p_1 and p_2, we always find that the steam quality x, after the mixture, differs but little from unity, so that in practical cases we may well assume that the steam which results from the mixture of dry saturated vapors is again dry saturated, and that its pressure p may be calculated with sufficient accuracy according to equation (20). Pursuing the problem still further in the manner indicated, we are led to the result that equation (20) can be employed for the calculation of the mixing-pressure p even when the steam before mixture is wet, in the one space as well as in the other, provided the quantity of water mixed with the steam is not too considerable and that the steam pressures do not exceed the limits customary in steam-engine practice.

For the mixing of dry saturated vapors the proposition given here may be developed into a somewhat more general form.

If v_1 is the specific volume of dry steam in space V_1 (Fig. 10, p. 129), v_2 the specific volume in space V_2, and finally v the specific

volume of the dry saturated steam after the mixture, then the relation

$$(G_1+G_2)v=G_1v_1+G_2v_2 \quad . \quad . \quad . \quad . \quad . \quad (21)$$

holds, and furthermore, according to equation (20),

$$(G_1v_1+G_2v_2)p=G_1v_1p_1+G_2v_2p_2. \quad . \quad . \quad . \quad . \quad (22)$$

Dividing both equations by G_1 and eliminating from both the ratio $G_2:G_1$, there is found

$$(a+p)v=C, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where the two constants a and C are determined from

$$a=\frac{p_1v_1-p_2v_2}{v_2-v_1} \quad \text{and} \quad C=\frac{v_1v_2(p_1-p_2)}{v_2-v_1}; \quad . \quad . \quad . \quad (24)$$

equation (23) is nothing but the equation of the limit curve of steam according to Pambour (see p. 113), which in fact reproduces the course of this curve with sufficient accuracy in approximate calculations when the pressures p_1 and p_2 do not lie too far apart. Had we used equation (23) as a basis we should have been able to derive equation (20) directly for dry saturated vapors of water.

In the problem of mixing wet vapors at constant total volume, treated here, it is assumed that heat has neither been supplied nor withdrawn during the mixing.

In order to extend the problem, however, let us suppose the heat quantity Q to be supplied from the outside during the mixing, then, according to the designations introduced on p. 130, the heat content J after the mixture is given by the formula

$$J=J_1+J_2+Q,$$

and from this is found, from given formulas,

$$Q=(G_1+G_2)(q+x\rho)-G_1(q_1+x_1\rho_1)-G_2(q_2+x_2\rho_2), \quad . \quad (25)$$

while here also is found

$$(G_1+G_2)xu=G_1x_1u_1+G_2x_2u_2. \quad . \quad . \quad . \quad . \quad (26)$$

If, therefore, the state of steam b e f o r e the mixture is known in each space (Fig. 10, p. 129) and the pressure p, after the mixture, is g i v e n, then the steam quality x, after the mixture, is calculated from equation (26) and the heat quantity Q from equation (25).

If we assume, in particular, that after the mixture the pressure in the whole space is $p=p_2$, i.e., is equal to the initial pressure in one of the spaces, then the two preceding equations give, after substituting $u=u_2$, $q=q_2$, and $\rho=\rho_2$,

$$x=\frac{G_1x_1u_1+G_2x_2u_2}{(G_1+G_2)u_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (26a)$$

and after some easily made reductions we get from equation (25)

$$Q=G_1\left[q_2-q_1+x_1u_1\left(\frac{\rho_2}{u_2}-\frac{\rho_1}{u_1}\right)\right]. \quad . \quad . \quad . \quad . \quad (25a)$$

It follows from this formula that we might have supplied Q, the heat at constant volume, b e f o r e the mixture, to the space V_1, or, if Q is negative, it might have been withdrawn till the pressure in V_1 became identical with the pressure prevailing in space V_2 and not till t h e n effect the mixture.

E x a m p l e. Let one space V_1 (Fig. 10) contain G_1 kg. [lb.] of dry saturated steam of the pressure 1.5 atmospheres, and let the other space contain $G_2=25\ G_1$ kg. [lb.] of steam and water of the steam quality $x_2=0.01$, and having a pressure of 0.1 atmosphere. After mixing at constant volume, suppose the pressure $p=p_2=0.1$ atmosphere. If we utilize the corresponding values of Table 11 of the Appendix, we get first, from equation (26a),

$$x=0.0126,$$

and then, from equation (25a),

$$Q=-511.45\ G_1\ [-920.61\ G_1].$$

Accordingly heat must be withdrawn. The example corresponds to conditions which may arise in condensing engines.

§ 18. THEORY OF SURFACE CONDENSERS.

Let A (Fig. 11) represent the cylinder of a steam engine; let the piston K be at the end; and let the cylinder be filled with G_1 kg. [lb.] of steam of pressure p_1 and of quality x_1.

Let C represent the surface condenser, a space enclosing a series of vertical tubes, through which the steam coming from the cylinders flows downward.

The tubes are surrounded by cold water which enters at a and flows away through the pipe b.

When the steam passes through the tubes condensation takes place; the water collects in the lower part of the condenser and is removed by the pump B, the air and hot-water pump.

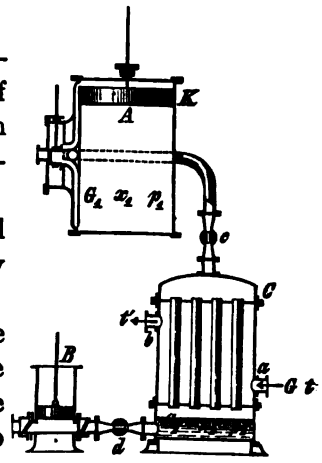

FIG. 11.

Let us first suppose the steam cylinder space to be shut off from the condenser space by the cock c, and, likewise, that the cock d cuts off communication between the condenser and the pump B, whose piston is in the lowest position.

Suppose G_2 kg. [lb.] of steam and water to be in the condenser; let the steam quality be x_2 and the pressure be p_2. Here we first determine the heat contents of the mass contained in condenser and steam cylinder,

$$J_1 = G_1(q_1 + x_1\rho_1) + G_2(q_2 + x_2\rho_2). \quad . \quad . \quad . \quad . \quad (27)$$

Suppose both cocks c and d to be opened and that in consequence there is a mixture of these masses in the condenser; because of the simultaneous cooling of the cooling water, the pressure in the two spaces settles down to the value p_2, the steam piston K going forward and pushing the whole steam mass into the condenser, where the weight G_1, in the form of water, is drawn out by the pump B.

At the end of the operation, we have for the heat contents J_2 of the whole mass in condenser and pump

$$J_2 = G_2(q_2 + x_2\rho_2) + G_1 q_2, \quad \ldots \ldots \quad (28)$$

for in the latter there is now the weight G_1 in the form of water, possessing the condenser temperature t_2, while in the condenser the condition at the end must be the same as at the beginning, for this development is intended to apply to the normal running condition of the engine.

During the transfer the whole mass has r e c e i v e d heat from the steam cylinder in the form of work, for the steam piston has swept through the volume $G_1(x_1u_1 + \sigma)$ under the constant pressure p_2. If L_1 is the corresponding work, then, in so doing, the heat

$$AL_1 = AG_1(x_1u_1 + \sigma)p_2 \quad \ldots \ldots \quad (29)$$

is g e n e r a t e d, and a b s o r b e d by the whole mass.

Simultaneously the pump piston has swept through the space $G_1\sigma$ under the constant pressure p_2. If L_2 designates the corresponding work, then in so doing the heat quantity

$$AL_2 = AG_1p_2\sigma \quad \ldots \ldots \quad (30)$$

has d i s a p p e a r e d, i.e., has been transformed into work.

Finally the heat quantity Q has been withdrawn from the whole mass during the assumed process by the cooling water, and hence there obtains the fundamental equation

$$J_1 + AL_1 - AL_2 - Q = J_2,$$

for we must get the heat contents at the end when we add to the heat contents at the beginning the absorbed heat quantities and subtract the rejected heat quantities. From the preceding equation there is found

$$Q = J_1 - J_2 + AL_1 - AL_2,$$

and, if we here utilize equations (27) to (30), we get

$$Q = G_1[q_1 - q_2 + x_1(\rho_1 + Ap_2u_1)]. \quad \ldots \ldots \quad (31)$$

Now it is this heat quantity which must be taken up by the cooling water. Let the weight of water which, during this process, enters the pipe a (Fig. 11) with the temperature t and is discharged through the pipe b with temperature t' be designated by G, then we also have

$$Q=G(q'-q). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

By equating this with the preceding formula we find

$$\frac{G}{G_1}=\frac{q_1-q_2+x_1(\rho_1+Ap_2u_1)}{q'-q}, \quad . \quad . \quad . \quad . \quad (33)$$

and in this equation there is now contained the principal result for condensers, for it is determined by the weight G of the cooling water which is necessary to condense the mixture G_1 of steam and liquid.

In practice, for the sake of safety, the values of the right member of this equation are so chosen that the mass of cooling water G becomes greater than is strictly necessary; therefore the quality x_1 at the end of expansion in the steam cylinder is assumed $x_1=1$, and the terminal pressure p_1 of expansion is also taken larger than is customary in condensing engines.

If we assume this pressure as $p_1=1$ atmosphere, then we must substitute in equation (33) $q_1=100.50$ [180.9] $\rho_1=496.30$ [893.34]; letting the pressure in the condenser be $p_2=0.1$ atmosphere, we get $Ap_2u_2=4.02$ [7.236]. If, in addition, we substitute at low temperatures the temperatures themselves in place of the heats of the liquid, we get for practical purposes the sufficiently accurate formula

$$\frac{G}{G_1}=\frac{600-t_2}{t'-t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33a)$$

$$\left[\frac{G}{G_1}=\frac{1112-t_2}{t'-t}\right].$$

This formula has the same structure and equal simplicity with the old Watt's formula, which is still to be found in all the technical manuals, but is falsely based from the thermodynamic point of view. Watt's formula contained, in place of the numerical value 600 [1080], the total heat of the steam correspond-

ing to the temperature $t = 100°$ [212°], which was assumed in round numbers to be 640 cal. [1152 B.t.u.]. Watt's formula therefore gives still greater values for the cooling water G than the preceding expression; but this circumstance will not justify applying a falsely established formula in place of a correct one.

Example. Let the terminal pressure of expansion in a steam engine be $p_1 = 0.6$ atmosphere and the steam quality $x_1 = 0.80$.

Let the pressure in the condenser be 0.1 atmosphere and let the cooling water be warmed from $t = 15°$ [59° F.] to $t' = 35$ [95° F.]. Here the exact formula (33) gives

$$\frac{G}{G_1} = 23.24$$

because the temperature in the condenser amounts to $t_2 = 46.21°$ [115.18° F.] according to the tables of the Appendix.

The approximate formula (33a) gives for this ratio 27.69 for general conditions, and Watt's formula gives 29.69.

§ 19. THEORY OF THE JET CONDENSER.

In Fig. 12, A represents the cylinder of a steam engine; suppose the piston to be at the upper end of the stroke and the cylinder to contain G_1 kg. [lb.] of steam of pressure p_1 and of steam quality x_1.

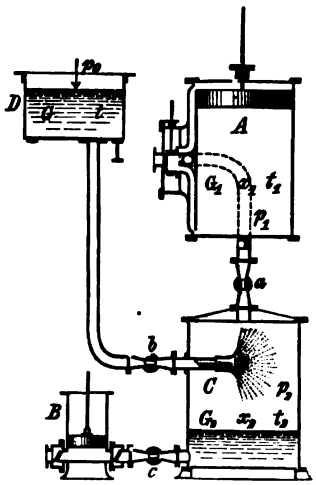

FIG. 12.

C represents the condenser in which there are G_2 kg. [lb.] of steam and water under the pressure p_2 and having the steam quality x_2.

A pipe opens into the side of the condenser and ends in a spray; the pipe is connected with the vessel D which contains G kg. [lb.] of cold water of a temperature t, under the atmospheric pressure p_0. The lower part of the condenser is connected by a pipe with a pump cylinder B.

First of all suppose the four spaces to be shut off from each other by the cocks a, b, and c; then the heat contents of the mass of steam and

water in cylinder A are determined by $G_1(q_1+x_1\rho_1)$, the heat of the mass in the condenser by $G_2(q_2+x_2\rho_2)$, and the heat in the jet water in the vessel D is determined by Gq. Accordingly the total heat contents J_1, in the three spaces mentioned, are

$$J_1=G_1(q_1+x_1\rho_1)+G_2(q_2+x_2\rho_2)+Gq. \quad . \quad . \quad . \quad (34)$$

Now let the three cocks be opened, let the piston move forward n the steam cylinder and push the mixture of G_1 kg. [lb.] of steam and liquid into the condenser, overcoming, in so doing, the constant pressure p_2. The jet water G passes under atmospheric pressure p_0 from the vessel D through the spray into the condenser, and finally the piston of the air and hot-water pump B receives and sucks in, under the pressure p_2, not only the jet water G but also the weight G_1 of steam and liquid in the fo m of water.

At the end of the occurrence the whole mass is united in the condenser and in the pump cylinder, and at this moment the total heat contents in the two spaces combined are

$$J_2=G_2(q_2+x_2\rho_2)+(G_1+G)q_2. \quad . \quad . \quad . \quad . \quad . \quad (35)$$

The term which represents the heat contents of the mass in the condenser must have the same value at the end as in the beginning, because we assume the engine to be running normally.

Moreover the mass in the steam cylinder has taken up the work L_1 and the jet water in the vessel D has absorbed the work L_2, and these quantities of work, exp essed in heat, amount to

$$AL_1=AG_1(x_1u_1+\sigma)p_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

$$AL_2=AG\sigma p_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Finally there nas been g i v e n o f f in the pump cylinder a work L_3 which, measured in units of heat, amounts to

$$AL_3=A(G+G_1)\sigma p_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

Evidently the heat contents J_2, at the end, are equal to the heat contents J_1 at the beginning, increased by the algebraic sum

of the quantities of work received, expressed in units of heat, and therefore amount to

$$J_2 = J_1 + AL_1 + AL_2 - AL_3.$$

If we here use the preceding equations (34) to (38), we get for the fundamental equation of the jet condenser,

$$\frac{G}{G_1} = \frac{q_1 - q_2 + x_1(\rho_1 + Ap_2u_1)}{q_2 - q - A\sigma(p_0 - p_2)}, \quad \ldots \quad (39)$$

from which the jet water G can be calculated. For the purpose of transforming this equation for practical use, i.e., for obtaining a simple approximate formula, the very small term $A\sigma(p_0 - p_2)$ in the denominator can be neglected; moreover we can take $x_1 = 1$ and therefore assume dry steam in the steam cylinder and also one atmosphere for the terminal pressure of expansion; if, besides, the condenser pressure is $p_2 = 0.1$ atmosphere, we must assume

$$q_1 = 100.50, \quad \rho_1 = 496.30, \quad Ap_2u_1 = \frac{p_2}{p_1} Ap_1u_1 = 4.02$$

$$[q_1 = 180.90, \quad \rho_1 = 893.34, \quad Ap_2u_1 = 7.236],$$

and therefore

$$\frac{G}{G_1} = \frac{600 - q_2}{q_2 - q}$$

$$\left[\frac{G}{G_1} = \frac{1080 - q_2}{q_2 - q}\right],$$

or, if we substitute for the heat of the liquid the temperature itself, we get

$$\frac{G}{G_1} = \frac{600 - t_2}{t_2 - t} \quad \ldots \quad (39a)$$

$$\left[\frac{G}{G_1} = \frac{1080 - (t_2 - 32)}{t_2 - t} = \frac{1112 - t_2}{t_2 - t}\right].$$

This formula differs from that which was found for the surface condenser, equation (33*a*), p. 137, only in this particular, that in the denominator of the right member t_2 appears in place of t'.

As regards the comparison of this formula with the customary W a t t formula, we may say that the remarks made on p. 137 apply here also.

E x a m p l e. In a steam engine let the terminal pressure of the expansion be $p_1 = 0.6$ atmosphere and the steam quality be $x_1 = 0.80$. Let the pressure in the condenser be 0.1 of an atmosphere, and the temperature of the jet water be $t = 15°$ [59°]. Here we have $p_0 = 10333$ [2116.31] and $p_2 = 0.1\ p_0$, and hence the term $A\sigma\ (p_0 - p_2)$, in the denominator of equation (39), becomes equal to 0.026, so that, using Table 11 of the Appendix, we get

$$\frac{G}{G_1} = 14.37.$$

Equation (39*a*) gives for average conditions

$$\frac{G}{G_1} = 17.69,$$

while, according to W a t t ' s formula, we have 18.97 if we substitute, in equation (39*a*), the value 640 [1152] in place of 600 [1080].

Into jet condensers there enters a large volume of air which is absorbed by the jet water, is carried along with it, and is mostly given off in the condenser, where a higher temperature and a far lower pressure prevails than outside; but the preceding theory of the condenser is not touched thereby or, to speak more accurately, only to a very subordinate degree.

In order to take account of this fact, it is only necessary to give more generous dimensions to the air and hot-water pump B (Fig. 12). Ordinarily it is estimated that the volume of the air, which is simultaneously withdrawn with the water from the condenser, by the pump amounts to 11/14 of the volume $G\sigma$ of the jet water.

§ 20. CALORIMETRIC INVESTIGATION OF THE GIFFARD STEAM JET PUMP (INJECTOR).

Let Fig. 13 schematically represent an apparatus from whose mode of action the occurrences in the Giffard injector may be derived.[1]

A represents a cylinder which contains a piston and G_1 kg. [lb.] of steam and water of pressure p_1 and corresponding temperature t_1; the steam quality there is x_1.

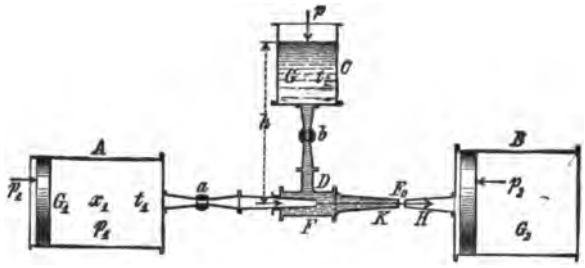

FIG. 13.

The cylinder is provided with a conical discharge pipe, whose discharge orifice lies in the interior of the casing D, which is provided with a large pipe K that discharges at F_0 into the open air.

Casing D is connected by a vertical pipe with an open vessel C which is under atmospheric pressure p and contains cold water of the temperature t. Let us at first suppose that both spaces A and C are shut off by the cocks a and b. If both cocks are now opened steam will flow through the nozzle F, but will immediately be condensed by the cold water coming from the vessel C. The jet passing through the outer orifice F_0 accordingly consists of a mixture of cold water with the condensed steam.

The pressure p_1 in cylinder A can be kept at a constant height during the outflow by supposing the piston in the cylinder to be pushed forward with the constant pressure p_1. Let us moreover suppose that the level in vessel C is always kept in the same posi-

[1] A first investigation of the problem treated in the text was given by me in Zivilingenieur, Vol. VI, 1860, p. 315.

tion by new water so that, during the whole occurrence, it may be at the height h above the axis of the pipe K, and so that the flow of water into the casing D and the flow toward the mixing pipe may likewise take place under constant pressure.

Now let the jet, discharging horizontally through the orifice F_0, be caught by a second pipe H (Fig. 13), and be guided by it to a second cylinder B where the transferred fluid mass must make place for itself by pushing back a piston and overcoming a constant back pressure p_2.

As an introduction to the general investigation let us assume that the cylinder B and the receiving tube H are removed, and that the water-jet flows directly into the open air with a velocity w_0 and a temperature t_2.

Before the opening of the two cocks a and b the heat contents of the wet steam in cylinder A were $G_1(q_1+x_1\rho_1)$, and Gq was the heat content of the cold water in vessel C; hence there follows, for the combined heat contents J_1 of both spaces,

$$J_1=G_1(q_1+x_1\rho_1)+Gq. \quad . \quad . \quad . \quad . \quad . \quad (40)$$

If the cocks are now opened and to simplify matters the weights G_1 and G referred to the second, then in this time there will discharge through the orifice F_0 the quantity of water (G_1+G) kg. [lb.], and since the temperature of the mixture is t_2 and its velocity w_0, the heat contents J_2 of the whole mass will be

$$J_2=(G+G_1)\left(q_2+A\frac{w_0^2}{2g}\right), \quad . \quad . \quad . \quad . \quad . \quad (41)$$

where $A\frac{w_0^2}{2g}$ represents the kinetic energy of the unit of weight of liquid measured in units of heat.

The heat contents J_1, at the beginning, are by no means identical with the value J_2 at the end of the procedure, for during its occurrence the whole mass has taken up and given off work.

In cylinder A the piston is moved, with constant pressure p_1,

through the space $G_1(x_1u_1+\sigma)$; the work L_1 taken up, measured in units of heat, therefore amounts to

$$AL_1=AG_1p_1(x_1u_1+\sigma). \quad . \quad . \quad . \quad . \quad . \quad (42)$$

In the vessel C the level of the water is under the atmospheric pressure p and moves through the space $G\sigma$; to this must still be added the work Gh of the force of gravity; accordingly the amount of work L_2 taken up, also measured in units of heat, here sums up to

$$AL_2=AG(p\sigma+h). \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

Finally the weight of liquid (G_1+G) flowing through the orifice F_0 has the volume $(G_1+G)\sigma$ and must overcome the constant atmospheric pressure p during the discharge. Accordingly the work L_3, expressed in heat which is given off, amounts to

$$AL_3=A(G_1+G)p\sigma. \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

Now the heat contents J_2, at the end, are evidently equal to the heat contents at the beginning, when increased by the heat equivalents of the work taken up, and diminished by the heat equivalents of the work given off; it follows, therefore, that

$$J_2=J_1+AL_1+AL_2-AL_3,$$

or, if we consider the relation $\rho_1+Ap_1u_1=r_1$,

$$(G+G_1)\left(q_2+A\frac{w_0^2}{2g}\right)=G_1[q_1+x_1r_1+A(p_1-p)]+G(q+Ah). \quad (45)$$

This is the first fundamental equation to which the proposed problem leads; the subsequent investigation will now be divided into two parts, the first of which includes a utilization of the preceding formula, and the second of which continues the investigation of the Giffard injector.

First Part. Suppose the jet, flowing through the orifice F_0 (Fig. 13) into the open air, is caught by a measuring tank in

which, at the start, there is G_2 kg. [lb.] of water of the temperature t_3; at the end the tank contains the weight (G_1+G_2+G) of liquid. Let the temperature be t_4.

Then if the heat contents at the end are J_4, we have

$$J_4=(G_1+G_2+G)q_4.$$

On the other hand, before the mixture, we have

$$J_3=G_2q_3+(G_1+G)\left(q_2+A\frac{w_0^2}{2g}\right).$$

Now since work was neither taken up nor given off during the mixture, we have $J_3=J_4$, or, according to the preceding equations,

$$(G+G_1)\left(q_2+A\frac{w_0^2}{2g}\right)=(G+G_1+G_2)q_4-G_2q_3, \quad . \quad . \quad (46)$$

which is the second fundamental equation for the present particular case.

Combining equations (45) and (46) gives

$$G_1[q_1-q_4+x_1r_1+A\sigma(p_1-p)]=G(q_4-q-Ah)+G_2(q_4-q_3). \quad (47)$$

All the quantities of this equation, with the exception of the steam quality x_1, can be found by observation.

If the injector is directly attached to a steam boiler we have here a means of ascertaining whether the steam, flowing from the boiler to the injector, is dry or wet, and in the latter case can ascertain steam quality x_1.

If the liquid quantities G and G_1 are taken per second, and if F_0 is the cross-section of the orifice of efflux in pipe K, measured in square meters [square feet], there follows, from the formula

$$F_0w_0=(G+G_0)\sigma, \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

the efflux velocity w_0, and from this the kinetic energy, measured in units of heat,

$$A\frac{w_0^2}{2g}=\frac{A}{2g}\left(\frac{(G+G_1)\sigma}{F_0}\right)^2. \quad . \quad . \quad . \quad . \quad . \quad (49)$$

Furthermore we find from equation (46) the heat q_2 of the liquid and the corresponding temperature t_2 of the water-jet in the orifice F_0.

In utilizing equation (48) there exists, to be sure, some uncertainty; the water jet, issuing from the water nozzle F_0 of an injector, has a milk-white appearance and seems to be loosely put together and swollen, so that the specific volume σ is probably g e a t e r than ordinary water. Many explain the phenomenon by saying that the jet, at this place, still carries with it uncondensed particles of steam, which is possible; to be sure I have observed precisely the same phenomenon of a milk-white jet in a water-jet apparatus [1] in which there was no steam at all, whenever one of the two jets engaged in the mixing possessed a great velocity. The coloring and opaque character may accordingly be due to eddies and to a peculiar motion of the particles in the interior of the jet, and may be due to the air which the water has absorbed, and which becomes free when the water is warmed and subjected to a lower pressure.

E x a m p l e. During an experiment with an injector which was directly attached to the steam boiler, the mixture of cooling water with condensed steam, which constituted the water jet flowing through the orifice F_0 (Fig. 13), was caught in a carefully constructed measuring tank, in which there existed at the beginning of the experiment $G_2 = 44.665$ kg. [98.47 lb.] of water of the temperature $t_3 = 32.6$ C. [90.68° F.].

The duration of the experiment was exactly 60 seconds, and at the end of this time the measuring tank contained $G + G_1 + G_2 = 92.977$ kg. [204.979 lb.] of water of the temperature $t_4 = 37.76°$ C. [96.24° F.].

In the same time there flowed from the vessel C (Fig. 13), which was likewise an accurately constructed measuring tank, the quantity of water $G = 46.266$ kg. [102.0 lb.] whose temperature amounted to $t = 12.0°$ C. [53.6° F.].

From this was first determined the weight G_1 of steam and liquid which passed, in 60 seconds, out of the boiler and through the injector, namely,

$$G_1 = (G_1 + G_2 + G) - G - G_2 = 2.046 \text{ kg. [4.511 lb.]}$$

The absolute steam pressure in the boiler was 1743.5 mm. [68.642 in.] of mercury, which pressure was read off from a mercury manometer. The

[1] Das Lokomotiven-Blasrohr. Zürich, 1863, p. 117.

boiler pressure was therefore 2.3 atmospheres and, according to Table 11 of the Appendix, the values corresponding to this pressure are:

$$t_1 = 125.07°, \quad q_1 = 125.97, \quad r_1 = 518.677$$
$$[t_1 = 257.13°, \quad q_1 = 226.75, \quad r_1 = 933.62].$$

If we substitute the given values in equation (47) and neglect the very small quantity $Ah = 0.003$ cal. [0.0054 B.t.u.], we get, from the quoted equation,

$$x_1 = 0.993,$$

which is so nearly equal to unity that we may assume that the steam is dry saturated which flows from the steam boiler to the injector.

The substitution of the given numerical observations in equation (46) gives, for the present case,

$$q_2 + A\frac{w_0^2}{2g} = 39.56 \; [69.408].$$

The cross-section of the orifice F_0 has a diameter of $d_0 = 10.2$ mm. [0.4014 in.], whose cross-section amounts to $F_0 = 81.71$ sq. mm. [0.12665 sq. in.]; through this flows in one minute the quantity of liquid $G + G_1 = 48.312$ kg [106.51 lb.], or, per second, 0.8052 kg. [1.775 lb.]; hence, according to equation (48), the velocity of efflux becomes

$$w_0 = 9.85 \text{ m.}$$
$$[w_0 = 32.3165 \text{ ft.}],$$

where, to be sure, σ was taken equal to 0.0010 [0.016]; we therefore have

$$\frac{w_0^2}{2g} = 4.458 \text{ m. } [14.626 \text{ ft.}] \quad \text{and} \quad A\frac{w_0^2}{2g} = 0.012 \text{ Cal. } [0.0216 \text{ B.t.u.}],$$

and hence, according to the preceding equation,

$$q_2 = 38.55 \quad \text{and} \quad t_0 = 38.5°$$
$$[q_2 = 69.39 \quad \text{and} \quad t_0 = 101.3°]$$

as the temperature of the jet in the orifice.[1]

[1] The example in the text was taken from an extended, experimental series, which I conducted, at the beginning of 1870, on the experimental boiler of the engineering apparatus belonging to the Polytechnicum at Zurich; it was made with a large injector, partly to study the remarkable occurrences in the injector, but, particularly, to study more thoroughly the pressure conditions in the condenser space, and also in part to get an experimental basis for the theory of the efflux of steam, to which latter question we will return later on. Fifty-three experiments were made under different conditions; the example treated in the text was picked at random from the set. Although the experiments may

Moreover, in this experiment, the adjustable steam nozzle was pushed so far forward that the influx cross-section for the cold water was greatly contracted and the ratio of the steam weight G_1 to the weight G of the sucked-in water seemed abnormal. As a feed apparatus, the injector would have failed to work with the data underlying this experiment, and this is evident from the value of the velocity head 4.458 m. [14.626 ft.] corresponding to the velocity w_0, because the excess of steam pressure in the boiler, of 1.3 atmospheres, corresponds to a water column of 13.4 m. [43.964 ft.].

Second Part. Let us now assume, in addition, that the jet flowing from the orifice F_0 (Fig. 13) is caught by the receiving tube H and is led to the cylinder B, where the mass passes into a condition of rest, pushing back the piston and overcoming, in so doing, the constant back pressure p_2.

At the end, after the spreading has taken place, let the temperature be t_2' and then the heat contents will be $J_2' = (G+G_1)q_2'$. The initial contents J_2, in the orifice F_0, are determined by equation (41). The work L, performed in cylinder B, is found to be

$$L = (G+G_1)(p_2-p)\sigma,$$

and there follows from equation

$$J_2 = J_2' + AL,$$

and with the help of the given expressions,

$$q_2 + A\frac{w_0^2}{2g} = q_2' + A(p_2-p)\sigma. \quad . \quad . \quad . \quad . \quad . \quad (50)$$

---

be characterized as successful ones, they have not, thus far, been reported, because I hoped again to take up and continue the experiments on a more extended scale and particularly with a higher boiler pressure and perhaps under still more favorable conditions; the experiments at Zurich only went to about 3.3 atmospheres, but the expectation that in the interval others would conduct similar experiments suitable for scientific needs has not been fulfilled. It is surely a curious fact that even at the present time we do not possess a theory of the injector that is based on reliable experiments, although this apparatus is working in numberless cases and the experiments, of the kind I mean, and which meet more than purely practical needs, would cost but little.

I will add that in my experiments at Zurich the steam entered the injector in the dry saturated condition. The tanks employed consisted of rectangular boxes which were jointed by planed iron plates and provided with verniers for the accurate determination of their water levels.

If we substitute this value in equation (45) we get

$$G[q_2' - q + A\sigma(p_2 - p) - Ah] = G_1[q_1 - q_2' + x_1 r_1 + A\sigma(p_1 - p_2)]. \quad (51)$$

If we regard the injector as a feeding apparatus and accordingly replace cylinder A and cylinder B by the steam boiler, then $p_2 = p_1$ and equation (51) is transformed into

$$G[q_2' - q + A\sigma(p_1 - p) - Ah] = G_1(q_1 - q_2' + x_1 r_1). \quad (51a)$$

If we now add $G(q_1 - q_2')$ to both members of this equation, we get

$$G[q_1 - q + A\sigma(p_1 - p) - Ah] = (G_1 + G)(q_1 - q_2') + G_1 x_1 r_1. \quad (52)$$

This equation constitutes the principal result of the calorimetric investigation of the Giffard steam-jet pump.

The right member of this equation represents nothing but the heat quantity needed for the running of the injector. For G_1 is the weight of a quantity of steam and water which in a given time, say in a second, passes from the boiler to the injector; united with the feed-water quantity G, it returns to the boiler and, after distributing itself there, has the temperature t_2'.

Now the weight $(G_1 + G)$ must be brought from the temperature t_2' to the boiler temperature t_1, which requires the heat quantity $(G_1 + G)(q_1 - q_2')$, and then the consumed steam quantity $G_1 x_1$ must again be evaporated under constant pressure, which demands a heat quantity $G_1 x_1 r_1$; in fact, as was maintained, there is needed for running the injector a heat quantity

$$Q = (G_1 + G)(q_1 - q_2') + G_1 x_1 r_1.$$

According to equation (52) there is then also found

$$Q = G[q_1 - q + A\sigma(p_1 - p) + Ah], \quad . \quad . \quad . \quad . \quad (53)$$

when the sign of h is changed and it is assumed that the feed-water is not forced on the injector from the pressure head h, but has to be sucked up the height h through the injector, as is ordinarily the case.

The preceding equation leads to very remarkable results; it shows that the heat needed for feeding with the injector is entirely

independent of the quantity and constitution of the steam (of G_1 and x_1) which is necessary for running it; it is also independent of the temperature of the jet in orifice F_0 (Fig. 13) and of the temperature t_2 at which the water enters the boiler, and, furthermore, is independent of all the separate dimensions of the injector, provided it works at all, and does not fail.

The two terms $A\sigma(p_1 - p)$ and Ah are in most practical cases so small that they may be neglected; the amount of the suction height h and the pressure p have accordingly a very subordinate influence on the heat quantity Q; it is therefore a matter of indifference whether the pressure p, as in the Giffard injector, is identical with the external atmosphere, or, as in the Schaus injector, is identical with the pressure in the mixing chamber.

To be sure, the formulas obtained give no clue whatever as to the several dimensions which are to be given to the injector; but, for the starting and setting into action, the suction head h and the temperature t of the feed-water will be of marked influence.

The comparison of the injector with the ordinary feed-pump is of technical importance.

Let A (Fig. 14) be the cylinder of a feed-pump, F the cross-section of the piston, S the piston stroke: let the pump suck water from the open vessel B through the height h. The cylinder is supposed to be open on the right-hand side, regarded as single-acting, and the atmospheric pressure p will prevail to the right of the piston; to the left of the piston there will exist the pressure $(p - h\gamma)$, where γ represents the specific weight of the water ($\gamma = 1000$) [$\gamma = 62.425$]. The difference of the two temperatures is accordingly $h\gamma$, and the piston force during suction is $Fh\gamma$, hence the work L' is

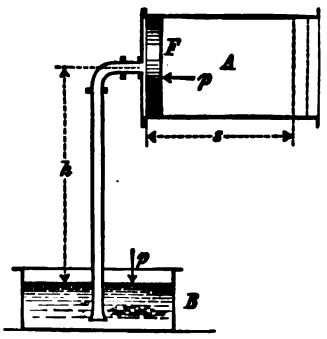

FIG. 14.

$$L' = Fh\gamma s.$$

During the return of the piston the water is forced against the back pressure p_1 into the boiler, and the work L'' of this forcing action is

$$L''=Fs(p_1-p).$$

Both works taken together give the whole pump work L (of course neglecting all frictional and hydraulic resistances), which then amounts to

$$L=Fs\gamma\left[\frac{p_1-p}{\gamma}+h\right].$$

Let $Fs\gamma=G$ be the weight of the feed-water, and, because the relation $\gamma\sigma=1$ obtains, we then have for the work of the pump

$$L=G[(p_1-p)\sigma+h]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

In the boiler the feed-water must be heated from t to t_1, which requires a heat quantity Q' which is determined by

$$Q'=G(q_1-q), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

but in the present case the heat quantity corresponding to the pump work L is not, say AL, but something greater, even if the pump is driven by a theoretically perfect engine. If we substitute the heat quantity ϕAL consumed in performing the work, where $\phi>1$, there follows for the quantity of heat Q, which is absorbed by feeding with the feed-pump,

$$Q=G(q_1-q)+\phi A[(p_1-p)\sigma+h],$$

while, according to equation (53), the amount found for the injector proved to be

$$Q=G(q_1-q)+A[(p_1-p)\sigma+h].$$

Accordingly the injector, as a feeding apparatus, is better than the feed-pump, indeed from the theoretical standpoint the injector must be regarded as the most perfect feeding apparatus conceiv-

able; this statement, however, only holds for the case of boiler feeding. As a water-raising apparatus the Giffard steam-jet pump is, mechanically, very unprofitable, because the unnecessary heating of the lifted water utilizes the steam in an unsuitable fashion. Finally, it is worthy of note that from all the formulas developed there can easily be derived approximate formulas, when the terms containing the factor A are left out.

In this case equation (45), p. 144, gives

$$(G+G_1)q_2=G_1(q_1+x_1r_1)+Gq,$$

or if we assume dry saturated steam, designating as before the total heat q_1+r_1 by λ_1, and finally, for the comparatively low temperatures t_2 and t, substitute the temperatures themselves, we get

$$t_2=\frac{G_1\lambda_1+Gt}{G_1+G}. \quad \ldots\ldots\ldots \quad (45a)$$

Accordingly we determine, approximately, the temperature t_2 of the jet at its entrance into the receiving tube H (Fig. 13, p. 142).

Neglecting as before, there follows from equation (50), p. 148, $q_2'=q_2$ or

$$t_2'=t_2, \quad \ldots\ldots\ldots \quad (50a)$$

according to which, the temperature t_2' of the jet, after its distribution and attainment of a condition of rest, can be assumed as nearly equal to its temperature t_2 at the entrance into the receiving tube, and probably in most cases can, with sufficient accuracy, be taken as of equal value.

Equation (45a) has been much used, but of course the users did not notice that it is only approximately correct.

If the temperatures t_1, t, and t_2 were known there would follow from equation (45a)

$$\frac{G}{G_1}=\frac{\lambda_1-t_2}{t_2-t}, \quad \ldots\ldots\ldots \quad (45b)$$

and this would determine the weight of water G which is sucked in by the steam weight G_1. If we wished to choose t_2, this formula would give the minimum quantity of water G necessary to effect complete condensation.

III. The Flow and the Efflux of Wet Vapors.

§ 21. FUNDAMENTAL FORMULAS FOR THE FLOW OF WET VAPORS.

The fundamental equations derived in general in Vol. I, § 40, p. 225, for the flow of any fluid, with the assumptions there made, can at once be converted into the form which will render them directly serviceable for the purpose of investigating the flow of wet vapors.

Suppose a fluid under the sole influence of gravity to flow through a vessel F_1F of continuously varying cross-section (Fig. 15) down through a vertical height $(h-h_1)$; suppose that for the upper section F_1 there are given pressure and volume p_1 and v_1, also the velocity w_1; and suppose that at the lower cross-section F the designations p, v, and w represent similar quantities,

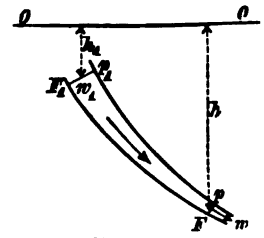

FIG. 15.

then, for the first fundamental equation (Vol. I, p. 229), there is found

$$Q=A[pv-p_1v_1+(U+H)-(U_1+H_1)-(h-h_1)], \quad . \quad \text{(I)}$$

or, written in the differential form,

$$dQ=A[d(pv)+dU+dH-dh], \quad . \quad . \quad . \quad . \quad \text{(Ia)}$$

where Q represents the quantity of heat which must be supplied from without to the unit of weight of the fluid along the path from F_1 to F; U_1 and U are the values of the inner work, and H_1

and H are the values of the energy of flow of the unit of weight in the first and second cross-sections, i.e.,

$$H=\frac{w^2}{2g} \quad \text{and} \quad H_1=\frac{w_1^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

As second fundamental equation there was found

$$Q+AW=A(U-U_1)+A\int_{v_1}^{v} p\,dv, \quad . \quad . \quad . \quad . \quad (II)$$

or, written in the differential form,

$$dQ+AdW=A[dU+p\,dv], \quad . \quad . \quad . \quad . \quad . \quad (IIa)$$

where W represents the work of the hurtful resistance encountered on the path from F_1 to F, reduced to the unit of weight of fluid.

Finally, if G is the weight of the fluid which passes through every cross-section under normal running conditions, we have in addition the equations

$$Gv=Fw \quad \text{and} \quad Gv_1=F_1w_1. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In applying these formulas to flowing vapors, we will, for simplicity, assume throughout that the axis of the vessel is horizontal as in Fig. 16, i.e., will assume $h=h_1$; let us also presuppose a d i a b a t i c f l o w, consequently the fluid neither receives nor rejects heat, and therefore $Q=0$ and $dQ=0$.

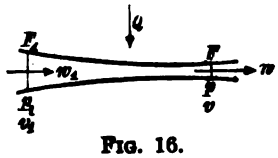

FIG. 16.

In spite of these limitations, which, moreover, may easily be dropped, the following formulas include all technically important cases.

We now get from equation (Ia)

$$AdH=-A[d(pv)+dU], \quad . \quad . \quad . \quad . \quad . \quad (Ib)$$

and from equation (II*a*)

$$AdW = A[dU + pdv]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (IIb)$$

Addition then produces the relation

$$d(H + W) = -vdp, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (III)$$

which equation was discussed graphically in Vol. I, p. 231.

First Transformation. In wet vapors $v = xu + \sigma$, and, according to equation (41), p. 54,

$$AdU = dq + d(x\rho);$$

hence, according to equation (I*b*), we have, if we utilize the relation $r = \rho + Apu$,

$$AdH = -[dq + d(xr) + A\sigma dp],$$

and from this, by integration,

$$A\frac{(w^2 - w_1^2)}{2g} = q_1 - q + x_1 r_1 - xr + A\sigma(p_1 - p). \quad . \quad . \quad (Ic)$$

This equation is still insufficient, however, for the solution of the problem; even if we know the quantities w_1 and x_1 in the first cross-section F_1 and the pressure p_1 with the attendant values q_1 and r_1, and in addition know the pressure p in the second cross-section F, still we cannot determine the velocity w in the latter section, because we do not know beforehand the steam quality x. For this purpose we must get the help of equation (II*b*).

According to the third transformation of the fundamental equations given on p. 58, equation (II*b*) can be written in the following form:

$$AdW = Td\left(\tau + \frac{xr}{T}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (IIc)$$

It is evident from this equation that it, also, does not lead to a solution, i.e., to the determination of the quality x, because, first of all, it is not known how the work of resistance AdW, that is transformed into heat, is connected with the independent variables x and p.

For the case in which the hurtful resistance may be neglected and hence $AdW=0$, the solution is of course at once rendered possible; we get from equation (IIc)

$$\tau+\frac{xr}{T}=\tau_1+\frac{x_1r_1}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From this equation the steam quality x can be determined, and if we substitute equation (3) in equation (Ic) [1] we get

$$A\frac{(w^2-w_1^2)}{2g}=x_1r_1\frac{(T_1-T)}{T_1}+q_1-q-T(\tau_1-\tau)+A\sigma(p_1-p). \quad (Id)$$

When w is found from this we can determine the weight G of steam and liquid which flows in one second through the cross-section F, from the equation

$$G(xu+\sigma)=Fw. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Second Transformation. According to the presentation given on p. 58, equation (52) there gives the inner work U in the following form:

$$AdU=\omega dt+d\left(v\frac{\rho}{u}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Let us now return to the general equations (Ia) and (IIa) and assume the axis of the vessel to be horizontal, as in Fig. 16, i.e., assume $dh=0$; in addition we will again take up the thought of

[1] The formulas given in the text for the efflux of wet steams, when the hurtful resistances are neglected, I developed first in my book "Das Lokomotivenblasrohr," Zürich, 1863, and later I continued the investigation in Zivilingenieur, Vol. X, 1864, p. 87, and Vol. XVII, 1871, p. 71.

heat supply, then, taking account of the relation $r=\rho+Apu$ and substituting equation (5) in equation (I*a*), we get

$$dH=dQ-\omega dT-d\left(\frac{r}{u}v\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III)}$$

Moreover there follows from equation (II*a*), with the help of equation (51*a*),

$$dQ+AdW=\omega dT+ATd\left(v\frac{dp}{dt}\right).$$

According to Clapeyron's equation, however,

$$\frac{dp}{dt}=\frac{r}{AuT},$$

which causes the preceding equation to take the following form:

$$dQ+AdW=\omega dT+Td\left(\frac{r}{uT}v\right). \quad . \quad . \quad . \quad . \quad \text{(IV)}$$

Equations (III) and (IV) include the result of the preceding second transformation.

If, as in the first transformation, we imagine flow to occur without heat supply, then $dQ=0$ and we get by integration, from equation (III),

$$A\frac{(w^2-w_1^2)}{2g}=\omega(T_1-T)+\frac{r_1}{u_1}v_1-\frac{r}{u}v, \quad . \quad . \quad \text{(III}a\text{)}$$

while equation (IV) can be written in the form

$$AdW=\omega dT+Td\left(\frac{r}{uT}v\right). \quad . \quad . \quad . \quad . \quad . \quad \text{(IV}a\text{)}$$

Here again applies what was said when discussing equation (I*c*) and (II*c*), which are of course identical with the preceding equations.

Not until we know how W changes with pressure and volume can we determine for the pressure p of cross-section F the corresponding volume and the velocity w.

For the special case in which the resistances are neglected, i.e., for $dW=0$, equation (IVa) gives, by integration,

$$\omega \log_e T+\frac{r}{uT}v=\omega \log_e T_1+\frac{r_1}{u_1T_1}v_1, \quad . \quad . \quad \text{(IV}b\text{)}$$

and from this equation we can calculate the specific volume v of the wet steam in cross-section F; the steam quality x, at this place, can then be found from the relation $v=xu+\sigma$.

Utilizing equation (IVb) in (IIIa) gives

$$A\frac{(w^2-w_1^2)}{2g}=\frac{r_1}{u_1T_1}(T_1-T)v_1+\omega\left[T_1-T-T\log_e\frac{T_1}{T}\right]. \quad \text{(III}b\text{)}$$

As soon as w is determined from this formula the weight G of the steam and liquid mass which flows per second through the cross-section F can be found from

$$Gv=Fw. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The last formulas are valid only under the supposition that no hurtful resistances exist, such as friction of the flowing fluids along the walls.

Strictly speaking, such resistances are, however, always on hand and the loss of work appears as a diminution of the kinetic energy, of the energy of flow, the corresponding work is transformed into heat, and the action is the same as if heat were supplied to the mass from the outside, and this is indicated by the structure of the above formulas (IIc) and (IVa).

As yet there is no way of representing AdW analytically without having recourse to certain hypotheses.

When investigating the efflux of gases and introducing resistances into the calculations, I have started from the assumption that the work of resistance dW may be taken proportional to the

change of temperature (see Vol. I, equation (28), p. 246). Now supposing that the same assumption may be made for flowing vapors, under certain conditions, for example during the efflux through simple orifices, we may then take

$$AdW = -\omega_0 dT,$$

where ω_0 is a constant which is to be determined by experiment for the case in question.

From equation (IV*a*) then follows

$$-\omega_0 dT = \omega dT + Td\left(\frac{r}{uT}v\right),$$

and from this, by integration,

$$\frac{r}{uT}v = (\omega+\omega_0)\log_e\frac{T_1}{T} + \frac{r_1}{u_1T_1}v_1, \quad \ldots\ldots \text{(V)}$$

and then, according to equation (III*a*),

$$A\frac{(w^2-w_1^2)}{2g} = \frac{r_1}{u_1T_1}(T_1-T)v_1 + \omega(T_1-T) - (\omega+\omega_0)T\log_e\frac{T_1}{T}. \quad \text{(VI)}$$

If there are no resistances, that is, $\omega_0=0$, then, according to equation (V), we must conclude that the resistances i n c r e a s e the specific volume v, and consequently also the steam quality x; on the other hand the velocity w, in the cross-section F, will be diminished, other things being equal.

If the difference of pressure in the two cross-sections is inconsiderable, then the temperature difference is still more so; we can therefore develop the logarithm occurring in equation (VI), according to a series:

$$\log_e\frac{T_1}{T} = \log_e\left[1+\frac{T_1-T}{T}\right] = \frac{T_1-T}{T} - \frac{1}{2}\left(\frac{T_1-T}{T}\right)^2 + -\ldots \quad (7)$$

and content ourselves with the first two terms of the series. Then equation (VI) gives

$$A\frac{(w^2-w_1^2)}{2g}=\left[\frac{r_1}{u_1T_1}v_1-\omega_0+(\omega+\omega_0)\frac{(T_1-T)}{2T}\right](T_1-T). \quad \text{(VI}a\text{)}$$

From equation (V) there is also found approximately, when we content ourselves with the first term of the series in equation (7),

$$\frac{r}{u}v=\frac{T}{T_1}\frac{r_1}{u_1}v_1+(\omega+\omega_0)(T_1-T). \quad . \quad . \quad . \quad . \quad \text{(V}a\text{)}$$

§ 22. EFFLUX OF HIGHLY HEATED LIQUIDS FROM SIMPLE ORIFICES UNDER CONSTANT PRESSURE.

Suppose a vessel, for example a steam boiler, to contain liquid and steam of the pressure p_1 and the temperature t_1 corresponding to the pressure. Let the lower part of the boiler in the water space be provided with a discharge-pipe well rounded on the inside and possessing a cross-section of efflux F sq. m. [sq. ft.]. Let the pressure in the plane of the orifice be p, its specific volume there v, and the efflux velocity w. Here we must put the initial velocity $w_1=0$, likewise the initial steam quality $x_1=0$, and then the formulas of the first transformation will give us the following, when we neglect resistances.

FIG. 17.

According to equation (Ia),

$$A\frac{w^2}{2g}=q_1-q-T(\tau_1-\tau)+A\sigma(p_1-p). \quad . \quad . \quad . \quad . \quad (8)$$

The steam quality x, in the plane of the orifice, is to be determined by equation (3) from

$$xr=T(\tau_1-\tau) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and then the specific volume v, at the same place, is to be found from

$$v=xu+\sigma. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The mass of steam and liquid, measured in kilograms [pounds], flowing through the orifice in a second, is

$$G=F\frac{w}{v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The weight D of the steam quantity which passes the orifice per second is finally

$$D=Gx, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

while the liquid passing the orifice is $G(1-x)$ kg. [lb.].

If the pressure p, in the plane of the orifice, is identical with the pressure that prevails outside of the boiler, then when the liquid leaves the orifice and during the passage into a condition of rest, a part of the liquid will be converted into steam under constant pressure p, and this will take place as if heat were supplied from the outside to an amount equivalent to the energy of flow given by equation (8).

If x' is the steam quality in the final condition of rest, then the relation

$$A\frac{w^2}{2g}=r(x'-x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

will hold and from it x' may be determined. From the combination of this equation with equations (8) and (9) there follows, moreover, for the direct calculation of the steam quality x', the formula

$$rx'=q_1-q+A\sigma(p_1-p). \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Example. In a boiler, water has the steam pressure of $p_1=5$ atmospheres and its temperature is $t_1=152.22°$ [305.996°]. Let the hot boiler water discharge into the open air, so that $p=1$ atmosphere and $t=100°$ [212°]. Using Table 11 of the Appendix we now calculate, by equation (9), the steam quality x in the plane of the orifice; then

$$x=0.0929,$$

and consequently $1-x=0.9071$ is the quantity of water; from equation (10) we find that the specific volume v at the orifice is

$$v=0.1532 \text{ [2.4541]};$$

furthermore from equation (8),

$$A\frac{w^2}{2g} = 3.516 \text{ Cal. } [6.3288 \text{ B.t.u.}],$$

and from this we get the efflux velocity

$$w = 171.02 \text{ m. } [561.097 \text{ ft.}].$$

The mixture of steam and water, measured in kilograms [pounds], which flows per second through F sq. m. [sq. ft.] is, according to equation (11),

$$G = 1108.99\, F$$
$$[G = 227.12\, F],$$

and of this there exists in the shape of steam

$$D = 130.0\, F$$
$$[D = 26.626\, F].$$

The steam quality x', after the distribution, is found from equation (13) or (14):

$$x' = 0.1068.$$

If, as in the preceding example, we carry through the calculations for different boiler pressures, and under the supposition that the pressure in the orifice in all cases is identical with the external atmospheric pressure, we get the peculiar result that the weight G of water and steam discharged per second is nearly c o n s t a n t for every boiler pressure; it is therefore approximately independent of the boiler pressure and thus depends only on the magnitude of the external pressure, the pressure of the orifice,[1] a proposition which can be theoretically justified, as will be shown at once.

Here the efflux velocity w and the steam quality x in the plane of the orifice appear greater the higher the boiler pressure considered.

Accordingly, if we allow water under steam pressure to flow from a calibrated boiler through the water cock, then, according to the preceding, equal quantities should be discharged in equal times, independently of the boiler pressure; whether the emptying

[1] See author's article: "Über den Ausfluss von Dämpfen und hocherhitzten Flüssigkeiten." Zivilingenieur, Vol. 10, 1864, p. 105.

of the boiler takes place under high or low pressure, the time of discharge must lead to the same value, provided that in each separate experiment the water level in the boiler is the same at the beginning and at the end of the blowing off.

It must be noted of course that these results are only valid under the express proviso that the resistances are so very small that they can be neglected.

When considering the resistances, we must make use of the results furnished above by the second transformation of the general equations.

For this case, equation (VI*a*) gives, when we substitute $w_1 = 0$ and $v_1 = \sigma$,

$$A\frac{w^2}{2g} = \left[\frac{r_1}{u_1 T_1}\sigma - \omega_0 + (\omega + \omega_0)\frac{(T_1 - T)}{2T}\right](T_1 - T), \quad . \quad (15)$$

and from equation (5*a*) there results

$$\frac{r}{u}v = \frac{T}{T_1}\frac{r_1}{u_1}\sigma + (\omega + \omega_0)(T_1 - T). \quad . \quad . \quad . \quad . \quad (16)$$

If we again neglect the resistances here, we must make $\omega_0 = 0$; if we also neglect the terms containing the factor σ there will follow, from equations (15) and (16),

$$w = (T_1 - T)\sqrt{\frac{\omega}{A}\frac{g}{T}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (15a)$$

$$\frac{r}{u}v = \omega(T_1 - T), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16a)$$

and therefore from the relation $Gv = Fw$ we get for the discharge G the approximate formula

$$G = F\frac{r}{u}\sqrt{\frac{g}{A\omega T}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

This formula, in fact, no longer contains any magnitude which depends upon the boiler pressure p_1, and thus we have shown theoretically that in the discharge of boiler water under steam pres-

sure the quantity of the discharge may be regarded as approximately independent of the boiler pressure. If the discharge takes place into the open air and we substitute in the preceding formula for r, u, and T the values corresponding to the atmospheric pressure, we find, because for water $\omega = 1$,

$$G = 1090.2\, F$$
$$[G = 223.287\, F],$$

or 0.1090 kg. [lb.] per sq. cm. [1.5512 lb. per sq. in.] of orifice area. (see example on p. 161).

The results of the preceding investigation depart completely from the results furnished by known hydraulic laws.

For the simple efflux of cold water, or when the changes of temperature during the flow toward the orifice are so small that steam formation does not occur, the equations (8) to (11), p. 160, give

$$\frac{w^2}{2g} = \sigma(p_1 - p);$$

furthermore $x = 0$, $v = \sigma$, and accordingly the weight of the water, which flows per second through the cross-section of F sq. m. [sq. ft.] (neglecting resistances), is

$$G = F\frac{w}{\sigma}.$$

From these formulas we have computed, for the different boiler pressures given in the first row, the water discharged in kilograms [pounds] per second through an orifice area of one square centimeter [square inch], cold water being assumed. The second row gives the discharge of water (measured in the interior of the boiler) which attains efflux under the corresponding steam pressure and is computed from equations (8) to (11).

Boiler pressure:	2	4	6 atmospheres.
	1.4238	2.4662	3.1838 kg.
	0.1095	0.1106	0.1113 kg.
[	20.251	35.077	45.283]
[	1.5574	1.5731	1.5830]

The difference of the numerical values of the two rows is evidently very marked, and in the second row the equality, already discussed, is apparent. The attempt that I made some years ago to test the foregoing propositions experimentally, by collecting in a receiving vessel the discharging mass of steam and water and cooling and condensing it there by a jet of cold water and then measuring it, did not succeed. Recently experiments of another kind, in the direction considered, have become known.

Sauvage[1] induced the engineers Pulin and Bonnin to make these experiments, which were conducted in 1890. An upright, cylindrical steam boiler (the Field boiler) was provided with a well-rounded orifice 22.4 mm. [0.882 in.] in diameter, and connected with the water space of the boiler by a pipe only about 200 mm. [7.874 in.] long, provided with a cock; the discharge therefore took place in a horizontal direction. The quantity of water discharged from the boiler during the period of efflux was determined from the boiler cross-section (above the heating tubes) and from the observed lowering of the water level (after boiler feed had been shut off).

The complete statement of this series of experiments must be looked for in Sauvage's article; in the following there are presented, for a few boiler pressures, the observed mean values for the discharge in kilograms [pounds] per sq. cm. [sq. in.] of orifice area.

Boiler pressure:	2	4	6 atmospheres.
	1.09	1.15	1.34 kg.
	[15.503	16.357	19.059 lb.]

The first value, corresponding to a boiler pressure of 2 atm., is reported as being quite unreliable. But if we compare the values here given with those in the second row of the preceding tabulation, it will be noticed that the observations differ greatly from the results of my calculations; the former, as Sauvage points out without further remark, give about ten times more discharge, but the enormous discrepancy is explained by the short length of

[1] Sauvage, "Écoulement de l'eau des chaudières." Annales des Mines, 1892. Ninth Series, Vol. II, p. 192.

the orifice of discharge. In deriving the formulas given above, it is assumed that there is reversible adiabatic expansion during the flow toward the orifice, and therefore that when discharge into the open air takes place both the water and the steam have the temperature of 100° C. [212° F.] in the plane of the orifice. This assumption, however, is evidently not realized with such s h o r t orifices as existed in these experiments.

The quantity of water discharged is disproportionately excessive, relatively to the steam discharged, and we must therefore assume that on this short path the water does not experience at all the prescribed lowering of temperature; for example, with 6 atmospheres of boiler pressure the temperature of the water does not fall from 159.2° to 100° C. [318.56° F. to 212° F.]. Consequently—always keeping in mind orifices inserted directly into the boiler wall—the water will reach the orifice itself in a h i g h l y s u p e r - h e a t e d condition and o u t s i d e of the orifice, corresponding to the temperature of 100° [212°], will rapidly evaporate. Doubtless with short orifices there exists, during the flow toward the orifice of efflux, a n o n - r e v e r s i b l e p r o c e s s. That this is the case is clearly shown by the reproductions which S a u v a g e gives in the photographs of the steam and water jet outside the orifice (see p. 198 and p. 199). The steam jet appears in the form of a p a r a b o l o i d o f r e v o l u t i o n whose vertex lies near the plane of the orifice, the jet spreading more the greater the boiler pressure.

The ordinary steam jet looks very different; from the orifice outward, for a long distance, it expands simply in c o n i c a l fashion.

Doubtless the observations on the discharge of boiler water would agree better with the results of the above theoretical investigation if we gave the hot water flowing toward the orifice a chance to cool down to a pressure corresponding to the orifice pressure, which could be realized by inserting a long pipe between the boiler wall and orifice. On the other hand, when discharging ordinary boiler steam the short orifice may be inserted directly in the boiler shell, because here the small amount of water mixed with the vapor can easily evaporate, and thus perfectly follow the change of temperature.

If the whole question of the discharge of boiler water were of special technical importance, and it may become so sometime, then the repetition of more careful experiments would appear desirable. It would then be well to extend more fully than was done above the theoretical investigation of the occurrences here taking place.

§ 23. EFFLUX OF DRY SATURATED VAPORS FROM SIMPLE ORIFICES UNDER CONSTANT PRESSURE.

Let the steam flow from a large steam space, in a boiler which is maintained at constant pressure, through a simple orifice to another space in which the pressure is kept at constant height (hence we must make $w_1 = 0$ in equation (I*d*), p. 156, and must substitute $x_1 = 1$, because it is assumed that dry saturated steam exists in the discharging vessel); we then get for the calculation of the efflux velocity w the equation

$$A\frac{w^2}{2g} = \frac{r_1}{T_1}(T_1 - T) + q_1 - q - T(\tau_1 - \tau) + A\sigma(p_1 - p), \qquad (18)$$

where the pressure p and the temperature T in the plane of the orifice are assumed as known.

The steam quality x in the plane of the orifice is found according to equation (3), p. 156, from

$$\frac{xr}{T} = \tau_1 - \tau + \frac{r_1}{T_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

the specific volume from

$$v = xu + \sigma, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

and then the weight G of steam and water, which flows per second through F square meters [square feet] of orifice section, is found from the equation

$$Gv = Fw. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

But here, as there, these equations presuppose adiabatic efflux and are also subject to the express assumption that the resistances may be neglected.

In place of the foregoing equations, however, use may be made of the equations of the second transformation (p. 156, etc.).

If we take the resistances into account, then, according to equation (VI), p. 159, we have for $w_1 = 0$:

$$A\,\frac{w^2}{2g} = \frac{r_1}{u_1 T_1}(T_1 - T)v_1 + \omega(T_1 - T) - (\omega + \omega_0)T\,\log_e \frac{T_1}{T}, \quad (22)$$

where $v_1 = u_1 + \sigma$ represents the specific volume of the dry steam in the discharging vessel. The value ω for steam must be put equal to unity when ω_0 represents an experimental value which depends upon the magnitude of the resistances.

The specific volume v, in the plane of the orifice, may be derived according to equation (V), p. 159, from the formula

$$\frac{r}{uT}\,v = (\omega + \omega_0)\,\log_e \frac{T_1}{T} + \frac{r_1}{u_1 T_1}v_1, \quad . \quad . \quad . \quad . \quad (23)$$

the steam quality x at this place follows from

$$v = xu + \sigma, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

and the discharge from the equation

$$Gv = Fw. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

If we neglect the resistances, we must substitute $\omega_0 = 0$ in these formulas; they then lead to the same results as equations (18) to (21).

Example. Suppose steam to flow from a boiler carrying the pressure $p_1 = 1.6$ atmospheres into the open air. In so doing it is assumed that the external atmospheric pressure extends up to the plane of the orifice, so that we may write $p = 1$ atmosphere. Here we must, for steam, place $\omega = 1$, and, according to Table 11 of the Appendix, substitute in the preceding formulas the quantities:

$T_1 = 386.69$, $r_1 = 526.786$, $u_1 = 1.0596$, $v_1 = 1.0606$ for $p_1 = 1.6 \times 10333$,
$T = 373.00$, $r = 536.500$, $u = 1.6495$, for $p = 10333$,
$T_1 = 696.042$, $r_1 = 948.215$, $u_1 = 16.974$, $v_1 = 16.990$ for $p_1 = 1.6 \times 2116.31$,
$T = 671.40$, $r = 965.700$, $u = 26.423$, for $p = 2116.31$.

Let us first work out the example without considering the resistances; i.e., for $\omega_0 = 0$, and next work it out on the supposition that the consideration of the resistances has led to the value $\omega_0 = 0.125$. We will put the approximate results for these two cases side by side.

Here we find for

$$\omega_0 = 0 \quad \text{or} \quad \omega_0 = 0.125,$$

according to equation (22),

$$A\frac{w^2}{2g} = 18.917 \quad \text{or} \quad A\frac{w^2}{2g} = 17.236$$
$$\left[A\frac{w^2}{2g} = 34.051 \quad \text{or} \quad A\frac{w^2}{2g} = 31.025\right],$$

and from this the efflux velocity:

$$w = 396.70 \quad \text{or} \quad w = 378.66$$
$$[w = 1301.5 \quad \text{or} \quad w = 1242.3].$$

The ratio of the second value to the first is called in hydraulics the coefficient of velocity; the ratio is 0.954. From equation (23) there now follows the specific volume in the orifice:

$$v = 1.6050 \quad \text{or} \quad v = 1.6218$$
$$[v = 25.710 \quad \text{or} \quad v = 26.040].$$

And the steam quality x at this place is

$$x = 0.9724 \quad \text{or} \quad x = 0.9826.$$

The resistances, therefore, increase the specific volume and the steam quality. Finally the discharge of steam and water in kilograms [pounds], according to equation (25), amounts to

$$G = 247.16\,F \quad \text{or} \quad G = 233.48\,F$$
$$[G = 50.621\,F \quad \text{or} \quad G = 47.82\,F]$$

The ratio of the second value to the first is designated in hydraulics as the coefficient of efflux, and is here

$$0.945.$$

The quantity of steam discharged per second is

$$Gx = 240.34\,F \quad \text{or} \quad Gx = 229.42\,F$$
$$[Gx = 49.225\,F \quad \text{or} \quad Gx = 46.988\,F],$$

and the quantity of water:

$$G(1-x) = 6.82\,F \quad \text{or} \quad G(1-x) = 4.06\,F$$
$$[G(1-x) = 1.397\,F \quad \text{or} \quad G(1-x) = 0.832\,F].$$

The example may well correspond to the conditions which exist with a conical adjutage that is well rounded on the inside.

With what has been given, the question of efflux of steam might be regarded as settled, were it not that here, on closely examining the given formulas, we sometimes encounter, as in the efflux of gases (see Vol. I, p. 240), numerical results which seem unacceptable, for example, if we hold fast, as in the preceding example, to the assumption that the pressure in the receiving vessel is always identical with the pressure in the orifice. For a fuller presentation of the question indicated, we will first convert the above formulas (22) to (25) to approximate ones.

If, in so doing, we neglect the resistances by putting $\omega_0 = 0$, then we find that in equation (22) the second and third terms of the right member are nearly equal, so that we can write

$$\frac{w^2}{2g} = \frac{r_1}{Au_1T_1}(T_1 - T)v_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Moreover, the first term of the right member of equation (23) can be neglected because it is very small and we can thus write

$$\frac{r}{uT}v = \frac{r_1}{u_1T_1}v_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

If we utilize Clapeyron's equation (22), p. 29, in these two formulas, namely, use

$$\frac{r}{AuT} = \frac{dp}{dt}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

abbreviating with ϕ in so doing the differential coefficient of the right member, we shall get

$$\frac{w^2}{2g} = \phi_1(T_1 - T)v_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26a)$$

also

$$\phi v = \phi_1 v_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27a)$$

and there accordingly follows, from the relation $Gv = Fw$ for the discharge G of steam and water, the approximate formula

$$\frac{G}{F} = \frac{\phi}{\phi_1}\sqrt{\frac{2g\phi_1}{v_1}(T_1 - T)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

or

$$\frac{G}{F}=\sqrt{\frac{2g}{\phi_1 v_1}}\left[\frac{dp}{dt}\sqrt{t_1-t}\right]. \quad . \quad . \quad . \quad . \quad . \quad (29a)$$

If we conceive of a series of experiments in which pressure p_1 in the discharging vessel is always the same, and if we assume the pressure p in the receiving vessel and the pressure in the plane of the orifice to be equal to the external pressure, then it may be expected that the preceding expression will make the discharge G greater as the external pressure becomes smaller.

But this expectation is not realized; instead of that, the value of the bracket in the preceding equation (29*a*) gives, for a particular temperature t_1, a value of t or p for which G becomes a maximum; with a further decrease of t or p, the discharge will accordingly again diminish.

Here, therefore, the same case occurs which was more fully discussed when investigating the efflux of gases (Vol. I, p. 240), and which led to the hypothesis of de Saint-Venant and Wantzel; this case arises at the instant in which the orifice pressure p can be no longer assumed as identical with the external pressure.

There is now no difficulty, with equation (29*a*) as a help, in discussing the question still further, but it is only necessary to discuss it approximately. Far more reliable formulas are obtained when we remember, in transforming the general equations (18) to (21), that the adiabatic curve of dry steams and wet steams has the same general course as for gases, with the difference that the exponent κ for steam is given by the relation $\kappa=1.035+0.1\,x_1$, and therefore for dry steam, for which $x_1=1$, we must use $\kappa=1.135$.

In such a case we use for short simple orifices the formulas which were developed in Vol. I, pp. 247 and 250. If we at the same time introduce the resistances, the adiabatic curve will pass over into the polytropic curve with the exponent n instead of κ, and designating the coefficient of resistance by ζ there obtains the relation (Vol. I, equation 32, p. 247)

$$\zeta=\frac{\kappa-n}{\kappa(n-1)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

According to the propositions there presented we calculate the orifice pressure p, for the maximum of discharge, from the equation

$$\frac{p}{p_1}=\left(\frac{2}{\kappa+1}\right)^{\frac{n}{n-1}}, \quad \ldots\ldots\ldots \quad (31)$$

and the specific volume v in the orifice from

$$\frac{v}{v_1}=\left(\frac{p_1}{p}\right)^{\frac{1}{n}}, \quad \ldots\ldots\ldots \quad (32)$$

where p_1 and v_1 represent the pressure and volume of the steam in the discharging vessel, which values can be regarded as given; the pressure p_2, in the receiving vessel outside of the orifice, is also regarded as known.

We must here distinguish between two cases according to the value of p resulting from equation (31).

First Hypothesis. Let $p_2>p$.

Here the external pressure establishes itself in the orifice, and hence $p=p_2$ and the specific volume v_2 in the orifice is

$$v_2=v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}. \quad \ldots\ldots\ldots \quad (32a)$$

The steam quality x_2 in the orifice is calculated from the formula

$$v_2=x_2u_2+\sigma,$$

because the value u_2, corresponding to the pressure p_2, is known.

The efflux velocity w is determined (Vol. I, equation (33), p. 247) by the formula

$$w=\sqrt{\frac{2g\kappa}{\kappa-1}p_1v_1\left[1-\left(\frac{p}{p_1}\right)^{\frac{n-1}{n}}\right]}, \quad \ldots\ldots \quad (33)$$

and the discharge of steam and water, in kilograms [pounds] per second, is found from

$$G=F\frac{w}{v}, \quad \ldots\ldots\ldots \quad (34)$$

taking $p=p_2$ and $v=v_2$ and letting F represent the cross-section

of the orifice in square meters [square feet]. The weight of the steam in the orifice is

$$D=Gx_2 \quad \text{with} \quad x=x_2.$$

Second Hypothesis. Let $p_2<p$ be assumed.

Here the pressure p of the orifice is at once found from equation (31); it is then independent of the external pressure and is valid even for discharge into a vacuum. The specific volume v in the orifice follows from equation (32) and the steam quality x from the formula

$$v=xu+\sigma,$$

where u corresponds to the pressure p.

The efflux velocity w is determined from the combination of equations (33) and (31):

$$w=\sqrt{\frac{2g\kappa}{\kappa+1}p_1v_1}, \quad \ldots\ldots\ldots \quad (33a)$$

the discharge is found from

$$G=F\frac{w}{v}, \quad \ldots\ldots\ldots \quad (34a)$$

and the corresponding weight of steam is

$$D=Gx.$$

In all these formulas we must substitute $n=\kappa=1.135$ for dry steam and substitute $n=1.035+0.1\,x_1$ for wet steam, where x_1 is the steam quality in the discharging vessel itself.

Example 1. Suppose dry saturated steam flows from a boiler into the open air, under the pressure $p_1=1.6$ atmospheres.

Let us first work out the example neglecting the resistances, and next, under the assumption $n=\kappa=1.135$ and side by side with this first calculation, let us work out the example for the efflux exponent $n=1.121$, which value, according to equation (30), corresponds to a coefficient of resistance $\zeta=0.1$, a value which might arise with, say, a conical orifice well rounded on the inside.

We then get for

$$\kappa=1.135 \quad \text{and} \quad n=1.121,$$

according to equation (31), the controlling pressure ratio:

$$\frac{p}{p_1}=\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}=0.5774 \quad \text{and} \quad \left(\frac{2}{\kappa+1}\right)^{\frac{n}{n-1}}=0.5460,$$

and from this, for $p_1=1.6$,

$$p=0.9328 \quad \text{and} \quad p=0.8736,$$

in both cases, therefore, values <1 or $p<p_2$; consequently we have before us the first of the two hypotheses mentioned above.

Since the specific volume, corresponding to the pressure of 1.6 atmospheres, amounts to $v_1=1.0606$ [16.990], there follows from equation (32)

$$v=1.6047 \text{ [25.706]} \quad \text{and} \quad v=1.6131 \text{ [25.840]},$$

and we then get from the relation $v=xu+\sigma$, because $u=1.6495$ [26.423] for one atmosphere of pressure, the steam quality in the orifice:

$$x=0.972 \quad \text{and} \quad x=0.977.$$

The efflux velocity w, calculated from equation (33), is found to be

$$w=397.02 \quad \text{and} \quad w=378.27$$
$$[w=1302.6 \quad \text{and} \quad w=1241.1];$$

finally the discharge per second, with an orifice of F sq. m. [sq. ft.], is

$$G=\frac{w}{v}F=247.41\,F \quad \text{and} \quad G=234.50\,F \text{ kg}.$$

$$[G=\frac{w}{v}F=50.673\,F \quad \text{and} \quad G=48.029 \text{ lb.}],$$

whereupon there can easily be found the steam weight Gx and the water weight $G(1-x)$.

Example 2. Let steam flow from a boiler, under the same assumptions as in the preceding example; let the boiler steam be at a pressure $p_1=5$ atmospheres and let it flow into the open air.

Here again, for $\kappa=1.135$ and 1.121 respectively, the controlling pressure ratio becomes, as in the preceding example,

$$\frac{p}{p_1}=0.5771 \quad \text{and} \quad 0.5460,$$

and hence the orifice pressure is

$$p=2.887 \quad \text{and} \quad 2.730 \text{ atmospheres},$$

and is therefore greater than one atmosphere; accordingly we have before us the second of the above hypotheses.

From equation (32) there is found

$$\frac{v}{v_1} = 1.6223 \quad \text{and} \quad 1.7157,$$

and hence the specific volume in the orifice, because $p_1 = 5$ atmospheres and $v_1 = 0.3636$ [5.8245], is

$$v = 0.5899 \quad \text{and} \quad 0.6238$$
$$[v = 9.4496 \quad \text{and} \quad 9.9926].$$

Corresponding to the pressure in the orifice we find, by interpolation from Table 11 of the Appendix, the values

$$u = 0.6081 \quad \text{and} \quad u = 0.6411$$
$$[u = 9.7411 \quad \text{and} \quad u = 10.270].$$

Accordingly we find from $v = xu + \sigma$ the steam quality in the orifice as

$$x = 0.969 \quad \text{and} \quad x = 0.971.$$

Now, from equation (33a), we compute the efflux velocity for both cases:

$$w = 442.64 \text{ m. } [1452.3 \text{ ft.}],$$

and then, from equation (34a), the discharge:

$$G = 750.36\,F \quad \text{and} \quad G = 709.59\,F \text{ kg.}$$
$$[G = 153.68\,F \quad \text{and} \quad G = 145.33\,F \text{ lb.}].$$

Of this there appears in the orifice in the form of steam:

$$Gx = 726.35\,F \quad \text{and} \quad Gx = 689.01\,F \text{ kg.}$$
$$[Gx = 148.76\,F \quad \text{and} \quad Gx = 141.12\,F \text{ lb.}],$$

where F represents the cross-section of the orifice in square meters (square feet].

By the methods of computation employed in the preceding examples, and by neglecting the efflux resistances and presupposing short, simple orifices, the results in the following table have been found. The computed results of the table differ from the statements of the table which I published in earlier works,[1] in which it was assumed that the external pressure p_2 extended to the plane of the orifice in all cases; it was in my later works that I used, as I now do, the hypothesis of de Saint-Venant and Wantzel, after I had convinced

[1] "Lokomotivenblasrohr." Zürich, 1863, p. 84. Zivilingenieur, Vol. X, 1864, p. 99. 2d edition of present book, 1866, p. 414. New reprint of the book, 1877, p. 414.

Discharge of Dry Saturated Steam in the Open Air through a Simple Rounded Orifice, Neglecting Resistances. (Metric Measure.)

	1.	2.	3.	4.	5.	6.	7.	8.
Pressure.	Specific Volume.	Pressure.	Specific Volume.	Steam Quality.	Efflux Velocity.	Discharge in Kilograms per Second. Orifice Area F in Square Meters.		
p_1	v_1	p	v	x	w			
in Boiler.		in the Plane of the Orifice.				Mixture.	Steam.	Water.
Atmospheres.		Atmospheres.			Meters.	$\frac{G}{F}$	$\frac{Gx}{F}$	$\frac{G(1-x)}{F}$
1.1	1.5088	1	1.6410	0.994	178.5	108.8	108.1	0.7
1.2	1.3902	1	1.6324	0.989	247.0	151.3	149.6	1.7
1.4	1.2025	1	1.6174	0.980	335.5	207.4	203.3	4.1
1.6	1.0606	1	1.6047	0.972	397.0	247.4	240.5	6.9
1.8	0.9494	1.039	1.5402	0.965	429.2	278.6	268.9	9.7
2	0.8599	1.155	1.3950	0.966	430.5	308.6	298.1	10.5
3	0.5875	1.732	0.9531	0.967	435.8	457.3	442.2	15.1
4	0.4484	2.310	0.7274	0.968	439.7	604.4	585.1	19.3
5	0.3636	2.887	0.5899	0.968	442.6	750.4	726.4	24.0
6	0.3064	3.464	0.4971	0.969	445.1	895.4	867.7	27.7
7	0.2652	4.042	0.4302	0.969	447.3	1039.7	1007.5	32.2
8	0.2339	4.619	0.3795	0.969	449.1	1183.3	1146.6	36.7
9	0.2095	5.197	0.3399	0.969	450.8	1326.2	1285.1	41.1
10	0.1897	5.774	0.3078	0.969	452.2	1469.0	1423.5	45.5
11	0.1735	6.351	0.2815	0.969	453.5	1611.1	1561.2	49.9
12	0.1599	6.929	0.2594	0.969	454.7	1753.1	1698.7	54.4
13	0.1483	7.506	0.2406	0.969	455.8	1894.5	1835.8	58.7
14	0.1383	8.084	0.2244	0.969	456.8	2035.7	1972.6	63.1

Discharge of Dry Saturated Steam in the Open Air through a Simple Rounded Orifice, Neglecting Resistances. (English Measure.)

	1.	2.	3.	4.	5.	6.	7.	8.
Pressure.	Specific Volume.	Pressure.	Specific Volume.	Steam Quality.	Efflux Velocity.	Discharge in Pounds per Second. Orifice Area F in Square Feet.		
p_1	v_1	p	v	x	w			
in Boiler.		in the Plane of the Orifice.				Mixture.	Steam.	Water.
Atmospheres.		Atmospheres.			Feet.	$\frac{G}{F}$	$\frac{Gx}{F}$	$\frac{G(1-x)}{F}$
1.1	24.1693	1	26.2871	0.994	585.6	22.3	22.1	0.14
1.2	22.2695	1	26.1490	0.989	810.4	31.0	30.6	0.35
1.4	19.2627	1	25.9090	0.980	1100.7	42.5	41.6	0.84
1.6	16.9896	1	25.7056	0.972	1302.5	50.7	49.3	1.41
1.8	15.1280	1.039	24.6723	0.965	1408.2	57.1	55.1	1.99
2	13.7747	1.155	22.3464	0.966	1412.4	63.2	61.1	2.15
3	9.411	1.732	15.2676	0.967	1429.8	93.7	90.6	3.09
4	7.1829	2.310	11.6525	0.968	1442.6	123.8	119.8	3.95
5	5.8245	2.887	9.4496	0.968	1452.1	153.7	148.8	4.82
6	4.9082	3.464	7.9630	0.969	1460.3	183.4	177.7	5.67
7	4.2482	4.042	6.8913	0.969	1467.5	212.9	206.4	6.60
8	3.7468	4.619	8.0722	0.969	1473.1	242.4	234.8	7.52
9	3.3560	5.197	5.4449	0.969	1479.0	271.6	263.2	8.42
10	3.0388	5.774	4.9306	0.969	1483.6	300.9	291.6	9.32
11	2.7793	6.351	4.5093	0.969	1487.9	330.0	319.8	10.22
12	2.5614	6.929	4.1553	0.969	1491.8	359.1	347.9	11.14
13	2.3756	7.506	3.9541	0.969	1495.4	388.0	376.0	12.02
14	3.2154	8.084	3.5946	0.969	1498.7	416.9	404.0	12.93

myself of its reliability by extensive experiments on the efflux of air.

At this place we may add, as the closing statement of the theoretical studies on the efflux of vapors, that the last given formulas for the efflux of saturated vapors can be converted into much simpler form for practical use.

For this purpose let us assume that during the efflux into the open air the pressure of the steam in the boiler does amount to more than 1.8 atmospheres, an assumption which will obtain in most practical cases, then the pressure p which is computed from equation (31), p. 172, where $n = \kappa$, will be established in the orifice.

It was shown on p. 37, by equation (33), that there exists for dry saturated steam, between the pressure limits occurring in practice, the relation

$$p_1 v_1^{\mu} = C, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

where $\mu = 1.0646$ and $C = 1.7049$ [32.67], when the pressure p_1 is given in atmospheres and v_1 is the corresponding specific volume of dry saturated steam in cbm. [cu. ft.].

If the pressure p_1 is expressed in kilograms per square centimeter [pounds per square inch], then we must take $C = 1.7617$ [480.144]. Eliminating v_1 from equation (35) we get, with the help of equations (32), (33a), and (34a),

for p_1 in atmospheres:		for p_1 in kg.: sq. cm. [lb.: sq. in.]:	
$w = 421.67\ p_1^{0.0303}$	(36)	$w = 421.36\ p_1^{0.0303}$	(36a)
$G = 157.55\ F p_1^{0.9696}$	(37)	$G = 152.59\ F p_1^{0.9696}$	(37a)
$\left[\begin{array}{l} w = 1383.4\ p_1^{0.0303} \\ G = 32.268\ F p_1^{0.9696} \end{array}\right.$		$\left.\begin{array}{l} w = 1275.2\ p_1^{0.0303} \\ G = 0.016545\ F p_1^{0.9696} \end{array}\right]$	

In the last formula for G the area F is in square inches.

From equation (35) there is found, approximately,

$$p u^{\kappa} = p_1 u_1^{\kappa}.$$

Moreover we get from $v = xu + \sigma$ and $v_1 = x_1 u_1 + \sigma$, after neglecting σ, the ratio

$$\frac{v}{v_1}=\frac{xu}{x_1u_1},$$

where x and x_1 represent the steam quality in the orifice and discharging vessel respectively. From the preceding equation there is found $u:u_1$, and from equation $pv^\kappa=p_1v_1^\kappa$ the ratio $v:v_1$; hence follows

$$\frac{x}{x_1}=\left(\frac{p}{p_1}\right)^{\frac{\kappa-\mu}{\kappa\mu}},$$

and finally, with the help of equation (31) and with $n=\kappa$,

$$\frac{x}{x_1}=\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa-\mu}{\mu(\kappa-1)}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

If we make $x=1$ and substitute the value $\kappa=1.135$ and $\mu=1.0646$, we have

$$x=0.9685,$$

in complete agreement with the statements of the table on p. 177.

For the boiler pressure of $p_1=5$ atmospheres we get, from equations (36) and (37), $w=442.7$ m. [1452.4 ft.] and $G=750.3\,F$ kg. [153.67 F lb.], which correspond almost exactly with the statements of the table.

The investigations presented here, we repeat, are valid under the supposition that the pressure p_1, in the discharging vessel, amounts to more than 1.8 times the external pressure in the receiving space; in this case the pressure p in the plane of the orifice is greater than in the receiving space, which latter pressure was designated above by p_2.

If we now follow the steam jet outwardly, away from the orifice, we find that it spreads out gradually and therefore at a certain distance, estimated from the plane of the orifice, there will exist a cross-section of the jet in which the pressure has just become identical with the external pressure p_2. Let us call this cross-section the "efflux cross-section" and designate it by F_0; this section is accordingly to be distinguished from the "orifice cross-section" F, the relation between the two being $F_0>F$; only for the case in which the inner pressure p_1 is less than 1.8 times the external pressure will the orifice pressure p and the external pressure p_2 be

identical; when this happens the efflux cross-section will coincide with the orifice cross-section.

If the first-mentioned case exists, we find the discharge for dry saturated steam, for $n=\kappa$, and the combination of equations (33), (34), and (32a), p. 172, to be

$$G=F_0\sqrt{\frac{2g\kappa}{\kappa-1}\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{\kappa}}-\left(\frac{p_2}{p_1}\right)^{\frac{\kappa+1}{\kappa}}\right]}; \quad . \quad . \quad . \quad (39)$$

moreover the combination of equations (33a), (34a), (32), and (31) and the orifice cross-section F gives

$$G=F\sqrt{\frac{2g\kappa}{\kappa+1}\frac{p_1}{v_1}\left(\frac{2}{\kappa+1}\right)^{\frac{2}{\kappa-1}}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Equating we get

$$\left(\frac{F_0}{F}\right)^2=\frac{\left(\frac{\kappa-1}{\kappa+1}\right)\left(\frac{2}{\kappa+1}\right)^{\frac{2}{\kappa-1}}}{\left(\frac{p_2}{p_1}\right)^{\frac{2}{\kappa}}-\left(\frac{p_2}{p_1}\right)^{\frac{\kappa+1}{\kappa}}}, \quad . \quad . \quad . \quad . \quad . \quad (41)$$

or, with $\kappa=1.135$,

$$\frac{F_0}{F}=\frac{0.1550}{\sqrt{\left(\frac{p_2}{p_1}\right)^{\frac{2}{\kappa}}-\left(\frac{p_2}{p_1}\right)^{\frac{\kappa+1}{\kappa}}}}, \quad . \quad . \quad . \quad . \quad (42)$$

according to which the efflux cross-section F_0 can be computed from the orifice cross-section F, when the inner and outer pressures p_1 and p_2 are given.

The last formulas, just developed, have recently found an important, technical application. Suppose a pipe is added to the short efflux orifice F, which gradually enlarges from F to F_0, then the steam will flow through the efflux cross-section F_0 with the external pressure p_2.

In this way de Laval has designed the steam nozzle for his steam turbine; the efflux through the nozzle I have completely discussed quite recently[1]; different experiments on the steam consumption of the de Laval steam turbine have already established the reliability of the calculations.

[1] Zeuner, "Vorlesungen über Theorie der Turbinen." Leipsic, 1889, p. 267.

§ 24. EXPERIMENTS ON THE EFFLUX OF STEAM.

There are but few experimental investigations on the efflux of steam. Kolster, reporting[1] on the older experiments, presents his own experiments, which relate especially to the efflux of steam from the safety valves of boilers, and in so doing mentions the oldest ones, those by Trémery,[2] who attempted to establish, experimentally, rules for the determination of the dimensions of safety valves; it is required of such a valve that when open it should blow off all the steam formed in the boiler, even when the boiler is excessively heated, so that an accumulation of steam, and the consequent rise of pressure connected with it, may be impossible.

For this purpose Tremery provided his experimental boiler with a large cock possessing an orifice of efflux in a thin plate whose cross-section he regulated by operating with nine different efflux cross-sections; the smallest efflux cross-section amounted to 0.7653 sq. cm. [0.11862 sq. in.] and each of the following ones increased successively by 0.2551 sq. cm. [0.039541 sq. in.]; the greatest cross-section was therefore 2.8061 sq. cm. [0.43495 sq. in.].

The steam boiler was heated as uniformly as possible and, inserting the separate orifices in their order, he observed for each the steam pressure at the manometer and, as soon as the reading became stationary, recorded it.

If F is the orifice cross-section in square centimeters [square inches] and p_1 the steam pressure in atmospheres [lb. per sq. in.], then, when we disregard resistances, equation (37), p. 178, gives the steam weight G in kilograms [pounds] discharged since the pressure became stationary:

$$G = 0.01575\,\alpha\, F p_1^{0.9696}$$
$$[G = 0.016545\,\alpha\, F p_1^{0.9696}];$$

[1] Rud. Kolster in Helsingfors, "Über das Ausströmen von Dampf und Luft aus Gefässmündungen und über die Dimensionen der gebräuchlichen Sicherheitsventile." Zeitschrift des Vereins deutscher Ingenieure, Vol. XI, 1867, pp. 433 and 711.—Vol. XII, 1868, p. 97.

[2] Trémery, "Recherches sur l'écoulement de la vapeur." Annales des mines, 1841, Vol. XX, p. 343.

in this formula the coefficient of contraction α must be substituted in the right member, because, in Trémery's experiments, the efflux orifice was in a thin plate.

Now if the coefficient α were independent of the steam pressure and orifice cross-section, then, according to the preceding equation, the quantity

$$F p_1^{0.9696}$$

should lead to the same, or nearly the same, value for every orifice section F and for the corresponding pressure p_1 of the individual series of experiments. Trémery conducted five such series under different rates of heating.

A closer examination of Trémery's experimental results, however, wnose reproduction we will here omit, does not confirm this proposition; on the contrary, they show that the said quantity diminishes with increasing steam pressure and diminishing orifice section. Consequently we ought rather to draw the conclusian that in the preceding equation the product

$$\alpha F p_1^{0.9696}$$

is a constant, and that accordingly the coefficient of contraction α must increase when the pressure grows and the orifice cross-section becomes smaller, which is confirmed by the fact that when air discharges through an orifice in a thin plate it does really behave so, as is shown by my older experiments. Unfortunately, the Trémery experiments do not permit a further examination, but we see from what has preceded that the experimental results in question confirm, at least in a general way, the views developed above on the efflux of vapors; if Trémery had not used orifices in a thin plate, but orifices rounded on the inside, according to the form of the contracted jet, his experiments would have furnished an excellent support for the hypothesis of de Saint Venant and Wantzel.

Other experiments pointed out by Kolster are those of Minary and Résal;[1] unfortunately these experiments

[1] "Recherches expérimentales sur l'écoulement des vapeurs," by Minary and Résal. Annales des mines, 1861 Vol. XIX, p. 379. German in Zivil-ingenieur, Vol. VIII, 1862.

furnish no solid ground for deciding the question and were conducted under circumstances which render an examination of the occurrences, in a theoretical way, very difficult.

Kolster believes, nevertheless, that he is justified in at least drawing the conclusion that the results of these experiments do not conflict with the assumptions made above; the constants in the above formula cannot be deduced from the experimental results, consequently we will not present those of Minary and Resal here and will content ourselves with referring to the source mentioned. There are also recorded the experimental results which Kolster himself obtained for the efflux of steam through the safety valve of a locomotive boiler; they, too, indirectly confirm the correctness of the hypothesis discussed.

A somewhat clearer light was finally thrown on the whole question of the efflux of saturated steam by the experiments of Napier (1869), and it is these experiments alone which form any basis for deciding the question.

The author of this book, about the same time and without knowing of Napier's experiments, sought to advance the question by an entirely different experimental method, but omitted to publish his own results, although they furnished a confirmation of Napier's investigations. In the following the principal results of these experimental series will be discussed so far as our limited space permits.

Napier's experiments were first made accessible to a larger circle by Rankine, who expressly notes, in the introduction to his article,[1] that he considers the fundamental principle of the

[1] The Engineer of November 26th and December 3d, 1869; also in German under the title "Über den Ausfluss des Dampfes" in Zivilingenieur, Vol. XVI, 1870, p. 35. In this article Rankine discusses Napier's work (see Engineer of Oct. 1st, 1869) without adducing the formulas or the experimental methods, and compares Napier's presentation with my investigations, "Über den Ausfluss von Dämpfen und hocherhitzten Flüssigkeiten," Zivilingenieur, Vol. X, 1864.—In 1870 Rankine sent me in his letters with some hand drawings the little article by Napier, "On the Velocity of Steam and other Gases, and the True Principles of the Discharge of Fluids," which I handed to Prof. Fliegner in Zurich to be worked up for Zivilingenieur. This work, with the reproduction of the hand drawings mentioned, can be found in Zivilingenieur, Vol. XVII, 1871, p. 215.

theory of these experiments as perfectly correct. The principle just referred to relates to the determination of the pressure in the plane of the orifice, as it has been discussed in the investigations presented above and in the complete discussion on the efflux of gases in Vol. I of this work; that these propositions were given by de Saint Venant and Wantzel, long before Napier, was doubtless unknown to Rankine.

In Vol. I, p. 255, of this work there was derived under equation (47*a*) the following approximate formula for the efflux of gases:

$$G = F\sqrt{\frac{2g}{1+\zeta}\,\frac{p_2(p_1-p_2)}{BT_1}}.$$

Here p_1 and T_1 are the pressure and temperature in the discharging vessel, p_2 the pressure in the receiving vessel and in the plane of the orifice, and ζ the coefficient of resistance in the sense there given. The formula gives the weight G of the air in kilograms [pounds] which is discharged, per second, through an orifice of F sq. m. [sq. ft.] cross-section; if we use in the formula the relation $p_1 v_1 = BT_1$, which is valid for gases, where v_1 represents the specific volume of the gas in the discharging vessel, there follows

$$\frac{G}{F} = \sqrt{\frac{2g}{1+\zeta}\,\frac{p_1}{v_1}\left(1-\frac{p_2}{p_1}\right)\frac{p_2}{p_1}}, \quad \ldots\ldots \quad (43)$$

and that is, at least in form, the equation which Napier wants to have applied to the efflux of steam for slight differences of pressure.

Closer consideration of this formula shows that the discharge G becomes a maximum when

$$\frac{p_2}{p_1} = \frac{1}{2}; \quad \ldots\ldots\ldots \quad (44)$$

we get in this case

$$\frac{G}{F} = \frac{1}{2}\sqrt{\frac{2g}{1+\zeta}\,\frac{p_1}{v_1}}. \quad \ldots\ldots\ldots \quad (45)$$

If the pressure p_2, in the receiving vessel, is smaller than $\frac{1}{2}p_1$, and if it extends to the plane of the orifice, then with a constant

inner pressure p_1 the discharge G will be smaller than is given by the formula (45).

Napier therefore proceeds on the assumption that equation (43) must be used whenever

$$\frac{p_2}{p_1} > \frac{1}{2},$$

and in this case the external pressure p_2 extends to the plane of the orifice; on the other hand, that equation (45) is to be used when the pressure in the receiving vessel is subject to the relation

$$\frac{p_2}{p_1} < \frac{1}{2},$$

and that in this case the pressure in the plane of the orifice settles down to the value $\frac{1}{2}p_1$, i.e., to half the internal pressure p_1.

We see that Napier's assumption agrees perfectly with the hypothesis of de Saint-Venant and Wantzel so completely discussed above; the only difference is that his formulas must be regarded as approximate expressions of the above-given formulas.

If w is the efflux velocity, we get by means of the relation $Gv_1 = Fw$ from equation (43)

$$w = \sqrt{\frac{2g}{1+\zeta} p_1 v_1 \left(1 - \frac{p_2}{p_1}\right) \frac{p_2}{p_1}} \quad \text{for} \quad \frac{p_2}{p_1} > \frac{1}{2}, \quad . \quad . \quad . \quad (46)$$

provided the volume in the plane of the orifice is taken as approximately equal to that in the discharging vessel; then equation (45) gives

$$w = \frac{1}{2}\sqrt{\frac{2g}{1+\zeta} p_1 v_1} \quad \text{for} \quad \frac{p_2}{p_1} < \frac{1}{2}. \quad . \quad . \quad (47)$$

These equations are also given by Napier, but with the difference that his formulas do not contain the coefficient of resistance.

My experiments on the efflux of air through a well-rounded orifice gave $\zeta = 0.066$ (Vol. I, p. 270) for the coefficient of resistance.

If we transfer this value to the efflux of steam and suppose the

pressure in atmospheres to be substituted in equations (43) and (45), by placing there $10333\,p_1$ [2116.3 p_1] in place of p_1, we shall get for such an orifice

$$\frac{G}{F}=436\sqrt{\frac{p_1}{v_1}\left(1-\frac{p_2}{p_1}\right)\frac{p_2}{p_1}} \quad \text{and} \quad \frac{G}{F}=218\sqrt{\frac{p_1}{v_1}}$$

$$\left[\frac{G}{F}=357.39\sqrt{\frac{p_1}{v_1}\left(1-\frac{p_2}{p_1}\right)\frac{p_2}{p_1}} \quad \text{and} \quad \frac{G}{F}=178.697\sqrt{\frac{p_1}{v_1}}\right].$$

Some of Napier's experiments do in fact give the coefficients standing in front of the radical, but in general Napier gives for well-rounded orifices the two values 420 and 210 [344 and 172] respectively. Neglecting the resistances, i.e., for $\zeta=0$, equations (43) and 45) furnish the two coefficients 450 and 225 [369 and 184.5].

Now, as regards Napier's experiments, unfortunately the report is very incomplete, and the description of the experimental apparatus gives no clear insight into all parts of the different experiments.

In the main the experimental apparatus was of the kind indicated by Fig. 18.

The steam left the boiler by the pipe A, and was discharged through the well-rounded orifice B, but at the orifice there was screwed on a pipe C of the same inside diameter which was chosen of different lengths in the different experiments. From this tube the steam entered an enlarged pipe D, which constituted the real receiving vessel and in which the pressure p_2 was measured by a manometer a. From the receiving vessel D the steam flowed through a pipe E bent down to a catch-basin G, filled with cold water. The lower end of the pipe E nearly touched the

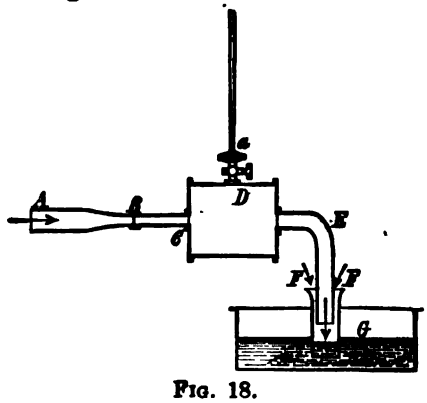

FIG. 18.

level of the water, and was surrounded by a somewhat larger pipe F which was open above and below; there was thus effected at the point of discharge of the pipe E a sudden change of cross-section so that air was sucked into pipe F from above, which caused the discharging steam to be forced under the water and completely condensed.

In the experiments it was a question of determining the boiler pressure p_1 corresponding to the cross-section F of the orifice B, to observe the pressure p_2 in the receiving vessel D, and besides to determine the weight of the steam discharged during the period of observation. The latter determination, however, was not effected by Napier by noting the water volume in the catch-basin G at the beginning and end of the experiment, and thus determining the volume and weight of the condensed steam. Instead, he calculated this weight from the weight of the water originally contained in the catch-basin and its original temperature, and also from the temperature exhibited at the end of the experiment by the water and condensed steam combined, and, in so doing, correctly took account of the heat quantity transferred to the walls of the apparatus. Whether this method of observation is more reliable than the direct observation of the increment of volume in the catch-basin, will not be discussed here. It is only of technical importance to know whether or not the hypothesis of de Saint-Venant and Wantzel is confirmed by the experiments of Napier, and the question as to the orifice pressure is particularly important, namely whether, when the external pressure is less than half the inner pressure, the discharge is the same for every value of the outer pressure, provided the inner pressure remains constant. For this case, therefore, the reliability of equation (45), which can be written in the form

$$\frac{G}{F}=\psi\sqrt{\frac{p_1}{v_1}}, \quad \ldots\ldots\ldots \quad (48)$$

must be investigated and the invariability of the coefficient ψ must be established in the manner attempted in Vol. I for the efflux of air.

In one series of experiments by Napier, whose results are presented in the following tabulation, and for which the apparatus indicated in Fig. 18 was employed, there was used a well-

1. Pressure p_2 in Receiving Space. Atmospheres.	2. $\frac{G}{F}$ kg.	3. ϕ
2.067	599.46	200.7
1.733	601.28	201.3 } 201 mean
0.733	599.25	200.6
1.800	668.32	223.8
1.800	663.24	222.1 } 224 mean
0.467	676.30	226.4
0.467	664.26	222.4

rounded orifice, whose diameter amounted to 14.3 mm. [0.563 in.]; the cross-section $F=0.000160606$ sq. m. [0.0017289 sq. ft.]. The boiler pressure p_1 in all the experiments was equal to four atmospheres, from which was found

$$\sqrt{\frac{p_1}{v_1}}=2.9867\ [=0.74625].$$

Col. 1 gives the observed pressure p_2 in the receiving space D; Col. 2 gives the value $G:F$ computed from Napier's statements, i.e., the discharge in kilograms [pounds] per second and per sq. m. [sq. ft.] of orifice area; Col. 3 gives the value of ϕ calculated from equation (48).

In the first three experiments the length of the pipe C (Fig. 18) amounted to 0.0953 m. [0.3127 ft.], but in the last four experiments to only 0.0127 m. [0.04167 ft.].

Those experiments in which the pressure in the receiving vessel was considerably smaller than half of the boiler pressure confirm the hypothesis with sufficient accuracy. That the value ϕ is smaller in the first three experiments than in the experiments of the second group is due to the greater length of the pipe C, which increases the coefficient of resistance ζ occurring in equation (45). The second group of experiments, on account of the shortness of

the pipe C, may be considered as representing almost exactly efflux through a well-rounded orifice and leads, as was shown on p. 185, to almost the same coefficient of resistance ζ found for the experiments on air discussed in Vol. I, p. 270.

In another experimental series made by Napier, with an orifice of 9.5 mm. [0.374 in.], the external pressure p_2, i.e., the pressure in the receiving vessel, was throughout equal to one atmosphere, the boiler pressure p_1 varying. The experiments furnished the following results:

Boiler pressure p_1. Atmospheres.	$\frac{G}{F}$	ϕ
1.800	292.22	160.3
2.067	333.12	159.9
3.000	467.79	156.4
4.000	623.03	157.6
		158.5 mean

In this experimental series also, the value of ϕ in the separate experiments differs but little from the mean value, so that they may be regarded as likewise confirming the proposition derived. That the value is here essentially smaller than in the preceding series is doubtless due to the much greater resistances existing in these experiments. Unfortunately nothing more definite can be found in Napier's paper, for the description gives no complete insight into the experimental method nor into the composition of the experimental apparatus.

§ 25. EFFLUX EXPERIMENTS WITH THE HELP OF THE STEAM JET PUMP.

The Giffard steam jet pump under certain circumstances provides us with the means of examining more fully the discharge of steam, provided we take away the receiving tube and allow the jet to flow through the mixing tube to a measuring tank. In this way, in 1870 (see reference on p. 147), the author conducted

a series of experiments on which he will now briefly report; it has not thus far been published, but we will reproduce it here because it confirms the propositions developed above.

Fig. 19 gives a schematic representation of the experimental apparatus; for the sake of clearness it differs in all parts from the proportions actually employed. The part *ABC* represents the injector whose axis is placed vertically. The steam pipe *A*, which supplies the steam to the boiler, ends below in a conical tube, the steam nozzle, and is there surrounded by a casing *B* which in its lower part becomes a conical pipe *C*, "the mixing chamber" or "water nozzle." A pipe *D* enters the casing *B* laterally, the pipe connecting with an upper vessel *G*, containing cold water. When the cock *E* in the pipe *D* is opened, the water enters into the casing *B*, passes through the mixing chamber and there comes in contact with the steam, which is thus condensed; the mixed jet then leaves the nozzle *C* and passes to the catch-basin *H* just below. The injector therefore does not need to suck up the water, for it is supplied under pressure. The two vessels *G* and *H* were very carefully finished boxes made of planed iron plates which were previously calibrated and provided with a vertical vernier scale which permitted the water levels at the beginning and end to be observed, and thus measured the quantity of water in the calibrated tanks at start and finish. The experiments were conducted in the following manner.

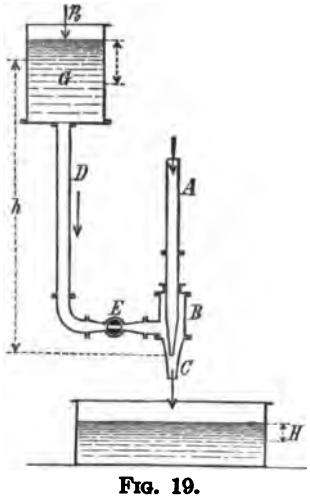

FIG. 19.

Before the steam pipe *A* and the cock *E* were opened the water level and the temperatures in the two vessels *G* and *H* were observed; the cock *E* was opened and five minutes later the steam pipe *A*, and from this time on the water jet discharged in milk-white condition from the mixing chamber (see remarks on p. 146). The discharge occurred under different boiler pressures and as a rule lasted 60 seconds; at the end the cock *E* and the steam valve

were quickly closed and the water level and temperature were observed both in the supply vessel G and in the catch-basin H.

In this way the weight of water, supplied to the injector in a unit of time, was measured, and from the excess of water in the catch-basin there was determined the weight of the steam that flowed through the steam nozzle and was discharged in the same time.

The steam pressure in the boiler was determined by a well-finished, open, mercury manometer, and of course the pressure was simultaneously read from the barometer; the mean was taken from the manometer readings observed at the beginning and end of the experiment, and these readings differed but little because of the great capacity of the steam boiler employed.

The diameter of the steam nozzle (discharge orifice of the steam pipe A, Fig. 19) was 10.6 mm. [0.4173 in.], and therefore its cross-section $F = 88.25$ sq. mm. [0.1368 sq. in.]; with this and the observed pressure p_1 in the boiler and the ascertained discharge G, the means were provided for testing whether or not the factor ϕ, in equation (48), p. 187, was really a constant, provided of course that the pressure p_2 outside of the discharge orifice of the steam, i.e., within the mixing chamber, was at most equal to half the boiler pressure. This pressure p_2 is smaller than the atmospheric pressure in all injectors; but its direct determination, say by inserting the tube of a vacuum gage into the mixing space C, would hardly accomplish the purpose. There is still another way, however, of determining this pressure, and this way was employed in all the experiments here mentioned.

For if h represents the mean pressure head (water column) under which the water flows to the steam nozzle from the measuring tank G, p_0 the specific pressure of the atmosphere, p_2 that below the orifice, and if w_1 is the velocity with which the water flows through the annular cross-section (round about the steam nozzle) into the mixing chamber (Fig. 19), then the following relation:

$$h + \frac{p_0 - p_2}{\gamma} = (1 + \zeta)\frac{w_1^2}{2g}, \quad \ldots\ldots \quad (49)$$

obtains, where γ represents the specific weight of the water ($\gamma = 1000$)

[$\gamma = 62.43$] and ζ the coefficient of resistance, which includes all the hydraulic resistances which the water must overcome on the way from the vessel G to the mixing chamber.

In all the experiments of the tabulation below, the annular cross-section $F_1 = 154.8$ sq. mm. [0.23995 sq. in.]; moreover in each separate experiment there was determined the weight G_1 of the water which was sucked in per second by the injector, or, to speak more correctly, was forced to the injector; these data render it possible to determine the velocity of efflux w_1.

On the other hand the coefficient of resistance ζ required for its determination special preliminary experiments. Thus the steam pipe A was closed and the quantity of water which then flowed in a unit of time from the measuring tank through the apparatus was measured. According to known hydraulic laws, a formula can easily be found which determines the coefficient of resistance ζ, but here we omit this determination. For the following experimental series it proved to be $\zeta = 1.227$.

From what has been given we can now easily find from equation (49) the value

$$\frac{p_0 - p_2}{\gamma},$$

i.e., find the excess of the external atmosphere over the pressure in the mixing chamber and thus determine the latter pressure itself (measured in water column).

Thus, in the following experimental series, $h = 1.303$ m. [4.275 ft.], and in the last experiment (No. 7 of the series) the quantity of water drawn in per second was $G_1 = 0.877$ kg. [1.9334 lb.], or the volume $V_1 = 0.000877$ cbm. [0.030934 cu. ft.]; consequently the velocity w_1 of the water, in the annular cross-section, was

$$w_1 = \frac{V_1}{F_1} = 5.66 \text{ m. [18.57 ft.]},$$

and hence for the data of the experiment considered, equation (49) gives

$$\frac{p_0 - p_2}{\gamma} = 2.333 \text{ m. [7.6543 ft.] water column,}$$

or 171 mm. [6.732 in.] of mercury; as the barometer read 728 mm. [28.661 in.], the absolute pressure in the mixing chamber was 557 mm. [21.929 in.] of mercury.

Since the boiler pressure in this experiment amounted to 1265.3 mm. [49.815 in.], and as this was the minimum boiler pressure in the following series of experiments, we see that here the pressure in the mixing chamber was always smaller than half the boiler pressure.

This method of determining the pressure in the mixing chamber of the injector has been introduced here in order that it may be used hereafter by others when performing similar experiments.

Let us now return to the question of the efflux of steam.

On the whole, eight experimental series were made [1]; in the following tabulation, however, the experimental and computed results are given for but o n e series, and this series is o n l y given so far as it bears on the question of the efflux of steam.

The principal purpose of the investigations concerned the behavior of the injector as a whole, but here we will not go into these general results more fully; a few remarks on them have already been inserted on p. 147.

For a steam-nozzle diameter of 10.6 mm. [0.4173 in.] and therefore for a cross-section $F = 88.25$ sq. mm. [0.13679 sq. in.], and for a barometer reading of 728 mm. [28.661 in.] the following results were found.

---

[1] In the injector employed, the steam nozzle A (Fig. 19) could be pushed in and out. Four experimental series were conducted (of which one is mentioned in the text) for four different positions of the nozzle, so that for each one of the several series another annular cross-section for the water supply was in use; in each series the cross-section of the steam nozzle was that given in the text, 88.25 sq. mm. [0.13679 sq. in.], but the efflux cross-section of the mixing chamber was 81.71 sq. mm. [0.12665 sq. in.]. In the other group of four sets of experiments there was employed in the steam pipe A, with the same four nozzle positions, a conical spindle which was pushed so far forward that the then annular-shaped stream of vapor had a cross-section of only 48.76 sq. mm. [0.07558 sq. in.] All the unmentioned sets of experiments, so far as steam discharge is concerned, furnished results which agree sufficiently well with those adduced here.

No. of Experiment.	1. Steam Pressure in the Boiler. Millimeters of Mercury. Observed.	2. Steam Weight, G kg. per Minute. Observed.	3. Value of $\sqrt{\frac{p_1}{v_1}}$ for p_1 in Atmospheres. Computed.	4. Value of ψ in Equation (48).	5. Steam Weight, G kg. per Minute. Computed.	6. Differences of the Values of Cols. 2 and 5.
1	2271.3	2.744	2.2510	230	2.682	+0.062
2	2197.8	2.534	2.1803	220	2.598	−0.064
3	2059.8	2.493	2.0474	230	2.439	+0.054
4	1961.8	2.394	1.9528	231	2.326	+0.068
5	1698.8	2.027	1.6885	227	2.012	+0.015
6	1420.3	1.645	1.4278	218	1.701	−0.056
7	1265.3	1.495	1.2764	221	1.521	−0.026
			Mean: $\psi=225$			

From the values of Cols. 2 and 3, the values of ψ in Col. 4 were computed with the help of equation (48), p. 187; they differ comparatively little from the given mean value $\psi=225$; if we substitute this mean value in equation (48) and substitute the nozzle cross-section F in square meters, then for the preceding experiments we calculate the weight of steam G, discharged per minute from the equation

$$G=1.1914\sqrt{\frac{p_1}{v_1}}.$$

The computed results are entered in Col. 5 and agree sufficiently well with the experimental results in Col. 2 for us to see in these experiments a confirmation of the propositions derived from the hypothesis of de Saint-Venant and Wantzel. Of course it would have been desirable to extend the experiments to still higher boiler pressures; unfortunately the experimental apparatus was inadequate for this purpose and the duration of the experiment was also restricted; only in a few of the experiments could the period of efflux be extended beyond 60 seconds. Although the measurements of the water in the supply and receiving vessels were made with great care, nevertheless some uncertainty attaches to the values of the discharge in Col. 2 on account of the short

period of observation, and in the preceding table this becomes manifest in the second decimal place.

The mean value of ψ, however, given in the table, agrees almost exactly with the one that Napier found for a well-rounded orifice (see p. 186). Of course this agreement is no proof that this is really the correct, absolute, value of ψ; according to the theoretical investigations presented above, too great a value of ψ may have been found by both Napier and me, because of the following circumstance.

There was no doubt in any of my experiments that the steam coming from the boiler was dry saturated, but in my injector experiments, considered here, the steam nozzle A was surrounded by cold supply water, as is evident from Fig. 19; doubtless, therefore, a partial condensation took place in the nozzle before the passage through the orifice of efflux, and this must have made the observed discharge greater than that given by the theoretical investigations; from the agreement of my experimental results with those of Napier, it must be concluded that in his experiments it was not dry saturated steam either but wet steam that was discharged. But the conclusion must not now be drawn that both series of experiments are without value; to decide the question before us it suffices that the factor ψ, in equation (48), p. 187, is shown to be approximately the same in all the separate experiments in which the pressure before the orifice is smaller than about half the boiler pressure, and this result I regard as sufficiently confirmed for steam, when I consider the results of my experiments (adduced in Vol. I) on the efflux of air. As regards the real value of the constant ψ, for different forms of orifices, we must await the results of further and more complete experiments; such experiments may well be made by the method employed by me, but instead of employing a real injector, it would be better to use an experimental apparatus similar to the injector in which the steam nozzle is protected from the cooling influence of the cold water sucked in.

§ 26. RECENT INVESTIGATIONS ON THE FLOW OF VAPORS.

(De Laval Nozzle.)

The experiments on efflux just discussed related to short, simple orifices of circular cross-section; experiments with such orifices (with and without the rounding off) and experiments with orifices of rectangular cross-section have recently been made, in large numbers, by Gutermuth[1]; they fully confirmed the correctness and reliability of the formulas developed above.

Moreover, recently, the question has been taken up in an extended form, with respect to the flow of gases and vapors through pipes with variable cross-section; here the investigations are confined exclusively to the de Laval nozzle which de Laval employed to introduce the steam into his steam turbines. If there is a short orifice well rounded on the inside, in the wall of the discharging vessel, to which is attached on the outside a conical tube that gradually widens toward the place of discharge, then such a conical diverging tube is called a de Laval nozzle (Fig. 20); for this the author[2] published more extensive investigations which bear on the relations between the dimensions of the tube, the quantities discharged, and the efflux of the steam.

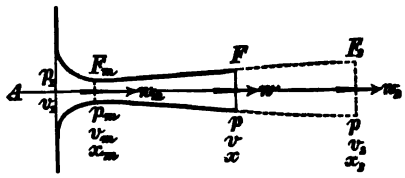

FIG. 20.

The calculations were made under the hypothesis of steady flow in which the same steam weight of G kg. [lb.] flows through every cross-section, but under the express proviso that the steam jet everywhere touches the wall of the diverging tube and that there is no separation anywhere from the wall; furthermore it is assumed that the steam pressure p_1, in front of the entrance to the tube, is

[1] Gutermuth, Versuche über den Ausfluss des Wasserdampfes. Zeitschr. d. Ver. d. Ingenieure, Vol. XLVIII, 1904, p. 75.

[2] Zeuner, Vorlesungen über Theorie der Turbinen, p. 267. Leipsic, 1899.

greater than 1.7318 p, where p represents the pressure in the discharge orifice; inversely we must also have $\frac{p}{p_1} < 0.5774$, a condition which is always fulfilled in the de Laval turbines. In addition saturated steam is presupposed for which $\kappa = 1.135$ is assumed. In the case of superheated steam we must make $\kappa = 1.333$, as will be shown later.

Although for further information concerning the de Laval nozzle the reader must refer to the just-mentioned book of the author, nevertheless we will briefly add to the above-mentioned efflux formulas the following remarks. Let p_1 be the pressure in the discharging vessel, v_1 the specific volume there, then for adiabatic change of state we get for the velocity w with which the steam flows through a cross-section F (Vol. I, p. 247)

$$w = \sqrt{\frac{2g\kappa}{\kappa - 1} p_1 v_1 \left[1 - \left(\frac{p}{p_1}\right)^{\frac{\kappa-1}{\kappa}}\right]}, \quad \ldots \quad (50)$$

where p represents the pressure in this cross-section. If v is the specific volume of the steam in this section, the relation

$$p v^{\kappa} = p_1 v_1^{\kappa} \quad \ldots \quad (51)$$

obtains, and the weight G of the steam, which flows through the cross-section F, is

$$G = F \frac{w}{v}. \quad \ldots \quad (52)$$

From the combination of these equations there results

$$G = F \sqrt{\frac{2g\kappa}{\kappa - 1} \frac{p_1}{v_1} \left[\left(\frac{p}{p_1}\right)^{\frac{2}{\kappa}} - \left(\frac{p}{p_1}\right)^{\frac{\kappa+1}{\kappa}}\right]}, \quad \ldots \quad (53)$$

which gives the connection between the pressure p and the corresponding cross-section F, because the weight G of the discharge must have the same value for all cross-sections. Moreover we see from the formula that, other things being equal with a variable cross-section, the expression in the bracket or under the radical becomes a maximum for a certain value of p, but then the

corresponding cross-section F is a **minimum**. Accordingly in the de Laval nozzle the initial cross-section of the diverging conical tube is the minimum cross-section mentioned, which we will designate by F_m. The pressure p_m there is found from the differentiation of the bracketed quantity in equation (53) and is

$$\frac{p_m}{p_1}=\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

and then follows from the same equation

$$G=F_m\sqrt{2g\frac{\kappa}{\kappa-1}\frac{p_1}{v_1}\left(\frac{\kappa-1}{\kappa+1}\right)\left(\frac{2}{\kappa+1}\right)^{\frac{2}{\kappa-1}}}. \quad . \quad . \quad . \quad (55)$$

If we substitute $\kappa=1.135$ for saturated steam we find from this

$$p_m=0.5744\,p_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54a)$$

and

$$\frac{G}{F_m}=199\sqrt{\frac{p_1}{v_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55a)$$

$$\left[\frac{G}{F_m}=215.5\sqrt{\frac{p_1}{v_1}}\right],$$

and from the combination of equations (53) and (55) we get

$$\frac{F}{F_m}=\frac{0.1550}{\sqrt{\left(\frac{p}{p_1}\right)^{\frac{2}{\kappa}}-\left(\frac{p}{p_1}\right)^{\frac{\kappa+1}{\kappa}}}}. \quad . \quad . \quad . \quad . \quad . \quad (56)$$

The efflux velocity is computed from equation (50), and for the smallest section we get (when p_1 is in atmospheres)

$$w_m=323\sqrt{p_1v_1} \quad [w_m=2641\sqrt{p_1v_1}], \quad . \quad . \quad . \quad (57)$$

while according to equation (50) we have

$$\frac{w}{w_m}=\sqrt{\left(\frac{\kappa+1}{\kappa-1}\right)\left(1-\left(\frac{p}{p_1}\right)^{\frac{\kappa-1}{\kappa}}\right)}. \quad . \quad . \quad . \quad . \quad (58)$$

If we suppose the conically diverging tube to be removed and the efflux of the steam into the receiving space to take place

directly from the smallest cross-section F_m, then equation (54a) gives the pressure in the orifice which here depends only on the pressure p_1 in the discharging vessel, whatever the pressure in the receiving space may be, only provided it is less than $0.5744\ p_1$.

The pressure p_m, according to equation (54a), also exists in the smallest cross-section when the conically diverging tube is not removed; the discharge G for the smallest cross-section is always calculated from equation (55a) and the velocity of flow w_m at this place, from equation (57).

In the conical tube the cross-section gradually increases from F_m to F, and therefore, according to equation (56), there occurs a d i m i n u t i o n of the mean cross-section pressure p and according to equation (58) there occurs an i n c r e a s e of the velocity of flow w.

If steam is to flow through such a nozzle into a space in which the prescribed pressure p prevails, and if this is also to be the pressure in the orifice, then the necessary orifice area F is calculated from equation (56); at this place, therefore, we must suppose the conically diverging tube to be cut off. The steam flows through this cross-section F with the greatest velocity; the energy of flow $\frac{w^2}{2g}$ is then a maximum, as further calculations show. The velocity and energy of flow diminish from there on if we extend the nozzle still more in an outward direction, as is indicated by dotted lines of Fig. 20. In the de Laval turbine p corresponds to the pressure at the entrance to the running wheel of the turbine, and also to the pressure in the buckets of the wheel and in the interior of the wheel as a whole, and consequently also to the condenser pressure or to the pressure of the atmosphere according as the turbine works with or without condensation. In designing the turbine we regard as known the pressure p_1, the pressure p, and the steam weight G; then from equation (55a) the smallest cross-section F_m is found; therefore, from equation (56), follows the efflux cross-section F, and from equations (57) and (58) the velocities w_m and w are determined; in so doing we can also substitute, according to the presentations on p. 37, equation (34),

$$v_1 = 1.7022\ p_1^{-0.9393}$$
$$[v_1 = 330.1\ p_1^{-0.9393}],$$

where p_1 is to be inserted in kilograms per square centimeter [pounds per square inch].

The given formulas, whose use is much facilitated by auxiliary tables, as the author has shown elsewhere, are of course only approximately correct, but experiments have shown that in designing steam turbines the quantities to be found can be determined with sufficient reliability.

Here we should chiefly consider the experiments of E. Lewicki,[1] who extended the experimental investigations to superheated steam and their application in the steam turbine. The corresponding efflux formulas are identical in structure with those derived above for saturated vapors, only the constants in the formulas are changed, for here we must make $\kappa = 1.333$ and we must apply the equation of condition for superheated steam, which is to be given later on.

Gutermuth (*ibid.*) likewise extended his experiments on saturated steam to the de Laval nozzle, but found slight differences from the calculated results given by the above formulas, and seeks to effect the agreement by introducing the coefficients of efflux and of contraction as is customary in hydraulics.

Since 1900 the de Laval nozzle has given rise to a great number of experimental and theoretical works, but we cannot discuss them separately here, for lack of space; we therefore emphasize only two special studies, those by Stodola and Hans Lorenz, which take up thoroughly the occurrences in the interior of the nozzle.

The investigations of Stodola,[2] which have received high and well-justified appreciation in engineering circles, are almost

---

[1] Ernst Lewicki, "Die Anwendung hoher Überhitzung beim Betrieb von Dampfturbinen." Zeitschrift des Vereins deutscher Ingenieure, Vol. XLVII, 1903, p. 441.

[2] Stodola, "Die Dampfturbinen und die Aussichten der Wasserkraftmaschinen." Zeitschrift des Vereins deutscher Ingenieure, 1902, p. 1. See also "Die Dampfturbinen, mit einem Anhang über die Aussichten der Wasserkraftmaschinen und über die Gasturbine." 2d edition. Berlin, 1904.

exclusively of the experimental kind and concern themselves with observations of the pressure in a steam jet flowing through a tube possessing variable cross-section, and particularly with the conically diverging tube having different pressures in the discharging vessel and in the receiving vessel. The experimental results are highly interesting and unique. The pressures in the jet, in an axial direction, approach, in the de Laval nozzle, the law given above by equation (53) when in the neighborhood of the efflux orifice, and the flow conditions in the minimum cross-section are sufficiently confirmed so that Stodola does not hesitate to make use of the formulas derived above for the quantity of steam discharged.

Hans Lorenz[1] discusses this problem clearly and comprehensively in an elegant mathematical presentation and proves the correctness of Stodola's observations, and at the same time refers to the additional work of Prandtl and Pröll.

[1] H. Lorenz, "Die stationäre Strömung von Gasen und Dämpfen durch Rohre mit veränderlichem Querschnitt." Zeitschrift des Vereins deutscher Ingenieure, 1903, Vol. XLVII, p. 1600.—See also Technische Wärmelehre, p. 122. Munich and Berlin, 1904.

B. THE BEHAVIOR OF SUPERHEATED OR UNSATURATED VAPORS.

§ 27. EXTENSION OF THE PROPOSITIONS CONCERNING THE LIMIT CURVE AND THE CRITICAL TEMPERATURE.

The difference between saturated and superheated vapors has already been pointed out in § 1, p. 4, with Fig. 1, p. 4, as a basis, and the diagram is repeated here in Fig. 21. The two limit curves b_1bb_2 and d_1dd_2, the upper and the lower limit curve, separate three areas from each other; suppose a unit of weight of steam is given possessing the specific volume v and the pressure p; lay off the former value as abscissa and the other value as ordinate, then the corresponding point c determines completely the state of the steam, as has already been explained elsewhere; if the point c, as in the figure, falls between the two limit curves, then this is a sign that we have a mixture of steam and liquid before us; on the other hand if the point c falls in the space to the left of the lower pressure curve, say at a, then liquid is present without steam, and if the point c, characterizing the state, falls into the space to the right of the upper limit curve, say at the point e, then we have before us steam without any liquid, superheated or unsaturated

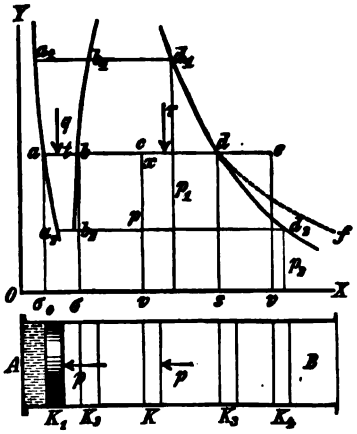

FIG. 21.

steam, and for this steam it now behooves us to find the "equation of condition," i.e., to establish the relation between the pressure p, volume v, and absolute temperature T.

In thus presenting the matter it has been tacitly assumed that the two limit curves do not intersect, although they doubtless approach each other in their upper portions.

Circumstances are different, however, if we assume that the two limit curves meet at a particular point and therefore pass into each other as represented in Fig. 22; here only one limit curve exists whose two branches have hitherto been distinguished as the upper and lower limit curve.

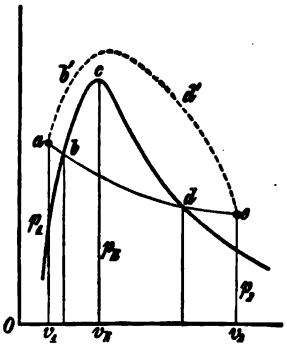

FIG. 22.

Recent experimental research has in fact shown that of the two cases discussed the latter, mentioned here, always exists.

Every point lying within a limit curve corresponds to a mixture of liquid and vapor, and indeed, according to the propositions for saturated steam developed above, there can be easily calculated from $v=xu+\sigma$ the proportions of the mixture, or the steam quality x, when the volume and pressure of the unit of weight are given.

If the point corresponding to the coordinates p and v falls upon the limit curve itself, then it indicates dry saturated steam when the point lies on the branch of the curve to the right of the culminating point c; in the other case, for example, with point b (Fig. 22), we have to do with liquid without any steam, and indeed this liquid is in a peculiar condition: it begins to evaporate the very instant there is the slightest increase of volume.

Finally if the point characterizing the instantaneous state lies in the space outside the limit curve, there exists either liquid or only dry superheated steam; the point a (Fig. 22), for example, would certainly be regarded as representing the liquid condition, while the point e would correspond to the vapor condition, or, as we briefly say, to the gaseous state, but it is evident from this that

at certain outside points, particularly above the limit curve, doubts may arise as to whether the substance is to be regarded as in the liquid or the gaseous state.

A better light is thrown on the question by the following consideration.

Let a unit of weight of a liquid be given in its initial state *a*, by its volume v_1 and by the pressure p_1 under which this volume exists; now let the liquid, with a suitable heat supply, expand along the pressure curve *abde* to the state *e* where the pressure has become p_2 and the volume v_2, so that we have before us only the gaseous mass as superheated steam, then the phenomena will be as follows, for the pressure traverses the inner space of the limit curve. At the moment in which the portion *ab* of the pressure curve has been traversed, evaporation begins at *b* and continues to the point *d*, where the last liquid element has passed into steam; along the whole path *bd*, if the occurrence takes place in a transparent vessel, we see liquid and steam separated and observe a continual diminution of the liquid mass; in the last portion *de* of the evaporation curve we have only steam before us, and the whole mass seems to be homogeneous, as was also the case during the initial portion *ab*.

The transition from the initial state *a* to the final state *e* appears very different when, with suitable heat supply, it follows the pressure curve *ab'd'e*, and when this curve, as indicated in Fig. 22, lies completely in the region beyond the limit curve. Here the mass seems to be alike (homogeneous) along the whole path, and at no place can we see where the state characterized as "liquid" ceases and the vapor or gaseous condition begins; the gas and liquid conditions are accordingly to be regarded as widely separated forms of one and the same state of aggregation, a proposition that was first enunciated by Andrews.

The whole area enclosed by the limit curve corresponds to the saturated state of the steam, and since the pressure is here only a function of the temperature, the culminating point *c*, whose coordinates we will designate by v_k and p_k, will also correspond to a particular temperature t_k, which also appears as a maximum value like the pressure p_k to which it belongs.

According to Andrews, the temperature t_k is designated as the "critical temperature," and it will be a perfectly definite but different value for each liquid, and so will the corresponding pressure p_k and volume v_k; p_k is called the critical pressure and v_k the critical volume, and the point having the coordinates p_k and v_k is called the critical point.

The mass in a vessel can only assume the form of a mixture of liquid and vapor when its temperature lies below the critical temperature; if it is greater than this, then no degree of diminution of volume or compression and no amount of increase of pressure will bring about liquefaction. The critical temperature is very different for the different vapors, and lies particularly low for those which were formerly designated as permanent gases; for example, the critical temperature for atmospheric air is $-140°$ C. [$-220°$ F.] and the critical pressure is 39 atmospheres; its liquefaction does not occur until its temperature has fallen below the given value, and the lower it falls the smaller the pressure of liquefaction will be; at $-191°$ C. [$-311.8°$ F.] liquefaction takes place at atmospheric pressure; liquid air, subjected to this pressure, evaporates at the given temperature (boiling-point). That a critical temperature exists, in this sense, for liquids, was first shown by Cagniard de la Tour[1]; he found for sulphuric ether $t_k=187°$ C. [368.6° F.] and the corresponding pressure $p_k=37.5$ atmospheres; furthermore he found for bisulphide of carbon $t_k=262.5°$ [504.5°] and $p_k=78$; for alcohol $t_k=259°$ [498.2°] and $p_k=119$; and for water the temperature was given that of melting zinc, which corresponds to about the temperature $t_k=420°$ [788°], all of which are values which have naturally been determined more accurately in recent times.

The first theoretical investigation of the critical temperature on the basis of the mechanical theory of heat was given by Avenarius[2]; starting with the idea, introduced by me, of the

[1] Annales de chim. et de phys., Vol. XXI, 1822, pp. 127 and 128; Vol. XXII, 1823, p. 410.

[2] "Über innere latente Wärme." Bulletin de la société impériale des naturalistes des Moscou, 1873, Vol. XLVI, part II, p. 117.

inner latent heat ρ and with the empirical formulas, established by me, for the computation of ρ for different liquids, Avenarius concluded that as ρ diminishes with increasing temperature, at a particular temperature t the inner latent heat must be zero and the corresponding temperature must be the critical temperature.

If we substitute $\rho=0$, in the formulas given on p. 31, we can calculate the corresponding temperature $t=t_k$; but this method of determination can only lead to rough approximations, the constants of the aforesaid formulas only holding for the narrow temperature limits for which Regnault determined, by observation, the liquid and the total heat, and these limits are very far from the corresponding critical temperature. This was recognized by Avenarius; he therefore determined, experimentally, the critical temperature for several varieties of vapor, and going back to Regnault's observations, modified the constants of the empirical formula for ρ in such a way that the formulas reproduced the experimentally found value for t_k whenever $\rho=0$. At the same time Avenarius also gave new constants for the empirical formula for the heat of evaporation r and for the external latent heat Apu; but since at the critical temperature $u=s-\sigma$ is also zero, his formulas should at the same time make $r=0$ and $Apu=0$, which is not the case; the way taken by Avenarius could therefore not lead to the goal, disregarding the fact that the values for t_k, given by him for several vapors, are shown to be inexact by other experiments made by Sajotschewsky, of which Avenarius himself makes mention.[1] The following table gives for a series of liquids the critical temperature t_k, the critical pressure p_k in atmospheres (an atmosphere equal to 10,333 kg./sq. m. [2116.31 lb./sq. ft.]), and the boiling temperature at atmospheric pressure.

[1] Beiblätter zu den Annalen der Physik und Chemie. Vol. III, 1879, p. 741.

	t_k	p_k	t_a
Water [1]	+365.0°	200.5	+100.0°
Bisulphide of carbon [2]	+271.8	74.7	+ 46.2
Chloroform [2]	+260.0	54.9	+ 60.2
Alcohol [2]	+234.3	62.1	+ 78.3
Acetone [2]	+232.8	52.2	+ 56.3
Ether [2]	+197.0	35.77	+ 35.0
Sulphurous acid [4]	+156.0	78.9	− 10.0
Ammonia [5]	+130.0	115.0	− 33.0
Nitrous oxide [5]	+ 35.0	75.0	− 89.0
Carbonic acid [6]	+ 31.35	72.9	− 80.0
Ethylene [7]	+ 10.0	52.0	−102.0
Nitric oxide [7]	− 93.0	71.0	−154.0
Oxygen [7]	−119.0	51.0	−182.0
Nitrogen [7]	−146.0	35.0	−194.0
Oxygen [7]	−234.0	20.0	−243.0
Air [7]	−140.0	39.0	−191.0

For some fluids the critical volume v_k, i.e., the specific volume of liquid and vapor in cubic meters [cubic feet], has been given for the critical temperature. Thus Battelli gives $v_k=0.00481$ [0.07705] for water; in so doing, however, he makes $t_k=364.3$ [687.74] and $p_k=194.6$ atmospheres.

For sulphurous acid $v_k=0.00192$ [0.030756] according to Cailletet and Mathias, and for carbonic acid according to Amagat $v_k=0.002155$ [0.03452].

Interesting observations, connected with the critical temperature, have been made on the liquid and gas enclosed in the microscopic cells of certain minerals, particularly in the topaz; that the carbonic acid contents of the latter are in the liquid and vapor state was found by Stelzner and Erhard.[8]

In the investigations a prepared specimen and the objective of the microscope were immersed in a water-bath, which was gradually brought to the temperature 31° C. [87.8° F.] and then, conversely, was again brought below this temperature. In the first case, say at 31° [87.8°], the surface of separation of the liquid

[1] By Cailletet and Colardeau. [2] Sajotschewsky. [3] Battelli. [4] Cailletet and Mathias. [5] Dewar. [6] Amagat. [7] Olszewski.

[8] Erhard and Stelzner, "Ein Beitrag zur Kenntnis der Flüssigkeitseinschlüsse im Topas." Mineralogische und petrographische Mitteilungen von G. Tschermak. New Series, Vol. I, 1878, p. 450.

carbonic acid confined in the topaz disappeared and reappeared in the second case at temperatures which varied for the different experiments between 28.74 and 29.19 and below what was expected. The observers supposed this deviation was due to the fact that the carbonic acid was impure and mixed with other gases.

Moreover, with these lowerings of temperature, at the instant at which the liquid reappeared, there occurred an unusual tumultuous motion of the mass in the cell, a seething froth; the experiments of Avenarius were conducted on a large scale, in glass tubes, and at the moment of change a peculiar coloring of the liquid occurred.

The last-mentioned experiments were conducted with ether, bisulphide of carbon, chloride of carbon, and acetone.

§ 28. THE EQUATION OF CONDITION OF DRY SATURATED, SUPERHEATED VAPORS.

If dry vapors behaved like gases the equation of condition of the latter, namely $pv = BT$, could also be used for the former; but it has long been known, and in fact we have already pointed it out when investigating saturated vapors, that dry steams depart more and more from the results of the preceding formula, the nearer they are to the point of condensation, and that consequently the equation of condition of gases must be regarded as only a first, rough, approximation when applied to vapors, and should only be used for high degrees of superheating, i.e., when the steam is as far from the point of condensation as possible.

Now as regards the possible form of the equation of condition of vapors and gases there exists a whole series of different proposals, of which the most important will be discussed more fully in the following, but we can say at once that in all cases it is only a question of an empirical formula, for no one has succeeded in deriving the true form of the equation of condition from theoretical considerations. The various forms proposed differ from each other only in reproducing, more or less accurately, the corresponding results of observation, without contradicting the ordinary views held concerning the nature of liquids, vapors, and gases.

From the purely physical standpoint the solution of the problem has been considerably advanced by the beautiful experiments of Andrews.[1]

Andrews especially subjected carbonic acid to investigation; he started with different initial temperatures, but in each experiment compressed, at constant temperature, and noted the gradual increase of pressure which took place; in other words, traced the course of the various isothermal curves of carbonic acid. In so doing it was shown that the course of these curves was very different according as the temperature was above or below 31° C. [87.8° F.], the critical temperature of carbonic acid. In Fig. 23, as in Fig. 22, the general course of the limit curve is indicated, but at the same time five different isothermals are sketched whose discussion will enable us to explain in a simple fashion the experiments of Andrews.

Let us assume at the point a (Fig. 23) a unit of weight of gaseous carbonic acid for which the volume v, the pressure p, and the corresponding temperature t_1 are given. Now suppose it to be compressed at constant temperature t_1, then the pressure curve ab will rise till it reaches the point b of the limit curve, at which the pressure is p_1 and where the carbonic acid exists as dry saturated vapor; during further compression along the isothermal the pressure remains constant along the path bcd till at the point d complete liquefaction obtains; with further compression the pressure curve de rises rapidly.

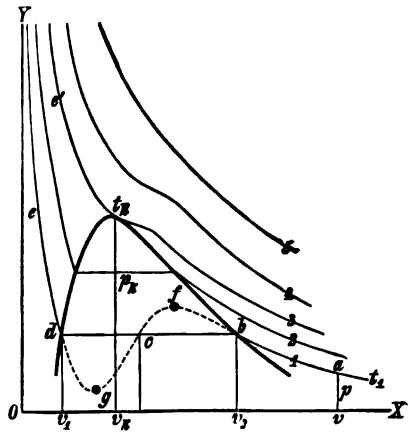

FIG. 23.

The course of isothermal No. 2 is similar; it corresponds to a higher temperature and traverses, inside of the limit curve, a shorter horizontal distance than bcd; for isothermal No. 3 this

[1] Andrews, Phil. Trans. for 1869, p. 575; also 1876, p. 421.

horizontal distance is 0, for it touches the limit curve at the highest point t_k, being horizontally tangent at the point of culmination.

The isothermals designated by 4 and 5 in Fig. 23 belong to temperatures which exceed the critical one, and we can see that with sufficiently high temperature the isothermal passes into an equilateral hyperbola, as is the case with an ideal gas. The essential result of the foregoing discussion is the fact that, when compression is taking place, condensation and gradual liquefaction of the vapor can only occur when the temperature corresponding to the isothermal is less than the critical temperature, and that in no other case can liquefaction in the ordinary sense be observed, a continuous change taking place instead; where the gaseous state ceases and the liquid condition begins is entirely arbitrary; for example, we can say with others that all those points shall be regarded as in the liquid state which lie in the area bounded on the right by the branch $t_k d$ of the limit curve, and by the branch $t_k e'$ of the isothermal belonging to the critical temperature, and on the left by the axis of ordinates OY.

The whole series of isothermals, whose general course has been indicated by a series in Fig. 23, can be represented by one and the same equation in which only certain constants change value when passing from curve to curve.

But now we see that those isothermals which pass through the inner space of the limit curve, cutting it in two places, appear to be bent at the intersections of the limit curve and run horizontally between these intersections; we can therefore draw the conclusion, to which James Thomson[1] first called attention, that, when passing through the space of the limit curve, a gradual transition from the gaseous to the usual liquid condition is conceivable, and that we can conceive that the broken curved line *abcde* (Fig. 23) is replaced by the course *abfcgde*, partly dotted in the figure; one must then assume that the second kind of transition represented can not really exist because it contains intermediate states which are in unstable equilibrium. The correctness of Thomson's hypothesis is made more probable by certain observations which make it appear that the dotted course of the curve is

[1] James Thomson, Proc. of Roy. Soc. of London, Nov. 1871.

really followed from the points of intersection b and d for a short distance inward.

The thought expressed by Thomson, concerning the course of isothermals between the points of intersection b and d, is the simplest and most obvious and has been taken as a starting point for further investigations by van der Waals, Clausius, and others, but it is easy to imagine that the portion of the curve lying between the intersections b and d constitutes, so to speak, one wave, the course *bfcgd* being made up of a whole series of waves and also joining the two branches *ba* and *de* outside the limit curve.

If we stick to the course given in the figure the equation of the isothermal, with respect to the volume v, is at least of the third degree and therefore must be put in the form

$$v^3+av^2+bv+c=0, \quad \ldots\ldots \quad (1)$$

where a, b, and c must be such functions of the pressure and of the temperature that the equation will give three real roots for the volume corresponding to the points b, c, and d in Fig. 23, provided the temperatures considered are below the critical value; the first value corresponds to the specific volume of saturated steam, and the third to that of the liquid under the same pressure and temperature. For temperatures above the critical value the preceding equation must furnish for v only one real value and two imaginary ones.

Equation by van der Waals.

Van der Waals[1] first made the attempt to establish an equation of condition which in fact makes the course of the isothermal like that given in Fig. 23; his equation has the following form:

$$p=\frac{BT}{v-\alpha}-\frac{C}{v^2}, \quad \ldots\ldots \quad (2)$$

where B, C, and α are constants which must be determined for each

[1] Van der Waals, "Die Kontinuität des gasförmigen und flüssigen Zustandes." Translated from the Dutch by Dr. Roth. Leipsic, 1887.

kind of vapor. On the basis of Andrews' experiments we must specially substitute for carbonic acid:

$$B=19.333,\ C=23.260,\ \text{and}\ \alpha=0.001167 \quad . \quad . \quad (2a)$$
$$[B=35.238,\ C=1222.45,\ \text{and}\ \alpha=0.018694],$$

provided that v represents the specific volume (vol. of 1 kg. measured in cbm. [1 lb. measured in cu. ft.]), and p the specific pressure (pressure in kg. per sq. m. [lb. per sq. ft.]).

On the other hand if the pressure is to be expressed in atmospheres (10,333 per sq. m. [2116.3 per sq. ft.]), then we must write [1]

$$B=0.001870,\ C=0.002251,\ \text{and, as above,}\ \alpha=0.001167 \quad (2b)$$
$$[B=0.016642,\ C=0.57762,\ \text{“ “ “}\ \alpha=0.018694].$$

For the rest we must regard T as the absolute temperature which obeys the relation $T=273+t$ [$459.4+t$].

Van der Waals (ibid.), with the help of equation (2), has made a series of investigations which are highly interesting from a physical standpoint, and he has theoretically justified the form of his equation, up to a certain degree, by the kinetic theory of gases. The introduction of the constants α and C represents

---

[1] Van der Waals gives other values for the constants because in place of the specific volume v he substitutes its ratio to the so-called normal volume. By the normal volume v_n is to be understood the volume of the steam at freezing and under the pressure of one atmosphere. If m is the molecular weight, then in equation $pv=BT$ we must substitute $B=\frac{845.182}{m}$ $\left[B=\frac{1540.52}{m}\right]$ (see Vol. I, p. 105, of the present book); consequently we get for $p=10{,}333$ [2116.3] and $T=273$ [491.4] the normal volume:

$$v_n=\frac{22.330}{m}\left[v_n=\frac{357.702}{m}\right].$$

For example, for carbonic acid, ammonia, and sulphuric acid we have, respectively, $m=44$, 17, 64, and therefore for these vapors, in their order,

$$v_n=0.5075,\quad 1.3135,\quad 0.3489$$
$$[v_n=8{,}1296,\quad 21.0408,\quad 5.5761].$$

If we substitute in equation (2) of the text 0.5075 v in place of v_1 we get, in place of the constants given under (2b), those values which van der Waals derived for carbonic acid from the experiments of Andrews.

the difference in the behavior of vapors from that of gases, for if we make these quantities equal to zero there remains the known equation of condition for gases.

The quantity α is what Hirn has already called "the sum of the volumes of the atoms"; α would represent the volume which would exist if the gas were compressed so far that the molecules would lie close to each other and a further compression would be impossible.

The second term of the right member of equation (2) is intended to express the influence of the mutual attraction of the molecules which can only be regarded as insignificantly small in a perfect gas. According to van der Waals this magnitude depends only on the specific volume v, a general assumption which has already been made by others. Other assumptions concerning the possible form of the equation of condition, which will be brought up presently, relate, in the main, to the form of this second term, which we may call the correction term and which expresses the deviations of vapors from the law of Mariotte and Gay-Lussac.

Let us examine the formula of van der Waals in at least one direction. Equation (2) can be written in the following form:

$$v^3-\left(\alpha+\frac{BT}{p}\right)v^2+\frac{C}{p}v-\frac{C}{p}\alpha=0, \quad \ldots \quad (3)$$

and comparison with equation (1) now shows what significance the quantities a, b, and c of cubic equation (1) possess, according to van der Waals.

If we characterize the volume, pressure, and temperature for the critical point by v_k, p_k, and t_k respectively, then because equation (3) only possesses one root for this point we have

$$(v-v_k)^3=0,$$

or

$$v^3-3v_kv^2+3v_k^2v-v_k^3=0,$$

so that combination with equation (3) will give

$$\left.\begin{aligned} 3v_k &= \alpha + \frac{BT_k}{p_k}, \\ 3v_k^2 &= \frac{C}{p_k}, \\ v_k^3 &= \frac{C\alpha}{p_k}, \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (4)$$

and from the combination of these three equations we easily find the three quantities corresponding to the vertical point:

$$v_k = 3\alpha, \quad p_k = \frac{C}{27\alpha^2}, \quad T_k = \frac{8C}{27\alpha B}, \quad \text{and} \quad p_k v_k = \frac{3}{8} BT_k. \quad (4b)$$

If we here use the constants given under (2*b*) we get for the critical point of carbonic acid:

the specific volume	$v_k = 0.003501$ cbm. [0.05608 cu. ft.],
" pressure	$p_k = 61.19$ atmospheres,
" critical temperature	$T_k = 305.40°$ [549.72°],
according to Celsius	$t_k = 32.40°$,
" " Fahrenheit	$t_k = 90.32°$.

Now of course these values differ not inconsiderably from Andrews' observations and from the statements made on p. 207. Concerning these deviations van der Waals says that small changes in the constants, particularly in the value of α, will bring about a better agreement. If his equation were theoretically well founded, observations of the magnitudes v_k, p_k, and T_k, at the critical temperature, with the help of equation (4*b*), would enable us to work backwards and calculate directly the constants B, C, and α of the equation of condition. As can easily be shown, we should get

$$\alpha = \frac{1}{3} v_k, \quad C = 3p_k v_k^2, \quad \text{and} \quad B = \frac{8}{3} \frac{p_k v_k}{T_k},$$

and then, according to equation (3),

$$\frac{p}{p_k}\left(\frac{v}{v_k}\right)^3 - \frac{1}{3}\left(\frac{p}{p_k} + 8\frac{T}{T_k}\right)\left(\frac{v}{v_k}\right)^2 + 3\left(\frac{v}{v_k}\right) - 1 = 0.$$

Accordingly, and van der Waals calls attention to it, the isothermal will be the same for all bodies if we express the pressure in parts of the critical pressure, the volume in parts of the critical volume, and the absolute temperature in parts of the absolute critical temperature.

The Equation of Clausius.

The deviations emphasized above and a further examination of the considerations which led van der Waals to the form of that term of his equation (equation 2) which takes into account the influence of the mutual attraction of the molecules, led Clausius[1] to the following form of the equation of condition:

$$p=\frac{BT}{v-\alpha}-\frac{C}{T(v+\beta)^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Considering especially carbonic acid and letting p represent the pressure in kg. per sq. m. [lb. per sq. ft.], and v the volume of one kilogram [pound] of carbonic acid in cubic meters [cubic feet], we must substitute, according to Clausius,

$$\left.\begin{array}{ll} \alpha=0.000426, & \beta=0.000494, \\ B=19.273, & C=5533. \\ [\alpha=0.006824, & \beta=0.007913,] \\ [B=35.129, & C=523426.] \end{array}\right\} \quad . \quad . \quad (5a)$$

On the other hand, if the pressure is given in atmospheres (10333 kg. per sq. m. [2116.3 lb. per sq. ft.]) we must employ the constants

$$\left.\begin{array}{lll} \alpha=0.000426, & & \beta=0.000494 \\ [\alpha=0.006824, & & \beta=0.007913] \\ \text{as before, and make} & & \\ B=0.001865 & \text{and} & C=0.535469 \\ [B=0.016597 & \text{and} & C=247.33]. \end{array}\right\} \quad . \quad . \quad . \quad (5b)$$

[1] Clausius, "Über das Verhalten der Kohlensäure in Bezug auf Druck, Volumen und Temperatur." Wiedemann's Annalen der Physik und Chemie, 1880. New edition, Vol. IX.

In computing these constants, Clausius utilized later experiments by Andrews; the older experiments referred to isothermals for the temperatures 13.1°, 31.1°, and 48.1° C., the more recent ones to 6.5°, 64°, and 100°. The results of all these experiments agree very satisfactorily, as regards the pressure and volume, with those derived by the formula of Clausius with the help of the given constants.

In order to derive from equation (5), with the help of the preceding constants, the pressure, volume, and temperature for the critical point, let us substitute

$$v+\beta=u \quad \text{and} \quad \alpha+\beta=\delta. \quad . \quad . \quad . \quad . \quad . \quad (5c)$$

We then get from equation (5)

$$u^3-\left(\delta+\frac{BT}{p}\right)u^2+\frac{C}{pT}u-\frac{C\delta}{pT}=0,$$

and then, by comparison with the relation

$$(u-u_k)^3=0,$$

we obtain the equations

$$3u_k=\delta+\frac{BT_k}{p_k}, \quad 3u_k^2=\frac{C}{p_kT_k}, \quad \text{and} \quad u_k^3=\frac{C\delta}{p_kT_k},$$

and hence

$$\left.\begin{aligned} v_k&=3\alpha+2\beta, & \frac{T_k}{p_k}&=\frac{8(\alpha+\beta)}{B}, \\ p_kT_k&=\frac{C}{27(\alpha+\beta)^2}, & p_kv_k&=\frac{3\alpha+2\beta}{8(\alpha+\beta)}BT_k. \end{aligned}\right\} \quad . \quad . \quad . \quad (5d)$$

Utilizing the values given under (5*b*) for carbonic acid, we get, according to the preceding formulas, $T_k=304°$ [547.2°] or $t_k=31°$ C. [87.8° F.]; the corresponding pressure being found to be $p_k=77$ atmospheres and the corresponding specific volume $v_k=0.002266$ [0.036299], which completely corresponds to Andrews' experiments; this of course is due to the fact that Clausius, in fixing his constants, doubtless made use of said experimental results.

Clausius himself stated that, in all probability, the temperature T should be introduced in some other form into the right member of his equation (5) and not simply as a divisor; thereupon Sarrau[1] showed, utilizing Andrews' experimental results and those of Cailletet and Hautefeuille,[2] that the following form for the equation of condition for carbonic acid gave very satisfactory agreement within the limits of the experiments:

$$p=\frac{BT}{v-\alpha}-\frac{C\varepsilon^{-T}}{(v+\beta)^2}, \quad \ldots\ldots\ldots \quad (6)$$

and this exponent represents another constant. The statement of the value of this constant and of the remaining constants will here be omitted, for, later on, we will proceed with different assumptions for carbonic acid.

In place of equation (5), we may, according to Clausius, write the general relation

$$p=\frac{BT}{v-\alpha}-\frac{C\phi}{(v+\beta)^2}, \quad \ldots\ldots\ldots \quad (7)$$

where $\phi=F(T)$ represents a function of the absolute temperature T which possesses the value 1 for the critical point, i.e., for $T=T_k$.

Van der Waals proposed for the form of this function

$$\phi=e^{1-\frac{T}{T_k}}, \quad \ldots\ldots\ldots \quad (7a)$$

but Clausius modifies equation (7), paying particular attention to the vapor of ether and to the technically all-important vapor of water.[3]

If we again substitute, as when we transformed equation (15),

$$v+\beta=u \quad \text{and} \quad \alpha+\beta=\delta,$$

---

[1] Sarrau, "Sur l'équation caractéristique de l'acide carbonique." Comptes rendus, Vol. CI, 1885, p. 1145.

[2] Comptes rendus, Vol. XCII, pp. 903 and 1087.

[3] Clausius, "Über die theoretische Bestimmung des Dampfdruckes und der Volumina des Dampfes und der Flüssigkeit." Wiedemann's Annalen der Physik und Chemie. New Series, Vol. XIV. 1881, pp. 279 and 692.

then equation (7) can be written

$$u^3-\left(\delta+\frac{BT}{p}\right)u^2+\frac{Cu}{p}\phi-\frac{C\delta}{p}\phi=0,$$

and comparison with the formula

$$(u-u_k)^3=u^3-3u^2u_k+3uu_k^2-u_k^3=0$$

then gives for the critical point, for which $\phi=1$,

$$3u_k=\delta+\frac{BT_k}{p},\quad 3u_k^2=\frac{C}{p_k},\quad u_k^3=\frac{C\delta}{pk}.$$

The combination of these formulas gives

$$u_k=3\delta,\quad 8\delta p_k=BT_k,\quad \text{and}\quad 27\delta^2p_k=C,$$

or

$$v_k=3\alpha+2\beta\quad\text{and}\quad C=\frac{27(\alpha+\beta)}{8}BT_k.$$

If we substitute the value of C in equation (7) we get

$$\frac{p}{BT}=\frac{1}{v-\alpha}-\frac{27(\alpha+\beta)}{8(v+\beta)^2}\left(\frac{T_k}{T}\phi\right).$$

Here Clausius introduces a new temperature function θ by making

$$\frac{8}{27(\alpha+\beta)}\frac{T}{T_k\phi}=\theta;\quad\ldots\ldots\quad(8)$$

hence, for the critical point, there follows

$$\frac{8}{27(\alpha+\beta)}=\theta_k,\quad\ldots\ldots\quad(9)$$

and then the equation of condition in the form

$$\frac{p}{BT}=\frac{1}{v-\alpha}-\frac{1}{\theta(v+\beta)^2},\quad\ldots\ldots\quad(10)$$

which is the form in which Clausius has given it for a particular vapor. Therefore it is here only a question of stating

the constants B, α, and β and the form of the function θ. In the earlier equation (5) given by Clausius it was assumed that $\phi = 1:T$.

Before we further modify equation (10) we ought to explain how the equation of condition enables us to infer the course of the limit curve.

Let us suppose the isothermal curve *abfcgde* (Fig. 24) to be drawn from the given equation of condition for a temperature t which lies below the critical temperature, then the two points b and d correspond to the two points on the limit curve (see also Fig. 23, p. 209); one ought therefore to be able to state the height p above the axis of abscissas OX at which the horizontal *bcd* must be drawn in order to obtain these points of intersection; the ordinate p will then also be the pressure which belongs to the isothermal for the saturated state corresponding to the temperature t. Now we know from experimental observations, for a large series of different kinds of vapors, the steam pressure $\dot{p}$ in the saturated condition which corresponds to the temperature t, and then the portion of the horizontal *bcd* will at once be given, but if we ignore this information, and seek to determine this position in a theoretical way, we must have recourse to an hypothesis.

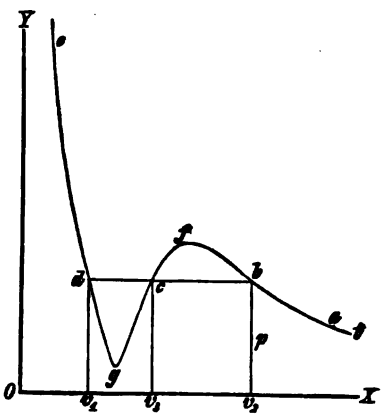

FIG. 24.

Now Clausius has expressed the view that the quantity of heat to be imparted is the same, whether the liquid corresponding to the point d is converted into dry saturated steam (state b) at constant pressure, and therefore under constant temperature, or whether we effect this change along the imaginary path *dgcfb* of the isothermal; the necessary quantity will then in both cases be given by the latent heat r. From this assumption it at once follows that the rectangle $p(v_2 - v_1)$ bounded by

the horizontal db must be equal to the area which is bounded by the curve between the points b and d; we shall then have

$$p(v_2-v_1)=\int_{v_1}^{v_2} pdv,$$

where we must substitute for the quantity p under the integral the value found from one of the above-given equations of condition.

Moreover here v_2 signifies the specific volume of dry saturated steam and v_1 the specific volume of the liquid at the instant at which evaporation begins, both taken at the temperature t and the corresponding steam pressure p; these magnitudes v_2 and v_1 in the above investigations for saturated vapors were designated by s and σ, and this notation we will retain in what follows, and, at the same time, let p represent saturation pressure corresponding to the temperature t.

If we utilize equation (10) in the preceding equation, we get, by integration,

$$\frac{p}{BT}=\frac{1}{s-\sigma}\log_e\frac{s-\alpha}{\sigma-\alpha}-\frac{1}{\theta(s+\beta)(\sigma+\beta)}. \quad . \quad . \quad . \quad (11)$$

Furthermore, we get from equation (10), at one time substituting $v=s$ and at another $v=\sigma$,

$$\frac{p}{BT}=\frac{1}{s-\alpha}-\frac{1}{\theta(s+\beta)^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\frac{p}{BT}=\frac{1}{\sigma-\alpha}-\frac{1}{\theta(\sigma+\beta)^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Supposing the function $\theta=F(t)$ is known, and that the constants B, α, and β are given, then for a particular temperature t these three equations will enable us to compute the corresponding quantities p, s, and σ, and thus determine the course of the limit curve from point to point.

Clausius has very ingeniously continued the discussion, by making

$$\log_e \frac{s-\alpha}{\sigma-\alpha} = \lambda;$$

then he expressed the several quantities p, s, σ, and θ in terms of the auxiliary quantity λ and set up an auxiliary table, which is valid for all vapors, for facilitating the calculations. We will give, later on, the final results of the calculations of Clausius, but, before doing this, will make the following remark:

If, as was done above, we substitute $s-\sigma=u$, we get at once, from equation (11),

$$Apu = ABT \log_e \frac{s-\alpha}{\sigma-\alpha} - \frac{ABTu}{\theta(s+\beta)(\sigma+\beta)}, \quad . \; . \quad (14)$$

from which the external latent heat can be computed.

It is also possible to establish an expression for the heat of evaporation r.

According to the second of equations (IIIa), Vol. I, p. 62, namely

$$dQ = \frac{A}{\frac{\partial t}{\partial p}}[Xdt + Tdv],$$

there can be found the heat quantity dQ which must be imparted if the change of state is to take place according to the isothermal curve, i.e., for the constant value of T, and we then get

$$dQ = AT\frac{\partial p}{\partial t}dv.$$

If we differentiate the equation of condition (10) with respect to t, there follows

$$dQ = ABT\frac{dv}{v-\alpha} - ABT\frac{d}{dt}\left(\frac{T}{\theta}\right)\frac{dv}{(v+\beta)^2},$$

and from this, by integration, for constant temperature from $v=\sigma$ to $v=s$,

$$r = ABT \log_e \frac{s-\alpha}{\sigma-\alpha} - \frac{ABTu}{(s+\beta)(\sigma+\beta)}\frac{d}{dt}\left(\frac{T}{\theta}\right). \; . \; . \; . \quad (15)$$

In like manner Planck[1] derived the heat of evaporation r, of course using the older equation of condition given by Clausius (the equation (5) mentioned above), in which

$$\theta = \frac{BT^2}{C}$$

was assumed.

The inner latent heat ρ can be found from the relation $\rho = r - Apu$, hence follows from the preceding equations, after a simple reduction,

$$\frac{\rho}{u} = -\frac{ABT^2}{(s+\beta)(\sigma+\beta)} \frac{d}{dt}\left(\frac{1}{\theta}\right). \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Now, according to Clausius, we must substitute in the preceding equations for the function θ the expression

$$\frac{1}{\theta} = CT^{-n} - D, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

the quantities C, D, and n being constant values which must be known for the vapor considered.

We finally get, according to Clausius, the equation of condition derived from equation (10):

$$\frac{p}{BT} = \frac{1}{v-\alpha} - \frac{CT^{-n} - D}{(v+\beta)^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and just as easily we can bring equations (15) and (16) and also equations (12) and (13) into the corresponding form, which can then be employed for calculating the specific volumes s and σ for the steam and the liquid in the saturated condition.

This second form of the equation of condition by Clausius contains six constants, given by him for the vapor of ether and the vapor of water.

For the vapor of water we are to substitute

---

[1] Planck, "Die Theorie des Sättigungszustandes." Wiedemann's Annalen der Physik und Chemie. New Series, Vol. XIII, 1881, p. 535.

$\alpha = 0.000754$	$C = 45.17$
$\beta = 0.001815$	$D = 0.00737$
$B = 47.05$ for p kg.: sq. m.	$n = 1.24$

$\alpha = 0.012078$	$C = 2733.6$
$\beta = 0.029074$	$D = 0.21519$
$B = 85.758$	$n = 1.24$

The equating will then furnish us the means of computing any one of the three quantities p, v, and T when the other two are known, and enable us to do this whether the steam is dry saturated or superheated. For example, if, according to Cailletet and Colardeau, we substitute for the critical point $t_k = 365°$ [698°] and $p_k = 200.5$ atmospheres (see page 207), then the formula gives $v_k = 0.005201$ [0.08331] for the critical temperature of water; while Clausius assumes $v_k = 3\alpha + 2\beta = 0.005892$ [0.09438] and, working backward, computes $t_k = 332.32°$ [630.2°] and $p_k = 134.1$ atmosheres, which differs from the newer experimental values. It follows from this that the equation given by Clausius must experience a fourth transformation, say in the manner suggested by Battelli.

There will be opportunity below of coming back to the forms of the equations of condition given in the preceding.

§ 29. OLDER APPROXIMATE FORMS FOR THE EQUATION OF CONDITION OF VAPORS.

The technically important vapors which play a part in engine running are the vapors of water, ammonia, sulphurous acid and carbonic acid, and these give rise to special investigations in the following. In so doing, we can make the distinction whether the vapor considered in its changes of state during its practical application experiences changes of pressure and temperature which are far distant from the critical values—and to this group the vapors of water and ammonia belong,—or whether, as is the case with carbonic acid, the changes ordinarily

occur in the neighborhood of the critical temperature, and therefore, in accordance with the circumstances, one of the above-given forms of the equation of condition must be applied. The circumstances are different with the vapor of water; here, in order to make the computations convenient for inspection, efforts have been made, even before van der Waals' propositions were known, to secure as simple forms as possible for the equation of condition, and in this direction there exist a number of different suggestions of which a few will be mentioned here.

For technical purposes the equation of condition must fulfill the following requirements:

1. It must only furnish the relation between v, p, and T for the steam state proper, estimated from the state of saturation, because, from the previously given and earlier presentations, the behavior of the steam and liquid mixtures is sufficiently known.
2. It must at least correspond to the experimental results thus far obtained for the vapor in question within the pressure and temperature limits occurring in practice.
3. It should allow the determination of the specific volume v of the steam from the pressure p and the temperature T to be made as easily as possible, i.e., without laborious computations; p and T are here mentioned because they are quantities which can easily be ascertained by observation and can therefore be ordinarily regarded as given.
4. Finally the heat equations, developed with the corresponding equation of condition as a basis, should be simple and perspicuous in order that, with its help, the change of state of the vapor connected with heat supply may be easily followed.

The requirement under No. 3 is not fulfilled by the equations of condition adduced in § 28. The determination of v calls for the solution of a cubic equation, a procedure that would be much too cumbrous for the frequently occurring cases of the determination of v. Still less is the requirement under 4 fulfilled; one here encounters equations of extremely complicated form which render a survey of the whole course of calculation impossible.

When investigating gases in Vol. I, p. 141, it was pointed out that the equation of condition of all bodies, without exception, could be brought into the form

$$pv = BT - R, \quad \ldots\ldots\ldots \quad (19)$$

where B is a constant depending on the body contemplated and R is a function of two of the variables p, v, or T. It is now a question of so choosing a form of the function R that the equation will fulfill the requirements first mentioned. For the equations of condition adduced in § 28, R could be derived for every individual case, but the problem now is to find other simpler forms. When investigating the variability of the specific heat of gases on pp. 141 and 143 of Vol. I, there were derived, from equations (65) to (68), with formula (19) as a basis, some propositions which can now be utilized for vapors; first, however, we will derive one more equation for the total heat λ (see p. 22) with the help of which we can determine the specific heat c of the liquid for vapors and from it the heat of the liquid q (see p. 22).

From the equation

$$dQ = AdU + Apdv$$

there follows, because

$$pdv = d(pv) - pv\frac{dp}{p},$$

$$AdU = dQ - Ad(pv) + Apv\frac{dp}{p}.$$

If we employ here for dQ the third of equations (III*b*), Vol. I, p. 143, there will result

$$AdU = c_p dt - Ad(pv) + Apv\frac{dp}{p} - \frac{AT}{\frac{\partial t}{\partial v}}dp. \quad \ldots \quad (20)$$

Now if J is the steam heat, i.e., the heat content for the condition (p, v), then the heat quantity λ necessary for the generation of steam at constant pressure is

$$\lambda = J + Ap(v - \sigma).$$

Differentiating this equation and remembering $dJ = AdU$, there follows

$$d\lambda = AdU + Ad(pv - p\sigma),$$

and hence, by combining with equation (20),

$$d\lambda = c_p dt + Ap(v-\sigma)\frac{dp}{p} - \frac{AT}{\frac{\partial t}{\partial v}}dp.$$

If, in the equation of condition (19), we consider the quantity R as a function of p and v there will follow from it

$$\frac{\partial t}{\partial v} = \frac{1}{B}\left(p + \frac{\partial R}{\partial v}\right),$$

and with this help the last equation but one gives

$$d\lambda = c_p dt + Ap(v-\sigma)\frac{dp}{p} - \frac{ABT}{p + \frac{\partial R}{\partial v}}dp. \quad . \quad . \quad . \quad . \quad (21)$$

In these developments of course we cling to the assumption made above that the specific volume σ of the liquid can be regarded as a constant, which is permissible in vapors that, at ordinarily occurring temperatures, lie far below the critical temperature.

The preceding equation moreover is still valid for every degree of superheating. If we think specially of the condition of saturation, then the total heat λ is composed of the liquid q and the latent heat r (see p. 22), and we get from the relation $\lambda = q + r$, by differentiation,

$$\frac{d\lambda}{dt} = c + \frac{dr}{dt}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

here in case the latent heat r is known as a function of the temperature we can find in combination with the preceding formula the specific heat c of the liquid.

As regards the function R of the equation of condition (19), different initial assumptions have been made; the simplest formulas

and those most serviceable for technical calculations, which fulfill the requirements made on p. 224, are obtained when R is assumed as only a function of p, i.e., when $\frac{\partial R}{\partial v}=0$ is assumed.[1] It has been found that $R=Cp^n$ is a suitable form for R, so that the equation of condition for the vapor of water becomes

$$pv=BT-Cp^n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

in which equation B, C, and n are constants still to be determined. The following investigations will establish the reliability of the formula; its special usefulness for technical calculations is due to the fact that with its help the specific volume v of the steam can be determined directly and easily from the pressure p and the temperature T.

Another form of the equation of condition for the vapor of water is

$$pv=BT-Cv^{-n}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

and was first proposed by Hirn[2] and was developed later by Gustav Schmidt.[3] Comparison with equation (19) shows that here the function R is regarded only as a function of v; the constants B, C, and n have here, of course, other values than those used in equation (23). But for the case occurring most often in technical calculations, in which the specific volume v is to be computed, equation (24) is very inconvenient, for the determination of v can only be found by trial; in this connection the equation presents the same difficulties as the equations of van der Waals, Clausius and Sarrau, given in § 28.

An equation of condition in which the term R appears only

---

[1] See the author's articles: 1. "Theorie der überhitzten Wasserdämpfe." Zeitschrift des Vereins deutscher Ingenieure, Vol. XI, 1867, p. 41.—2. "Über das Verhalten der überhitzten und gemischten Wasserdämpfe." Zivilingenieur, Vol. XIII, 1867, p. 343.—3. "Zur Theorie der Kaltdampfmaschinen." Anhang: Verhalten der Ammoniakdämpfe. Zivilingenieur, Vol. XXVII, 1881, p. 449.

[2] Hirn and Cazin, "Mémoire sur la détente de la vapeur d'eau surchauffée." Ann. de Chim. et de Phys., 4th series, Vol. X, 1867, p. 365.

[3] Gustav Schmidt, "Die Zustandsgleichung des Wasserdampfes." Zeitschrift des Vereins deutscher Ingenieure, Vol. XI, 1867, pp. 649 and 771.

as a function of p, as in equation (23), has also been given by Weyrauch[1] and has the form

$$pv = BT - (BT' - ps);$$

here T' and s represent the temperature and specific volume of the dry saturated steam, possessing the same pressure as the superheated steam under consideration; but the last-mentioned values must be taken from existing steam tables, when using the preceding formula, and in any case must be regarded as known.

Finally, for the sake of completeness we will mention a few other proposals with respect to the form of the equation of condition of vapors. Rankine[2] gives the following equation:

$$p = \frac{BT}{v} - \frac{C}{Tv^2},$$

where B and C are constant quantities; if we substitute in the first term of the right member $v - \alpha$ in place of v, and in the second term $v + \beta$ in place of v, we get the equation of condition of Clausius (equation (5), p. 215), provided we here regard α and β as constants.

Güldberg[3] suggests the following formula:

$$pv = BT - Cp^{\frac{\epsilon-1}{\epsilon}} - Dv^{1-\epsilon},$$

in which equation B, C, D and ϵ are constant magnitudes, and he shows that it satisfactorily reproduces, for the vapor of water and ether, the specific volume v for the saturated condition and for different pressures within the limits of experiment. Finally mention may be made of Ritter's[4] equation for the vapor of water, namely,

---

[1] Weyrauch, "Von den überhitzten Dämpfen." Zeitschrift des Vereins deutscher Ingenieure, Vol. XX, 1876, pp. 1 and 71.

[2] Phil. Trans. for 1854, p. 336.

[3] Güldberg, "Über die Zustandsgleichungen der Körper." Zeitschrift des Vereins deutscher Ingenieure, Vol. XII, 1868, p. 673.

[4] Ritter, "Uber die Temperaturfläche des Wasserdampfes." Wiedemann's Annalen der Physik und Chemie. New Series, Vol. III, 1878, p. 447

$$pv = BT - \frac{C}{pv^n};$$

in this formula B, C and n are constants, and when p is expressed in kg.:sq. m. [lb.:sq. in.] we get

$$B = 46.968, \quad C = 1315.11 \quad \text{and} \quad n = \tfrac{4}{3}$$
$$[B = 85.609, \quad C = 56657.4 \quad \text{and} \quad n = \tfrac{4}{3}].$$

§ 30. SUPERHEATED STEAM.

The advantage of employing superheated steam to run steam engines has long been known in Thermodynamics; the first experiments in this direction were made by Hirn, who was probably inspired in his practical investigations by theoretical considerations.

In recent years the use of superheated steam for engines has been quite extensive, so that it seems appropriate that the basis for the calculation of such engines should be established. Of course, in so doing, we must start with certain hypotheses which must be justified.

We start from the general form of the equation of condition

$$pv = BT - R, \quad \ldots\ldots\ldots \quad (25)$$

and assume that the supplementary term R may be regarded as a function of p alone in the case of the vapor of water.

Furthermore let us assume, always remembering the four requirements mentioned on p. 224, that the specific heat c_p, at constant pressure for the vapor of water, may be regarded as a constant magnitude within the present limits of technical investigations.

This assumption is sufficiently justified by Regnault's experimental results; he finds from four experimental series, for the vapor of water,

$$c_p = 0.46881, \quad 0.48111, \quad 0.48080, \quad 0.47963,$$

and says that only the first of these values is not perfectly reliable; as a mean of the rest there is found

$$c_p = 0.4805,$$

which value will be taken as the basis of the following consideration.

In Fig. 25 let two adiabatics A_2 and A_1 be passed through the two points T_2 and T_1 on the right-hand portion of the limit curve for steam, and let these points correspond to the temperatures T_2 and T_1, and let the adiabatics extend into the superheated field. Let p_2 and v_2 be the pressure and volume of dry saturated steam, corresponding to the point T_2, and let the point T_1 be given by p_1 and v_1.

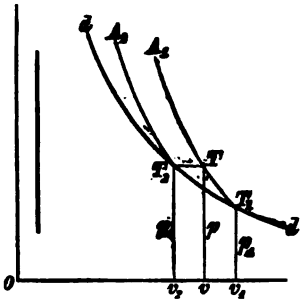

FIG. 25.

Starting from the formerly proved proposition that for every transition from a particular adiabatic curve to a prescribed second one the heat weight or the entropy,

$$P = \int \frac{dQ}{AT},$$

has the same value, then for the present case we get from equation (58), p. 61, for the transition on the limit curve from T_2 to T_1,

$$AP_1 - AP_2 = \left(\tau_1 + \frac{r_1}{T_1}\right) - \left(\tau_2 + \frac{r_2}{T_2}\right), \quad . \quad . \quad . \quad (26)$$

because on the limit curve the steam quality is constant, and we may make $x=1$ for all its points. On the other hand if we start from the point T_2 on the first adiabatic A_2, in a horizontal direction T_2T (and therefore under constant pressure $p=p_2$), and pass to the point T of the second adiabatic A_1, we find, because $dQ = c_p dT$,

$$AP_1 - AP_2 = \int_{T_2}^{T} \frac{c_p dT}{T} = c_p \log_e \frac{T}{T_2}. \quad . \quad . \quad . \quad . \quad (27)$$

Combination with equation (26) then gives the relation

$$c_p \log_e \frac{T}{T_2} = \left(\tau_1 + \frac{r_1}{T_1}\right) - \left(\tau_2 + \frac{r_2}{T_2}\right), \quad . \quad . \quad . \quad . \quad (28)$$

and accordingly we can find, for every point of a particular adiabatic for which the pressure $p=p_2$ is given, the temperature T of the corresponding superheated steam.

Differentiating equation (25) with respect to v, under the supposition that R is only a function of p, there follows

$$\frac{\partial t}{\partial v}=\frac{p}{B},$$

and according to the third of the generally valid equations (IIIb), Vol. I, p. 143, we get

$$dQ=c_p dt-ABT\frac{dp}{p}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

From this we have for the adiabatic change of state, i.e., for $dQ=0$,

$$\frac{dT}{T}=\frac{AB}{c_p}\frac{dp}{p}.$$

Assuming c_p as constant and abbreviating, by making

$$\frac{AB}{c_p}=m, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

we get for the adiabatic A_1 (Fig. 25)

$$\frac{T}{T_1}=\left(\frac{p}{p_1}\right)^m. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

It follows from this that, under the assumptions made, superheated steam behaves exactly like a gas so far as change of temperature with the pressure is concerned, when experiencing adiabatic change of state. According to equation (61), Vol. I, p. 139, there was found for adiabatic change of gases the relation

$$\frac{T}{T_1}=\left(\frac{p}{p_1}\right)^{\frac{\kappa-1}{\kappa}},$$

and from this follows that if we substitute for steam in place of the constant m the constant κ (equation 30), i.e., make

we get

$$\left.\begin{aligned} m &= \frac{\kappa-1}{\kappa}, \\ \kappa &= \frac{1}{1-m}. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (32)$$

In so doing of course the question arises whether the constant κ in vapors, as in gases, represents the ratio of the specific heats ($c_p : c_v$); but we may even now remark that the following investigations show that this assumption is not permissible, but rather that in steam the ratio $c_p : c_v$ is a variable magnitude.

Let us substitute in equation (28) the value of T, resulting from equation (31), remembering that in Fig. 25 the pressure p is identical with p_2, then there follows, after a simple transformation,

$$\tau_1 + \frac{r_1}{T_1} - c_p \log_e \frac{T_1}{p_1^m} = \tau_2 + \frac{r_2}{T_2} - c_p \log_e \frac{T_2}{p_2^m},$$

with the result that within the limits of the whole development the expression

$$\tau + \frac{r}{T} - c_p \log_e \frac{T}{p^m}$$

must have one and the same value for every point in the limit curve.

If we designate the constant in question by K we must have

$$\tau + \frac{r}{T} = c_p \log_e \frac{T}{p^m} + K. \quad \ldots\ldots\ldots \quad (33)$$

The question now is whether, with $c_p = 0.4805$ and with the values of this left member given in Col. 7 of the table on p. 63, the results of the preceding expression are reliable when the quantities m and K are correctly chosen. We really do get an excellent agreement, as the following table shows, if we get

$$m = 0.25 = \frac{1}{4}$$

and, according to equation (32),

$$\kappa = 1.3333 = \frac{4}{3},$$

while substituting Briggs' logarithms in place of the natural logarithms makes

$$\tau + \frac{r}{T} = 0.2766 \log \frac{T^4}{p} - 1.0933. \quad . \quad . \quad . \quad . \quad (33a)$$

TABLE FOR VAPOR OF WATER.

Pressure in Atmospheres (1 Atm. = 10333 kg.).	Temperature (Celsius) according to Regnault.	Temperature (Fahrenheit) according to Regnault.	Value of $\tau + \frac{r}{T}$.	
			According to Col. 7, Table p. 63.	According to Equation (33a).
0.1	46.21°	115.178°	1.9548	1.9538
0.2	60.45	140.810	1.8929	1.8912
0.5	81.71	179.078	1.8116	1.8111
1	100.00	212.000	1.7519	1.7520
2	120.60	249.080	1.6940	1.6946
3	133.91	273.038	1.6612	1.6619
4	144.00	291.200	1.6384	1.6391
5	152.22	305.996	1.6209	1.6217
6	159.22	318.596	1.6071	1.6076
7	165.34	329.612	1.5955	1.5958
8	170.81	339.458	1.5856	1.5858
9	175.77	348.386	1.5769	1.5769
10	180.31	356.558	1.5691	1.5691
11	184.50	364.100	1.5625	1.5621
12	188.41	371.138	1.5563	1.5557
13	192.08	377.744	1.5506	1.5503
14	195.53	383.954	1.5454	1.5445

Consequently for steam, between the wide range of pressure limits of 0.1 to 14 atmospheres, limits which are rarely overstepped at this time in steam engineering, we can regard the constant $m = 0.25$ as sufficiently safe and then, according to equation (30), substitute for steam

$$AB = 0.1201 \quad \text{and} \quad B = 50.933 \quad . \quad . \quad . \quad . \quad (34)$$
$$[AB = 0.1201 \qquad B = 92.835].$$

The reliability of the assumption made is apparent from the following consideration. If we differentiate equation (33) and consider the relation

$$\tau = \int \frac{dq}{T} = \int_0^t \frac{cdt}{T}$$

(see p. 59), where c represents the specific heat of water, there follows

$$c + \frac{dr}{dt} - \frac{r}{T} = c_p - mc_p \frac{Tdp}{pdt},$$

or designating the temperature function of the left member by h, as was done in equation (56), p. 60, we have

$$h = c_p - mc_p \frac{Tdp}{pdt}. \quad \ldots\ldots \quad (35)$$

From this equation m can be computed for different values of the temperature, as the following survey shows. Here $c_p = 0.4805$ was used and h was computed from the formula given for steam on p. 68, with the help of the values given in Col. 6 in Table 1a of the Appendix.

t	h	$T\frac{dp}{pdt}$	m
0° C.	−1.9168	19.520	0.255
50	−1.4648	16.083	0.252
100	−1.1333	13.344	0.251
150	−0.8789	11.359	0.249
200	−0.6766	9.851	0.244
		Mean:	0.250

The value assumed for m also finds confirmation here; moreover there exist direct experiments which serve for its determination.

Hirn and Cazin[1] have proved the reliability of equation (31), p. 231, by interesting experiments; they allowed superheated steam of different initial pressures and observed initial temperature to flow from a wide orifice in a reservoir immersed in an oil bath till the inner pressure had fallen to the external atmos-

[1] Comptes rendus, 1866.

pheric pressure and fog appeared in the interior of the vessel, which was provided on opposite sides with glass plates. The appearance of fog was a proof that at the lower pressure the state of saturation had just been reached and this determined the corresponding lower limit of temperature.

Grashof[1] computed from equation (31), p. 231, and from the experiments by the physicists mentioned, the exponent m, and found nearly the same value for all experiments; in the mean there was found $m=0.236$. But even at that time[2] I pointed out that the value calculated from these experiments must turn out too small, for Grashof's computation assumed that in the experiments of Hirn and Cazin just mentioned there was adiabatic expansion.

Short as the duration of each individual experiment was, some heat was imparted by the walls of the vessel to the expanding, discharging steam; the real curve of expansion in such an experiment will not coincide with the adiabatic, but will pass above the latter; this fact is taken into account if we make the exponent of the real adiabatic somewhat greater than that given by the experiments.

The value $m=0.25$, assumed above, may therefore be regarded as sufficiently confirmed by the experiments referred to.

For the purpose of further discussing the question we will now draw on equation (21), p. 226; if we think of dry saturated vapor and again take $\frac{\partial R}{\partial v}=0$, and also, as before, $u=v-\sigma$, we get

$$\frac{d\lambda}{dt}=c_p-(ABT-Apu)\frac{dp}{pdt}. \quad . \quad . \quad . \quad . \quad . \quad (36)$$

According to Regnault, the total heat of saturated steam is

$$\lambda=606.5+0.305\,t$$
$$[\lambda=1091.7+0.305(t-32)],$$

[1] "Versuche über die Ausdehnung des überhitzten Wasserdampfes." Zeitschrift des Vereins deutscher Ingenieure, Vol. XI, 1867, p. 404.

[2] Remarks on the Experiments of Hirn and Cazin, "Über die Ausdehnung überhitzten Wasserdampfes." Ibid., p. 523.

and therefore we will have from the preceding equation, for the specific heat c_p of steam, estimated from the state of saturation, the following expression:

$$c_p = 0.305 + (ABT - Apu)\frac{dp}{pdt}. \quad . \quad . \quad . \quad . \quad . \quad (37)$$

If steam followed the law of Mariotte and Gay-Lussac up to the point of saturation, we could assume $pu = BT$ with sufficient accuracy and consequently take $c_p = 0.305$, and this was in fact pointed out by Redtenbacher and Kirchhoff, from entirely different considerations.

Equation (37) gives us the means of computing the specific heat c_p, for we know for steam, from Table 1a of the Appendix, the value of

$$Apu \quad \text{and} \quad \frac{dp}{pdt}$$

for different temperatures, and according to equation (34), p. 233, we may take $AB = 0.1201$.

Calculation gives for

$t =$	0°	50°	100°	150°	200°
$c_p =$	0.4258	0.4650	0.4685	0.4842	0.5055;

of course this shows that the specific heat c_p of steam is by no means a constant quantity but rather that, in the vicinity of the limit curve, it increases slowly with growing temperature. That the specific heat c_p of steam is really variable is doubtless true; but we would not be justified in judging of its variability by the preceding tabulation, for the bases for the calculations instituted are themselves fluctuating. In steam engineering the pressure limits lie between 0.1 and 15 atmospheres, and the temperature limits therefore lie between 50° C. [122° F.] and 200° C. [392° F.].

If we take the mean of the last four values of the preceding tabulation we get $c_p = 0.4808$, almost exactly the same value which resulted from the experiments of Regnault, a value to which we will hold fast in what follows.

These preparations were intended to show the reliability of the

assumptions, namely, that for steam the specific heat c_p at constant pressure is a constant, and that the supplementary term R of the equation of condition for superheated steam can be regarded as a function of the pressure alone. We now add to the following investigations the further hypothesis that the correction term R can be used in the form

$$R = Cp^n, \quad \ldots\ldots\ldots \quad (38)$$

whereupon the equation of condition appears in the form

$$pv = BT - Cp^n, \quad \ldots\ldots\ldots \quad (39)$$

an equation to which attention was called on p. 227.

In the preceding formula C, n and B as well, represent constant quantities for the steam considered. The constant B for steam is found from equation (30) with $c_p = 0.4805$ and $m = \frac{1}{4}$, and then follows from equation (32) $\kappa = \frac{4}{3}$.

If we suppose the specific pressure p of the steam to be given (in kilograms per sq. m. [in pounds per square foot]), and the specific volume v (in cubic meters per kilogram [in cubic feet per pound]), then the above-established equation of condition reproduces almost everything which has thus far been experimentally established relatively to steam, when we substitute

$$B = 50.933, \quad C = 192.50, \quad \text{and} \quad n = \frac{1}{4}$$

$$\left[B = 92.835, \quad C = 938.82, \quad \text{and} \quad n = \frac{1}{4}\right].$$

On the other hand, if the pressure is given in atmospheres (one atmosphere taken at 10333 kg. [2116.3 lb. per sq. ft.]) we must make

$$B = 0.0049287, \quad C = 0.187815, \quad \text{and} \quad n = \frac{1}{4}$$

$$\left[B = 0.043862, \quad C = 3.00859, \quad \text{and} \quad n = \frac{1}{4}\right].$$

Finally, if the pressure is given in millimeters of mercury [pounds per square inch] we must assume

$$B=3.74616, \quad C=27.1876, \quad \text{and} \quad n=\frac{1}{4}$$

$$\left[B=0.644592, \quad C=22.581875, \quad \text{and} \quad n=\frac{1}{4}\right].$$

The fact that the exponent n arising in the equation of condition coincides for steam with the magnitude of the value m, as will appear from further investigation, is a particularly favorable circumstance.

1.	2.	3.	4.	[4]	[1]
Pressure in Atmospheres (10333 kg.)	Specific Volumes of Saturated Steam. According to Table 11 of Appendix.	Specific Volumes of Saturated Steam. According to Equation (39).	Value of $\frac{C}{B}p^n$ ($n=0.25$).	Value of $\frac{C}{B}p^n$ p=lb. per sq. in. ($n=0.25$).	Pressure in lb. per sq. in.
0.1	14.533	14.677	21.429	39.021	0.147
0.2	7.543	7.583	25.483	45.873	0.294
0.5	3.172	3.181	32.043	57.682	7.348
1	1.6505	1.6506	38.106	68.599	14.697
2	0.8599	0.8583	45.316	81.575	29.393
3	0.5875	0.5861	50.151	90.278	44.090
4	0.4484	0.4474	53.891	97.010	67.496
5	0.3636	0.3630	56.982	102.576	73.485
6	0.3064	0.3060	59.640	107.359	88.180
7	0.2652	0.2650	61.983	111.577	102.887
8	0.2339	0.2339	64.087	115.465	117.572
9	0.2095	0.2096	66.002	118.812	132.270
10	0.1897	0.1900	67.764	121.983	146.97
11	0.1735	0.1739	69.398	124.925	161.66
12	0.1599	0.1604	70.924	127.378	176.36
13	0.1483	0.1489	72.357	130.252	191.06
14	0.1383	0.1383	73.711	132.688	205.5

In order to recognize the reliability of equation (39), let us first apply it to the state of saturation. In the preceding table there is inserted for the different values of the pressure (Col. 1) the specific volume of dry-saturated steam, as adduced in Table 11 of the Appendix; Col. 3 contains this same quantity calculated by formula (39) with the help of the constants just given; in Col. 4 the corresponding values of

$$\frac{C}{B}p^n = \frac{C}{B}\sqrt[4]{p}$$

are incorporated, because these values are frequently used in practical calculations.

Comparison of the values of the second and third columns shows excellent agreement; only with very low pressures do differences appear; by a very slight variation of the constants in the case of low pressure there could be produced here also an agreement which would be very satisfactory in every respect, if there were any need for so doing.

According to equation (39), we can also calculate the volume of the superheated steam for any pressure whatever, and for any temperature; if we substitute $T=273+t$ and $t=100, 110, 120$, etc., we get, for example, for superheated steam of one atmosphere pressure the following tabulation of the values of the specific volumes:

$t=100°$	$v=1.6506$ cbm.	$t=160°$	$v=1.9463$ cbm.
110	1.6999	170	1.9956
120	1.7492	180	2.0449
130	1.7984	190	2.0942
140	1.8477	200	2.1435
150	1.8970	210	2.1927

Hirn has observed the specific volume for a few values of the pressure and of the temperature; the following little survey shows how excellently his experimental results agree with the results of our formula.

Pressure in Atmospheres	Temperature Celsius. t	Temperature Fahrenheit. t	Specific Volumes, Cubic Meters. According to Hirn's Experiments.	According to Equation (39).
1	118.5	245.3	1.7400	1.7417
1	141	285.8	1.8500	1.8526
3	200	392.0	0.6970	0.6947
4	165	329.0	0.4822	0.4733
4	200	392.0	0.5220	0.5164
4	246	474.8	0.5752	0.5731
5	162.5	324.5	0.3758	0.3731
5	205	401.0	0.4140	0.4150

If we compute for the same temperature and the same pressure the specific volume v' of atmospheric air, then the ratio $v':v$ gives the weight of the steam relatively to the air; for example, for one atmosphere pressure and temperatures of 100°, 150°, and 200° C., we get, respectively, 0.6401, 0.6316, and 0.6250; this shows that, as the superheating grows, these ratios diminish and have by no means constant values, as would result if the steam followed the law of Mariotte and Gay-Lussac.

Now here is the place to return to the extensive experimental determinations of Battelli of which mention was made on p. 49, where saturated steam was considered.

For each of the isothermals of the series of temperatures given in table on p. 48 Battelli observed and recorded the specific volume of superheated steam at pressures which were different and below the saturation pressure of temperature under consideration. 242 observations are reported in all; Tumlirz' empirical formula contains p in millimeters of mercury [pounds per square inch] and is as follows (see p. 49):

$$pv = 3.4348\,T - 0.008402\,p \quad . \quad . \quad . \quad . \quad . \quad (40)$$

$$[pv = 0.59111\,T - 0.13459\,p].$$

He used it to compute v and compared the results obtained with those of Battelli.

On the other hand our equation, given on p. 237, takes the following form:

$$pv = 3.74616\,T - 27.1876\sqrt[4]{p} \quad . \quad . \quad . \quad . \quad . \quad (41)$$

$$[pv = 0.64459\,T - 22.5819\sqrt[4]{p}].$$

A few cases, taken at random, give us the following results.

From this survey we may conclude that equation (41) or (39) is well suited for the solution of technical problems, particularly for such higher temperatures as occur with steam engines, and may be continued to be used because it confirms observations along other lines existing for superheated steam.

Pressure p mm.	Volume v.		
	According to Battelli. Observed.	According to Equation (40). Computed (Tumlirz).	According to Equation (41). Computed.
Isothermal for $t=144.21°$ C. [291.578° F.].			
393.2	3.6204	3.6352	3.6663
788.6	1.8042	1.8088	1.7992
1511.4	0.9376	0.9397	0.9219
3061.9*	0.4572	0.4596	0.4544
Isothermal for $t=182.90°$ C. [361.22° F.].			
708.7	2.2043	2.2012	2.2064
1518.1	1.0212	1.0231	1.0132
4955.0	0.3104	0.3076	0.2987
7971.4*	0.1876	0.1880	0.1819

The pressures designated by * relate to the state of saturation.

Nevertheless it is worthy of note that the empirical formula (40) by Tumlirz, within the wide region covered by Battelli's experiments[1]; it can also be written in the form

$$pv=BT-Cp, \quad . \quad . \quad . \quad . \quad . \quad . \quad (40a)$$

where the term of correction R of the general form (25), p. 229, also appears as only a function of the pressure p; disregarding the changed values of the constants B and C, it is simply necessary to assume $n=1$ in equation (41) or (39); for p in kg.:sq. m. [lb.:sq. ft.] we take $B=46.700$ [$B=74.808$].

As a supplement to the preceding investigations we will add a few remarks.

In saturated steam the pressure p and the volume v are connected within wide limits by the equation

$$pv^{\mu}=D, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

as was shown with equation (33), p. 37, where the constants μ

[1] Sitzungsberichte der Kaiserl. Akademie der Wissenschaften in Wien. Mathem.-naturw. Klasse, Vol. CVII, Section II, October 1899.

and D were given; if we determine p from this and substitute the value in the supplementary term $R=Cp^n$ of equation (39), there follows

$$R=CD^n v^{-\mu n},$$

and then we get from equation $pv=BT-R$ the equation of condition in the form in which it was given by Hirn and Gustav Schmidt (see p. 227). From this we can conclude that in the vicinity of the limit curve this equation will furnish nearly the same results as equation (39).

If we determine the value v from equation (42) and substitute it in equation (39) there follows

$$BT=D^{\frac{1}{\mu}}p^{\frac{\mu-1}{\mu}}+Cp^n,$$

which can be written in the form

$$T=\alpha p^\nu+\beta p^n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

If we use the values given above for the several constants we can substitute in this formula

$$\nu=\frac{\mu-1}{\mu}=0.0607, \quad n=0.2500,$$

$$\log_{10}\alpha=2.5249094 \quad \text{and} \quad \log_{10}\beta=1.5809979.$$

From this we get, for example:

Pressure p in Atmospheres.	Temperature of Steam.	
	According to Equation (43).	According to Regnault (Table 11 of the Appendix).
1	100.00°	100.00°
3	135.14	133.91
6	160.00	159.22
9	175.68	175.77
12	187.34	188.41

The difference between formula (43) and the numerous other empirical formulas which have been set up for the relation between pressure and temperature of saturated steam consists in its constants having a definite significance resulting from the above developments. By slightly changing the constants occurring in equation (43) a better agreement of the calculated results with Regnault's experiments could be brought about, but this would efface the difference indicated.

§ 31. THE HEAT EQUATIONS OF SUPERHEATED VAPORS.

With the equation of condition

$$pv = BT - Cp^n \quad \ldots\ldots\ldots \quad (44)$$

as a basis and with the assumption that the specific heat c_p of steam is constant within the assumed limits, we can transform the always valid fundamental equations (III*b*), Vol. I, p. 143, into forms which are valid for the vapor of water. The three identical equations are:

$$\left.\begin{aligned} dQ &= c_v\frac{\partial t}{\partial p}dp + c_p\frac{\partial t}{\partial v}dv \\ &= c_v\left[dt + \frac{AT}{c_v\dfrac{\partial t}{\partial p}}dv\right] \\ &= c_p\left[dt - \frac{AT}{c_p\dfrac{\partial t}{\partial v}}dp\right]; \end{aligned}\right\} \quad \ldots\ldots \quad (45)$$

moreover, equation (II*b*) was there found to be

$$(c_p - c_v)\frac{\partial t}{\partial v}\frac{\partial t}{\partial p} = AT. \quad \ldots\ldots\ldots \quad (46)$$

By differentiation of the equation of condition (44) we find

$$B\frac{\partial t}{\partial p}=v+nCp^{n-1} \quad \text{and} \quad B\frac{\partial t}{\partial v}=p. \quad . \quad . \quad . \quad (47)$$

From equation (30) we get the auxiliary constant

$$m=\frac{AB}{c_p},$$

and with this the second of equations (47) becomes

$$c_p\frac{\partial t}{\partial v}=\frac{A}{m}p. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

From equation (46) follows

$$c_v\frac{\partial t}{\partial p}=c_p\frac{\partial t}{\partial p}-\frac{AT}{\frac{\partial t}{\partial v}},$$

and from this, with the help of equations (47) and (44), we get, after some transformations,

$$c_v\frac{\partial t}{\partial p}=\frac{A}{m}[(1-m)v+(n-m)Cp^{n-1}]. \quad . \quad . \quad . \quad . \quad (49)$$

If we now use equations (48) and (49) in equations (45) we shall get the three identical equations which are valid for the superheated vapor of water. But as we must substitute for this vapor

$$m=n=\frac{\kappa-1}{\kappa}=\frac{1}{4},$$

we get for this case, in place of the fundamental equations (45),

$$\left.\begin{aligned} dQ&=\frac{A}{\kappa-1}(vdp+\kappa pdv),\\ dQ&=c_v\left[dT+(\kappa-1)T\frac{dv}{v}\right],\\ dQ&=c_p\left[dT-\frac{\kappa-1}{\kappa}T\frac{dp}{p}\right]. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (50)$$

These, however, are the same equations which were established before for gases (Vol. I, p. 134); the sole difference consists in the fact that here the specific heat c_v for constant volume is a variable quantity which must be determined more exactly and that consequently the constant κ does not represent the ratio $c_p : c_v$ as is the case with gases.

With the fact emphasized here, which for the present is of course only valid for the vapor of water, there is connected a great advantage for technical computations; this is so considerable that, as long as the exact form of the equation of condition is not known, we may use the preceding formulas and the approximate equation of condition which was taken as a basis, even if the latter does reproduce less perfectly the experimental results.

In the discussion of the whole problem we might have started with the preceding equations (50) as results given by the theory of gases and derived the above form of the equation of condition of superheated vapor from the assumptions that c_p and κ are constant but that c_v is variable. The other assumption that c_v and κ are constant while c_p is variable would have led, in the same way, to Hirn's form of the equation of condition.[1]

As regards the variable value of the specific heat c_v we can determine it from equation (46) with the help of equations (47) and (44), after some simple transformations:

$$c_p - c_v = \frac{AB}{1-(1-n)\frac{C}{B}\frac{p^n}{T}}. \quad \ldots\ldots \quad (51)$$

The formula shows that with increasing temperature T, i.e., with growing superheat, the difference of the two specific values approaches the constant value AB; the limit value for the vapor of water is

$$c_p - c_v = 0.1201,$$

[1] See author's article, "Über das Verhalten der überhitzten und gemischten Wasserdämpfe." Zivilingenieur, Vol. XIII, 1867, p. 347.

and from this, because

$$c_p = 0.4805,$$

we get the limit value of c_v:

$$c_v = 0.3604 \quad \text{and} \quad \frac{c_p}{c_v} = 1.3333.$$

As we must substitute for the vapor of water $n = m = 0.25$ we get for the state of saturation, from equation (51), the following values belonging to the assumed pressures p:

$p =$	0.1	0.5	1	5	10 atmospheres.
$c_p - c_v =$	0.1267	0.1289	0.1301	0.1335	0.1353
$c_v =$	0.3538	0.3516	0.3504	0.3470	0.3452
$\frac{c_p}{c_v} =$	1.3581	1.3664	1.3713	1.3849	1.3920

We see that the specific heat c_v changes but slowly so that we may sometimes, in certain approximate computations, make use of a constant mean value; there are no experimental investigations on hand for c_v nor for the ratio $c_p : c_v$; indeed the former value for gases was not determined directly, and in determining the aforesaid ratio for gases the results differed as much from each other as the values just found for the vapor of water.

In any case the results of the foregoing calculations do not contradict any known fact, and may therefore be regarded as valid and acceptable for technical investigations under the limitations fixed beforehand on p. 224.

Let us now join to the preceding investigation one on the magnitude of the coefficient of expansion of superheated vapors.

If we substitute the supplementary term $R = Cp^n$ of the equation of condition in equations (67) and (68), Vol. I, p. 142, there follows

$$B(a - a_v) = (1 - n)Cp^n, \quad \ldots\ldots (52a)$$

$$B(a - a_p) = Cp^n. \quad \ldots\ldots\ldots (53a)$$

Here $a=273$ [491.4] means the reciprocal value of the coefficient of expansion of a perfect gas, α_v the reciprocal value of the so-called coefficient of expansion at constant volume, hence with change of pressure, and α_p the coefficient of expansion at constant pressure, hence with change of volume; if the latter coefficient itself is designated by α_v and the other by α_p, then we have for the superheated vapor of water

$$\alpha_v = \frac{1}{a-(1-n)\frac{C}{B}p^n} \quad \ldots\ldots\ldots \quad (52)$$

and

$$\alpha_p = \frac{1}{a-\frac{C}{B}p^n}. \quad \ldots\ldots\ldots \quad (53)$$

Both formulas give for gases, for which $C=0$,

$$\alpha_v=\alpha_p=\frac{1}{a}=0.003665\ [0.002035].$$

On the other hand the formulas make the two coefficients of expansion greater for vapors than for gases and different from each other; it is at once evident from the two formulas that the coefficient α_p for change of volume is g r e a t e r than the value α_v for change of pressure; finally both equations show that both kinds of the coefficients of expansion are greater the higher the pressure considered.

These are all propositions which R e g n a u l t derived from his experiments on the behavior of gaseous bodies and concerning which an outline was given in Vol. I, pp. 97 and 98, of this treatise.

According to equations (52) and (53) and with the help of the tabulation on p. 238, for the vapor of water in a saturated condition, the aforesaid coefficients of expansion have been computed for several pressure values.

Pressure in Atmospheres.	Coefficients of Expansion for Change of Pressure.	for Change of Volume.
$p=0.1$	$\alpha_v=0.003892$	$\alpha_p=0.003975$
0.5	0.004017	0.004150
1	0.004090	0.004257
5	0.004343	0.004629
10	0.004501	0.004872

The two coefficients, according to equations (52) and (53), are only functions of the pressure; the preceding numerical values therefore are valid at the given pressure not only for the state of saturation, but also for the region of superheat.

Finally we must derive, for superheated vapors, the values of those heat quantities which were designated as steam heat and as total heat when considering saturated vapors.

From equation (20), p. 225, and with the help of equations (30) and (47), i.e., if we employ

$$c_p=\frac{AB}{m} \quad \text{and} \quad \frac{\partial t}{\partial v}=\frac{p}{B},$$

we get the increment of energy dU:

$$dU=\frac{d(BT)}{m}-\frac{BT}{p}dp-d(pv)+pv\frac{dp}{p},$$

and if we eliminate BT, with the help of the equation of condition (44), we get

$$dU=\frac{1-m}{m}d(pv)+\frac{n-m}{m}Cp^{n-1}dp. \quad . \quad . \quad . \quad . \quad (54)$$

Here the two constants m and n are still assumed as different; but for the vapor of water these constants were found to be alike, and if in addition we substitute the constant κ in place of m, according to equation (32), p. 232, we shall get for the vapor of water the equation

$$dU=\frac{d(pv)}{\kappa-1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

the same formula which is valid for gases (Vol. I, p. 132).

If we designate by J the inner work AU, expressed in units of heat and occurring in equation (55), and likewise the constant AU_0 by J_0, then this equation can also be written as

$$J = J_0 + \frac{Apv}{\kappa - 1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

If we compare the superheated vapor of water of pressure p and volume v with water of 0° [32°] temperature, i.e., assume that this steam was formed from this water, then the preceding formula states "how much more heat is contained in this steam than in the equal weight of water at 0° [32°]." The quantity J is therefore what was designated as "steam heat" when considering saturated vapor on p. 27; this quantity was expressed there by equation (20):

$$J = q + \rho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56a)$$

For the state of saturation of the vapor of water the last two formulas for the steam heat must give the same value, and here the heat of the liquid q and the inner latent heat ρ should be taken from Steam Table 11 of the Appendix.

We get a thoroughly satisfactory agreement if we substitute $J_0 = 476.11$ [857] in equation (56). The so-called total heat λ, i.e., the heat quantity necessary in order to produce superheated steam of volume v from water at 0° [32°] under constant pressure p, is found by adding the heat quantity Apu to the right member of equation (56), which quantity is here converted into external work; as the value u differs but little from v for the vapor of water, we can put Apv in place of Apu, and then get

$$\lambda = J_0 + \frac{\kappa}{\kappa - 1} Apv, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

which equation is also valid for dry saturated steam for which Regnault found the relation

$$\lambda = 606.5 + 0.305\, t$$
$$[\lambda = 1091.7 + 0.305(t - 32°)],$$

or, according to the earlier presentations, we can write

$$\lambda = q + \rho + Apu. \quad . \quad . \quad . \quad . \quad . \quad . \quad (57a)$$

As $\kappa = \frac{4}{3}$ for the vapor of water, the two equations (56) and (57) can also be written

$$J = 476.11 + 3\, Apv, \quad . \quad . \quad . \quad . \quad . \quad . \quad (56b)$$

$$\lambda = 476.11 + 4\, Apv. \quad . \quad . \quad . \quad . \quad . \quad . \quad (57b)$$

$$\left[\begin{matrix} J = 856.998 + 3\, Apv, \\ \lambda = 856.998 + 4\, Apv. \end{matrix}\right]$$

From these two formulas there were determined for saturated vapor of water the values contained in the second and fourth columns of the following tabulation, while the values in the third and fifth columns were found from equations (56*a*) and (57*a*), with the help of Table 11 of the Appendix.

1.	2.	3.	4.	5.
Pressure in Atmospheres (10333 kg.) p	Steam Heat J		Total Heat λ	
	according to Equation (56*b*).	according to Equation (56*a*).	according to Equation (57*b*).	according to Equation (57*a*).
0.5	592.39	592.78	631.15	631.42
1	596.79	596.80	637.01	637.00
2	601.61	601.42	643.44	643.28
3	604.66	604.47	647.51	647.34
4	606.95	606.81	650.56	650.42
5	608.81	608.73	653.04	652.93
6	610.34	610.39	655.08	655.06
7	612.91	613.18	658.52	658.60
8	615.02	615.49	661.32	661.49
9	616.83	617.49	663.74	663.96

We see, from a comparison of the numerical values obtained by the two methods of calculation, that the agreement, within the

assumed pressure limits, is a thoroughly satisfactory one; outside of these limits the empirical formulas by Regnault, which serve as the starting point of all our investigations, are themselves unreliable.

If we use the equation of condition $pv = BT - Cp^n$ in equation (56), and also use equation (30), p. 231, remembering that with the vapor of water we substitute

$$m = n - \frac{\kappa - 1}{\kappa},$$

we also get for the steam heat

$$J = J_0 + \left(1 - \frac{C}{B}\frac{p^{\frac{\kappa-1}{\kappa}}}{T}\right)\frac{c_p}{\kappa}T,$$

or, after using equation (51), p. 245, we find after some transformations

$$J = J_0 + \left(1 - \frac{1}{\kappa}\frac{C}{B}\frac{p^{\frac{\kappa-1}{\kappa}}}{T}\right)c_v T. \quad . \quad . \quad . \quad . \quad . \quad (56c)$$

For very great superheating or for the gaseous state there follows from this

$$J = J_0 + c_v T,$$

the same equation which was found earlier for the gas heat or "heat contents" of a gas (Vol. I, p. 132).

If, in the same way, we transform equation (57) which was first found for the total heat λ, we get for the vapor of water

$$\lambda = J_0 + c_p\left(T - \frac{C}{B}p^{\frac{\kappa-1}{\kappa}}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (57c)$$

and this equation differs from that given by Regnault in that it is simultaneously valid for saturated and superheated steam, while Regnault's empirical formula is only valid for dry saturated steam.

Let us assume superheated steam of pressure p and tempera-

ture T and let T' be the temperature of dry saturated steam of the same pressure, then the total heat λ' of the latter is

$$\lambda' = J_0 + c_p\left(T' - \frac{C}{B}p^{\frac{\kappa-1}{\kappa}}\right).$$

If we subtract this equation from equation (57*c*) we get

$$\lambda = \lambda' + c_p(T - T'),$$

or, utilizing Regnault's formula and the temperature according to Celsius [Fahrenheit], we get

$$\lambda = (606.5 + 0.305\,t') + c_p(t - t') \quad . \quad . \quad . \quad . \quad . \quad (57d)$$
$$[\lambda = 1091.7 + 0.305(t' - 32) + c_p(t - t')],$$

an equation which has long and often been used for the calculation of the total heat of superheated steam when the superheat is $(t - t')$.

§ 32. THE VAPOR OF AMMONIA.

The vapor of ammonia has become important in mechanical engineering because of its use in the production of cold and ice, through the medium of Carré's absorption machine and Linde's compression machine.

For the saturated state of this vapor Regnault also determined, experimentally, the relation between pressure and temperature and expressed it by empirical formulas; this has already been brought out on pp. 14 and 17, so that for saturated vapor of ammonia there can be found, from Table 8 of the Appendix for assumed temperatures, the values of p and also those of

$$\frac{dp}{pdt} \quad \text{and} \quad \frac{r}{u} = AT\frac{dp}{dt}.$$

The study and design of the aforesaid machines made it apparent from the beginning that a knowledge of the behavior of ammonia vapor should rest on as certain a foundation as possible.

In building the engine first mentioned the existing information desired was very scanty, there was a complete lack of observation on the heat of the liquid q and on the latent heat r, and not even the coefficient of expansion of the superheated vapor of ammonia was approximately known.

Ledoux[1] therefore sought to establish the fundamental equations for the vapor of ammonia on the basis of certain hypotheses and with the help of some superficially known physical values. In so doing he started from the assumption that this vapor is, within certain bounds, subject to the equation of condition

$$pv = BT - Cp^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

and determined the constants B, C, and n of this formula from the experimental material on hand; I had already shown that this formula was permissible for water.

Later on the author again took up the investigations in a similar manner,[2] and determined the constants of the preceding equation in a way closely corresponding to a certain series of experiments by Regnault. There resulted for p in kg./sq. cm. [lb./sq. in.]:

$$B = 52.642, \quad C = 29.783, \quad \text{and} \quad n = 0.3655$$
$$[B = 6663.27, \quad C = 2571.44, \quad \text{and} \quad n = 0.3655].$$

so that the preceding formula enabled us to compute the specific volume $v = s$ for different pressures and different temperatures; the specific volume σ of liquid ammonia is to be taken on the average as $\sigma = 0.0016$ [0.0256] within the temperature limits occurring with ammonia compressors.

The article quoted below gives the course of the calculation of the constants of equation (58), and in this connection we need only remark that the factor B depends mainly on the coefficient of expansion α_p of ammonia vapor under constant pressure, for which

[1] Ledoux, "Théorie des machines à froid." Paris, Dunod. 1878. Extrait des Annales des Mines, livraison de juillet-août, 1878.

[2] Zeuner, Zur Theorie der Kalt-Dampfmaschinen. With a supplement, "Über das Verha'ten der Ammoniakdämp'e" Zivilingenieur 1881, p. 449.

value there was assumed $\alpha_p = 0.00390$, which of course involves great uncertainty because no experiments were on hand; the two other constants C and n were derived from the results of some experiments which Regnault had given for the compression of ammonia gas at constant temperature. Then the following empirical formulas were found for the saturated vapor of ammonia:

the outer latent heat:

$$Apu = 30.248 + 0.06938\,t - 0.000235\,t^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$
$$[Apu = 54.4464 + 0.06938(t-32) - 0.000131(t-32°)^2];$$

the inner latent heat:

$$\rho = 284.617 - 0.71241\,t - 0.004479\,t^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$
$$[\rho = 512.3106 - 0.71241(t-32°) - 0.00249(t-32°)^2].$$

The specific heat of liquid ammonia was found to be

$$c = 1.0135 + 0.008378\,t \quad . \quad . \quad . \quad . \quad . \quad (61)$$
$$[c = 1.0135 + 0.00468(t-32°)],$$

and the heat of the liquid

$$q = 1.01235\,t + 0.004189\,t^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$
$$[q = 1.01235(t-32°) + 0.00233(t-32°)^2].$$

With the help of these formulas the several columns for saturated vapor of ammonia in Table 8 of the Appendix were calculated; the values in the upper half of the table relate to temperatures below 0° [32°] (from −5° to −30° C. [from +23° to −22° F.]). In the earlier editions of this book the table is continued for temperatures from 0° to 40° C. [32° to 104° F.], and have since been much used in calculating refrigerating machines, and not without success, although we must always be inclined to employ the numerical values with some hesitation, for the underlying formulas were developed under hypotheses against which many an objection could be raised from the beginning.

Although ammonia vapors have been used a great deal, we had

to wait longest for clarifying experiments concerning them; finally, quite recently, such experiments were conducted by C. Dietrici and were made public.[1] Unfortunately these experiments are confined to temperatures above 0° C. [32° F.].

The second part of Table 8 of the Appendix contains the data furnished by Dietrici, and they naturally inspire more confidence than the older results based on hypotheses.

In calculating refrigerating machines, however, we must go below 0° [32° F.], and therefore at the present time there is no other resource in such cases but to fall back on the old tables, for which purpose the first part of Table 8 is adduced with the older results.

As regards Dietrici's data the values in column 5 represent the specific volume s of saturated vapor of ammonia for the different temperatures 0° to +40° [32° to +104°] according to the experimental results; we omit connecting the given values by an empirical formula.

As here we only think of utilizing the table for the calculation of refrigerating machines and therefore have only a purely technical purpose in mind, we can substitute on the average $\sigma = 0.0016$ [0.0256] for the specific volume of liquid ammonia. Dietrici gives elsewhere the values s and σ up to the temperature $t = +130.5°$ [266.9°], the critical temperature of ammonia.

Dietrici found for the specific heat of liquid ammonia

$$c = 1.118 + 0.00208\, t$$
$$[c = 1.118 + 0.001156(t - 32°)],$$

and hence for the heat of the liquid

$$q = \int_0^t c\,dt = 1.118\, t + 0.00104\, t^2$$
$$\left[q = \int_0^t c\,dt = 1.118(t - 32°) + 0.000578(t - 32°)^2\right],$$

from which column 6 is calculated.

[1] Über die thermischen und kalorischen Eigenschaften des Ammoniaks. Zeitschrift für die gesamte Kälte-Industrie, 1904.

The latent heat r, and Apu and ρ as well, are calculated by Dietrici in columns 7, 8, and 9 under the assumption of his more exact values for s and σ, using for this purpose $u=s-\sigma$, according to the known formula

$$\frac{r}{u}=AT\frac{dp}{dt}.$$

There is nothing special to be said about the calculation of the values in columns 10 and 11; we will only say that, according to Dietrici, the latent heat can be expressed with sufficient accuracy by the empirical formula

$$r=27.65\sqrt{t_k-t}$$
$$[r=36.426\sqrt{t_k-t}]$$

when we take $t_k=130.5°$ [266.9°].

§ 33. THE VAPOR OF SULPHUROUS ACID.

For sulphurous acid Ledoux made the attempt to establish the equations of condition to derive the equations for the heat of the liquid, for the inner and outer latent heat, and to set up a table in the form discussed when ammonia vapor was under consideration; he used Regnault's compression experiments as a basis and the few remaining data that were known.

Later there became known the experimental results of Cailletet and Mathias[1] on the specific weight of liquid sulphurous acid and of its vapor in the condition of saturation; these results will be made the starting-point of the following investigations; they were used in the 3d edition of the present book for setting up a steam table. In so doing, however, it was assumed that the equation of condition of the vapor of sulphurous acid could be given the same form as with the vapors of water and of ammonia,

[1] Cailletet and Mathias, "Recherches sur la densité de l'acide sulfureux a l'état de liquide et de vapeur saturée." Comptes rendus, 1887, Vol. CIV, p. 1563.

and under this assumption formulas were derived for the specific heat c of liquid sulphurous acid for the heat of the liquid q and for the entropy τ, for up to this time no reliable experiments on the value of c existed. Meanwhile this gap has also been filled by the recent experiments of Mathias,[1] so that by drawing on the experimental results mentioned, the Steam Table 9 of the Appendix can be calculated without recourse to any hypothesis. Table 9 is only valid for saturated vapor and is calculated for the limited temperature interval $-30°$ to $+40°$ [$-22°$ to $+104°$], because this is sufficient for the application which these vapors have found in engine running; we have here omitted a special investigation of superheated vapor of sulphurous acid, in this respect departing from the presentations given in the third edition.

The following tabulation includes the experimental results of Cailletet and Mathias, and gives the specific weight γ (weight of a cu. m. in kg. [1 cu. ft. in lb.]) for different temperatures.

Saturated Vapor of Sulphurous Acid.

t	γ	t	γ
+ 7.3°	6.24	+100.6°	78.6
16.5	8.58	123.0	134.0
24.7	11.2	130.0	160.7
37.5	16.9	135.0	188.7
45.4	21.8	144.0	249.5
58.2	31.0	152.5	342.6
78.7	46.4	154.9	401.7
91.0	62.6		
+ 45.14	0.389	+213.08	4.91
61.70	0.532	253.40	8.37
76.46	0.699	266.00	10.04
99.50	1.055	275.00	11.78
113.72	1.361	291.20	15.58
136.76	1.935	306.50	21.39
173.66	2.900	310.82	25.08
195.80	3.908		

[1] Comptes rendus, 1894, Vol. CXIX, pp. 404–407.

Liquid Sulphurous Acid under the Steam Pressure Corresponding to the Temperature:

$t = +\ 0.0°$	$\gamma = 1433.8$	$t = +120.45°$	$\gamma = 1016.6$
21.7	1375.7	130.30	956.0
35.2	1337.4	140.8	869.0
52.0	1287.2	146.6	806.5
62.0	1252.3	151.75	731.7
82.4	1184.5	154.3	670.6
102.4	1104.1	155.05	637.0
$t = +\ 32.0$	$\gamma = 89.507$	$t = +248.81$	$\gamma = 63.316$
71.06	85.879	266.54	59.679
95.36	83.489	285.44	54.248
125.60	80.355	295.88	50.347
143.60	78.176	305.15	45.677
180.32	73.943	309.74	41.873
215.33	68.766	311.09	39.766

The critical temperature was found to be $t_k = 156°$ [312.8°]; here the specific weight of vapor and liquid is the same and amounts to 520 kg. per cbm. [32.46 lb. per cu. ft.].

The values of γ given in the preceding tabulation determine through their reciprocals the specific volume s of the saturated steam and σ of the liquid. If, in addition, we know the steam pressure p corresponding to the temperature t we can draw the limit curve for sulphurous acid in the manner discussed on p. 209 when considering carbonic acid. Cailletet and Mathias give a picture of the limit curve, for they lay off the temperatures t as abscissas and the specific weights γ as ordinates.

For the calculation of the saturation pressure p of sulphurous acid use was made of Regnault's empirical formula given on p. 14 when Table 9 of the Appendix was set up; up to the temperature of +50° [122°] this formula, according to Sajotschewsky,[1] gives perfectly reliable values, but at higher tem-

[1] Sajotschewsky, "Dampfspannungen gesättigter Dämpfe." Beiblätter zu Wiedemann's Annalen, 1879, Vol. III, p. 741

peratures it is said to differ from the new observations; we will omit the statements of Sajotschewsky in this connection and will only remark that he placed the critical temperature of sulphurous acid at $t_k = 155.4°$ [279.72°] and the corresponding pressure at $p_k = 78.9$ atmospheres; the first value agrees closely with that given by Cailletet and Mathias.

From the preceding experimental results and with the help of Clapeyron's equation (22), p. 29, namely,

$$r = Atu\frac{dp}{dt},$$

Cailletet and Mathias derive the following values for the latent heat r of sulphurous acid:

$t =$	0°	+10°	+20°	+30°	+40°
$r =$	91.2	88.7	84.7	80.5	75.5

$t =$	32°	50°	68°	86°	102°
$r =$	164.16	159.66	152.46	144.90	135.90

and these values can be reproduced with sufficient accuracy by

$$r = 91.2 - 0.2575\,t - 0.003375\,t^2 \quad . \quad . \quad . \quad . \quad (63)$$
$$[r = 164.16 - 0.2575(t - 32°) - 0.001875(t - 32°)^2].$$

By its means we get for

$t =$	0°	+10°	+20°	+30°	+40°
$r =$	91.2	88.3	84.7	80.4	75.5

$t =$	32°	50°	68°	86°	102°
$r =$	164.16	159.66	152.46	144.90	135.90

The experiments and the preceding formula to be sure confine themselves to temperatures above the freezing point of water, but it is probably permissible to also employ equation (63) below 0° [32°]; in cold-vapor engines the lower temperature limit is rarely below −15° C. [+5° F.].

From the preceding formula of Clapeyron we get

$$\frac{r}{Apu} = T\frac{dp}{pdt},$$

and from this, with the help of the values of Col. 4, Table 9, we calculate the following values for Apu:

$t=$	$-30°$	$-20°$	$0°$	$+20°$	$+30°$
$Apu=$	7.355	7.716	8.230	8.376	8.308

$t=$	$-22°$	$-6°$	$32°$	$68°$	$86°$
$Apu=$	13.239	13.8888	14.8140	15.0768	14.9544

There is a good agreement with the empirical formula

$$Apu = 8.230 + 0.0165\,t - 0.000460\,t^2 \quad . \; . \; . \quad (64)$$
$$[Apu = 14.814 + 0.0165\,(t-32°) - 0.000256\,(t-32°)^2],$$

for this gives for

$t=$	$-30°$	$-20°$	$0°$	$+20°$	$+30°$
$Apu=$	7.321	7.716	8.230	8.376	8.311

$t=$	$-22°$	$-6°$	$32°$	$68°$	$86°$
$Apu=$	13.1778	13.8888	14.8140	15.0768	14.9598

The combination of the two equations (63) and (64) and the relation $r = \rho + Apu$ furnish the following formula for the inner latent heat:

$$\rho = 82.970 - 0.2740\,t - 0.002915\,t^2 \quad . \; . \; . \; . \quad (65)$$
$$[\rho = 149.346 - 0.2740(t-32°) - 0.00162(t-32°)^2].$$

We can compute the value u from equation (64) for different values of the saturation pressure p and for the corresponding temperature t; for the closely lying temperature limits which arise here ($-30°$ to $+40°$) [$-22°$ to $104°$] we can treat the specific volume σ of sulphurous acid as constant, and can take it to be

$\sigma = 0.0007$ [0.00112]; we then get the specific volume v and from it the specific weight of the saturated vapor, as follows:

$$v = u + \sigma \quad \text{and} \quad \gamma = \frac{1}{u+\sigma}.$$

The corresponding values of Table 9 of the Appendix have been computed from the above formulas.

For the specific heat c of liquid sulphurous acid Mathias gives the formula

$$c = \frac{dq}{dt} = 0.31712 + 0.0003507\, t + 0.000006762\, t^2 \quad . \; . \; . \quad (64a)$$

$$\left[c = \frac{dq}{dt} = 0.31712 + 0.0001948(t-32°) + 0.00000209(t-32°)^2\right]$$

as valid for the wide temperature limits $-20°$ to $+130°$ [$-4°$ to $266°$]; for the narrow temperature limits $-30°$ to $+40°$ [$-22°$ to $+104°$] of our table we can, with sufficient accuracy, calculate c according to the simple formula

$$c = 0.3171 + 0.000554\, t$$
$$[c = 0.3171 + 0.000308(t-32°)],$$

and with this help we get for the heat of the liquid

$$q = \int_0^t c\,dt = 0.3171\, t + 0.000277\, t^2 \quad . \; . \; . \; . \; . \; . \; . \quad (65a)$$

$$\left[q = \int_0^t c\,dt = 0.1762(t-32°) + 0.0000855(t-32°)^2\right],$$

and for the entropy of liquid sulphurous acid, at the pressure of saturation, we have

$$\tau = \int_0^t \frac{c\,dt}{T} = 0.00055\, t - 0.93061 + 0.3820 \log_{10} T \quad . \; . \; . \quad (66)$$

$$\left[\tau=\int_0^t\frac{cdt}{T}=0.000306(t-32°)-0.83310+0.3820\log_{10}T\right].$$

Columns 6 and 11 of Table 9 were calculated from formulas (65) and (66); we see from the values in Col. 11 that the differences are nearly constant, so that we may approximately set $\tau=\alpha t$, as was suggested by Mollier, where α represents a constant quantity.

§ 34. THE VAPOR OF CARBONIC ACID.

It was stated on p. 209 that carbonic acid vapor was first subjected by Andrews to experimental examination, and these experiments furnished the basis for the theoretical studies of van der Waals and Clausius, and for the equations of condition set up by them (pp. 211 and 217). Later Sarrau had occasion to utilize more recent experiments by Cailletet and Hautefeuille, and still further experiments by Cailletet and Mathias[1] were utilized in the earlier editions for the treatment of the question. In so doing, however, the starting-point was the assumption that the same approximate form of the equation of condition could be used with carbonic acid as with the vapors of water and ammonia. This equation was found to answer completely for technical calculations concerning the vapor of water.

But conditions are different with carbonic acid; although I did not want the formulas and numerical values in question to be employed beyond the upper temperature limit of 20° C. [68° F.], and therefore not in the vicinity of the critical temperature of 31.35° [88.43° F.], still the observations on the carbonic acid engine (cold-vapor), which in the interval had been extensively employed, soon showed that assumptions which were based on the simple equation of condition were not tenable, for, with the engines mentioned, temperature values arise which are close to the critical temperature, and will often exceed it.

[1] Cailletet and Mathias. "Recherches sur les densités des gaz liquéfiées et de leurs vapeurs saturées." Comptes rendus, Vol. CII, 1886, p. 1202.

In the interval, Amagat's comprehensive experimental results have become known,[1] and thus the bases have been supplied for presenting the behavior of carbonic acid in more reliable fashion; Mollier,[2] on the basis of such experiments, has set up a steam table which is reproduced in the Appendix under No. 10 on pages xxii and xxiii, and this table can be employed with great confidence in calculating and judging cold-vapor engines of the carbonic acid variety.

Amagat gives, for liquid carbonic acid, and for saturated vapor, for every degree from 0° [32° F.] up to the critical temperature 31.35° C. [88.43° F.], the relative weight with respect to water; if this value is designated by ε, then 1000 ε is the specific weight, and the specific volume is calculated from

$$v=\frac{1}{1000\,\varepsilon}\left[\frac{0.016}{\varepsilon}\right].$$

From this were obtained the values of Column 7 and Column 8 of Table 10, and, furthermore, the values of u in Column 9 were determined for $u=s-\sigma$. Mollier finds that these are reproduced by the equation of condition

$$p=\frac{BT}{v-\alpha}-\frac{Ce^{1-\frac{T}{T_k}}}{(v+\beta)^2}, \quad \ldots\ldots\ldots \quad (67)$$

which was proposed by van der Waals. If p is estimated in atmospheres, we must substitute:

$B=19.32$, $C=19.36$, $\alpha=0.002037$, and $\beta=0.0007719$
[$B=171.936$, $C=4970.45$, $\alpha=0.032631$, and $\beta=0.012365$].

T_k signifies the absolute temperature at the critical point, so that

[1] Amagat, "Sur la détermination de la densité des gaz liquéfiées et de leurs vapeurs saturées.—Éléments du point critique de l'acide carbonique." Comptes rendus, Vol. CXIV, 1892, p. 1093. See also Vol. CXIII, 1891.

[2] Mollier, "Über die kalorischen Eigenschaften der Kohlensäure und anderer technisch wichtiger Dämpfe." Zeitschrift für die gesamte Kälte-Industrie. II. year. 1895. pp. 66 and 85.

$$T_k = 273 + 31.35 = 304.35° \text{ [547.8°]}.$$

As Amagat's data relate only to temperatures beyond 0° [32°], Mollier, to supplement the table for temperatures below 0° C. [32° F.], drew on the older experiments and they furnished a good connection.

As regards the saturation pressures adduced in columns 3 and 4, these are not calculated according to Regnault from the data upon pages 14 and 17, but the saturation pressures p were newly observed by Amagat; the values of Table 10 in question differ somewhat from those of Regnault, and increase less rapidly between the temperature limits employed. Mollier finds that the formula

$$p = 2.9674\left(\frac{T}{100} - 1\right)^{\frac{1}{0.221}} \quad \ldots\ldots \quad (68)$$

$$\left[p = 42.2056\left(\frac{T}{180} - 1\right)^{\frac{1}{0.221}}\right]$$

gives a good agreement with Amagat's data where p is expressed in kg. : sq. cm. [lb. per sq. in.]. With this the values of Columns 5 and 6 were calculated; and there follow, furthermore, the values of Columns 11, 12, and 13, because u is known, i.e., we have the values

$$r = AuT\frac{dp}{dt}, \quad Apu = \frac{r}{\frac{T}{p}\frac{dp}{dt}}, \text{ and } \quad \rho = r - Apu.$$

The same difficulty encountered above with ammonia now arises here; as the specific heat c of liquid carbonic acid has not yet been determined experimentally, we cannot compute directly the heat of the liquid q and the entropy τ, and must therefore try to determine these quantities in an approximate way. In a clever manner Mollier obtains here a fundamental formula which will be derived somewhat more briefly in what follows, and this may be regarded for the present as thoroughly reliable for technical

purposes, for here the hypothetical assumptions have been reduced to the minimum.

Of the three identical heat equations (45), p. 243, or equation (III*b*) of Vol. I, p. 143, we utilize the second form in the following investigations, namely,

$$dQ = c_v dt + \frac{AT}{\frac{\partial t}{\partial p}} dv. \quad \ldots\ldots\ldots \quad (69)$$

In this formula p and v have hitherto been regarded as independent variables, as has been our practice throughout the present book. For the following purposes, however, it is simpler to treat t and v as independent variables, as Clausius did in all his works; naturally t and p can also be treated as such.

In the case $t = f(p, v)$ or $p = F(t, v)$ we obtain, respectively,

$$dt = \left(\frac{dt}{dp}\right)_v dp + \left(\frac{dt}{dv}\right)_p dv, \quad dp = \left(\frac{dp}{dt}\right)_v dt + \left(\frac{dp}{dv}\right)_t dv,$$

where, to avoid confusion, the subscript affixed indicates the magnitude which is to be treated as constant during differentiation.

For constant v, hence for $dv = 0$, the two preceding formulas give

$$dt = \left(\frac{dt}{dp}\right)_v dp \quad \text{and} \quad dp = \left(\frac{dp}{dt}\right)_v dt,$$

and hence, for a combination of the two,

$$\left(\frac{dt}{dp}\right)_v = \frac{1}{\left(\frac{dp}{dt}\right)_v}.$$

Since $\left(\frac{dt}{dp}\right)_v$ is identical with the earlier notation $\frac{\partial t}{\partial p}$, there follows for our further purposes, from equation (69), with t and v as independent variables,

$$dQ = c_v dt + AT\left(\frac{dp}{dt}\right)_v dv. \quad \ldots\ldots \quad (69a)$$

Accordingly there follows for the heat weight or entropy P, from our former notation (Vol. I, p. 134),

$$dP=\frac{dQ}{AT},$$

$$dP=\frac{c_v dt}{AT}+\left(\frac{dp}{dt}\right)_v dv, \quad \ldots\ldots\ldots \quad (70)$$

an expression which can always be integrated and which will furnish the entropy P as a function of t and v; the determination of function P is the aim, in order that we may obtain the fundamental equation of Mollier. But in so doing we must remark that here P must be conceived as measured in units of work, and then AP is the entropy in units of heat.

We have, from equation (70), for constant temperature,

$$dP=\left(\frac{dp}{dt}\right)_v dv,$$

and from this

$$P=\int\left(\frac{dp}{dt}\right)_v dv+f(t), \quad \ldots\ldots\ldots \quad (71)$$

where $f(t)$ represents a temperature function to be determined.

By differentiation of this equation with respect to t and a constant v there is found

$$\left(\frac{dP}{dt}\right)_v=\int\left(\frac{d^2p}{dt^2}\right)_v dv+\frac{df(t)}{dt},$$

and on the other hand, from equation (70),

$$\left(\frac{dP}{dt}\right)_v=\frac{c_v}{AT},$$

hence from a combination of the two formulas we have

$$c_v=AT\int\left(\frac{d^2p}{dt^2}\right)_v dv+AT\frac{df(t)}{dt}. \quad \ldots\ldots \quad (72)$$

If we now proceed from the hypothesis that carbonic acid in the highly superheated condition approaches the equation of condition of gases $pv=BT$ and c_v approaches the constant value c_v', there follows for this state

$$\left(\frac{dp}{dt}\right)_v=\frac{B}{v} \quad \text{and} \quad \left(\frac{d^2p}{dt^2}\right)_v=0,$$

and with this, from equation (72),

$$c_v'=AT\frac{df(t)}{dt};$$

accordingly,

$$f(t)=\frac{c_v'}{A}\log_e T. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

Substitution in equation (71) then gives

$$AP=A\int\left(\frac{dp}{dt}\right)_v dv+c_v'\log_e T. \quad . \quad . \quad . \quad . \quad (74)$$

From equation (72) follows

$$c_v=c_v'+AT\int\left(\frac{d^2p}{dt^2}\right)_v dv, \quad . \quad . \quad . \quad . \quad . \quad (75)$$

where, according to J o l y, we should substitute $c_v'=0.16577$.

If we now return to the equation of condition which will be chosen in the form of equation (67), its differentiation gives

$$\left(\frac{dp}{dt}\right)_v=\frac{B}{v-\alpha}+\frac{Ce^{1-\frac{T}{T_k}}}{T_k(v+\beta)^2}, \quad . \quad . \quad . \quad . \quad . \quad (76a)$$

and the use of this expression in equation (74) gives, after integration, the entropy, measured in units of heat,

$$AP=AB\log_e(v-\alpha)-\frac{ACe^{1-\frac{T}{T_k}}}{T_k(v+\beta)}+c_v'\log_e T+K, \quad . \quad (76)$$

and this is the equation given by Mollier. In this we should, according to Joly, take $c_v' = 0.16577$ and the integration constant K should be so chosen that for liquid carbonic acid, i.e., for $v = \sigma$, the value $AP = \tau$, and therefore when $t = 0$ we should also have $AP = \tau = 0$. In like manner if we substitute for the value $v = s$ different temperatures, according to Table 10, we get the entropy of the saturated vapor of carbonic acid, i.e.,

$$AP = \tau + \frac{r}{T}.$$

In this way Mollier, with the help of equation (76), calculated for different temperatures a series of values τ for liquid carbonic acid and of $\tau + \frac{r}{T}$ for carbonic acid vapor, and laid off the result graphically (Fig. 26), making the entropies the abscissas, and the temperatures the ordinates; the adjacent curve ABC shows the law of change. If we imagine a mixture of liquid and vapor and let x be the steam quality, then for this mixture the entropy is

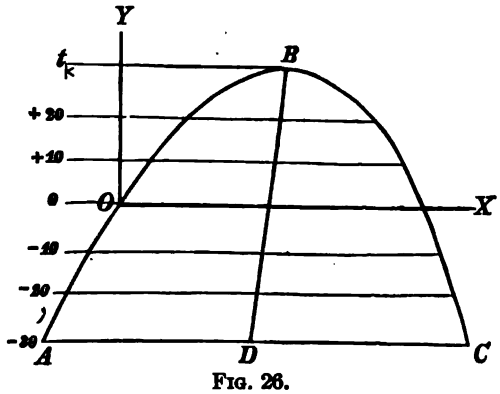

FIG. 26.

$$AP = \tau + \frac{xr}{T}$$

(see p. 64), so that now we can draw the curves for different values of x. In so doing it is seen that for $x = \frac{1}{2}$ the two curves become almost exactly a straight line BD (Fig. 26), which passes through the point of bisection D of the distance AC and through the point B corresponding to the critical temperature. With it we find

$$\tau = 0.10155 + 0.000333t - \frac{1}{2}\frac{r}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

$$\left[\tau = 0.10155 + 0.000185\,(t - 32°) - \frac{1}{2}\,\frac{r}{T}\right],$$

and from this simple formula are calculated the values of Col. 15, Table 10.

Mollier calculated the values of the heat of the liquid q of Col. 10 by multiplying the differences of the values of Col. 15 by the mean temperature of the interval and by summing up the products from 0° [32°] on.

The specific heat c of liquid carbonic acid is calculated from

$$c = T\,\frac{d\tau}{dt},$$

and with equation (77) it becomes

$$c = 0.000333\,T - \tfrac{1}{2}T\frac{d}{dt}\left(\frac{r}{T}\right) \quad . \quad . \quad . \quad . \quad . \quad (78)$$

$$\left[c = 0.000185\,T - \tfrac{1}{2}T\frac{d\left(\frac{r}{T}\right)}{dt}\right].$$

Mollier finds that r can be sufficiently well reproduced by the empirical formula

$$r = aT^n(T_k - T)^n,$$

when we substitute $a = 1.13$ [0.6816], $n = 0.43$, and $T_k = 304.35$ [547.83], and with this there follows from equation (78)

$$c = 0.000333\,T + 0.285\frac{r}{T} + 0.215\frac{r}{T_k - T} \quad . \quad . \quad . \quad (78a)$$

$$\left[c = 0.000185\,T + 0.285\frac{r}{T} + 0.215\frac{r}{T_k - T}\right]$$

For example, we get from this formula for

$t =$	$-20°$	$-10°$	$0°$	$+10°$	$+20°$	$+30°$	$+31°$	$+31.35°$
$c =$	0.437	0.473	0.529	0.621	0.833	2.505	5.269	∞

Finally we can calculate for carbonic acid vapor the specific heat c_v at constant volume.

From equation (76*a*) there is determined

$$\left(\frac{d^2p}{dt^2}\right) = -\frac{C}{T_k^2(v+\beta)^2}e^{1-\frac{T}{T_k}},$$

and from this follows with the help of equation (85)

$$c_v = c_v' + \frac{ACT}{T_k^2(v+\beta)}e^{1-\frac{T}{T_k}}, \quad \ldots\ldots \quad (79)$$

in accordance with which c_v can be calculated for different values of t and v.

For example, with the above given constants C, α, and T_k, and with $c_v' = 0.16577$, we get for

$t =$	0°	+10°	+20°	+30°	+31.35°
$c_v =$	0.1791	0.1842	0.1908	0.2052	0.2118
[$t =$	32°	50°	68°	86°	88.43].

These values of c_v relate to dry saturated vapor and of course cannot be regarded as very certain, for they depend essentially on the form of the temperature function assumed in the second term of the right member of the equation of condition (see § 28, p. 208) and which was here taken, according to van der Waals, as $1-\frac{T}{T_k}$. If we pass over into the superheated region we may perhaps, at the entrance to this region, which is only considered in technical calculations, use the constant mean value $c_v = 0.182$ for the specific heat c_v, at constant volume, of superheated carbonic acid vapor, as was proposed by Mollier.

The following is to be noted.

I have added to Mollier's Table 10 a Col. 17, containing the values of the steam heat J (see p. 28) according to the formula

$$J = q + \rho.$$

These values represent the magnitude AU, where U is the inner

work or energy of dry saturated carbonic acid vapor. These tabular values vary greatly only in the vicinity of the critical temperature.

In a later article[1] Mollier subjected to closer investigation the behavior of carbonic acid in the superheated field, bearing in mind the theoretical requirements involved in calculating and judging carbonic acid machines of the cold-vapor variety; in so doing he made use of the comprehensive experimental results of Amagat which give, in tabular form, the specific volumes v of the superheated vapor for widely differing pressures and temperatures.

In preparation for the following consideration of cold-vapor engines we will now give the formulas for the energy and entropy.

If we start from a particular point of the limit curve, corresponding to a certain steam volume, and designate its energy by $q'+\rho'$, and its entropy by $\tau'+\frac{r'}{T'}$, and the temperature by t', then for heating under constant volume the energy at the temperature t will have the value

$$AU=q'+\rho'+c_v(t-t'), \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

and the entropy will be

$$AP=\tau'+\frac{r'}{T'}+c_v \log_e \frac{T}{T'}, \quad . \quad . \quad . \quad . \quad . \quad (81)$$

which values can therefore be computed for a particular v and for different temperatures. Similar calculations can be made for every other volume v, and now it is only a question of representing the values obtained as functions of v and t in order to directly determine the magnitudes AU and AP.

When the volumes are greater than 0.00167 Mollier gives for the energy

---

[1] Mollier, "Über die kalorischen Eigenschaften der Kohlensäure ausserhalb des Sättigungsgebietes." Zeitschrift für die gesamte Kälte-Industrie, III. year, 1896, pp. 65 and 90.

$$AU = 56.6 - \frac{0.1}{v + 0.001} + 0.182\,t \quad . \quad . \quad . \quad . \quad . \quad . \quad (82a)$$

$$\left[AU = 101.88 - \frac{2.88}{v + 0.016} + 0.182(t - 32)\right],$$

for $v < 0.00167$

$$AU = 56.6 - \frac{0.0625}{v} + 0.182\,t \quad . \quad . \quad . \quad . \quad . \quad . \quad (82b)$$

$$\left[AU = 101.88 - \frac{1.8}{v} + 0.182(t - 32^{\circ})\right],$$

and for the entropy

$$AP = 0.128 \log_{10}(v - 0.00085) + 0.42 \log_{10} T - 0.562 \quad (83a)$$
$$[AP = 0.128 \log_{10}(v - 0.013616) + 0.42 \log_{10} T - 0.8234].$$

When we make $m = 0.3048$ and $v_0 = 0.00085$, the last equation may be written

$$AP = 0.42 \log_{10} T(v - v_0)^m - 0.562 \quad . \quad . \quad . \quad . \quad (83b)$$
$$[AP = 0.42 \log_{10} T(v - v_0)^m - 1.1844].$$

Since the entropy is constant for the adiabatic curve, it follows that the equation of the adiabatic of superheated carbonic acid vapor is

$$T(v - v_0)^m = \text{Constant.} \quad . \quad . \quad . \quad . \quad . \quad (84a)$$

If we also make AU constant in equations (82*a*) and (82*b*), we will get, approximately, the course of the isodynamic curve.

If we start from any point of the limit curve for which the temperature is T' and for which the specific volume s is designated by v', we get from equation (84*a*)

$$\frac{T}{T'} = \left(\frac{v' - v_0}{v - v_0}\right)^m. \quad . \quad . \quad . \quad . \quad . \quad . \quad (84b)$$

Accordingly if we compress dry saturated carbonic acid vapor

adiabatically, then for any value v whatever there can be computed the temperature T at the end of the compression, and with the help of the equation of condition (67), p. 263, there can also be found the terminal pressure p. There will be opportunity later on to return to the use of these formulas.

APPLICATIONS.

Reversible and Non-reversible Changes of Superheated Vapor of Water.

§ 35. THE ISOTHERMAL, ISODYNAMIC, AND ADIABATIC CURVE FOR SUPERHEATED STEAM.

The equation of condition which we have found in an approximate form at once gives the course of the isothermal curve. Let p_1, v_1, and T_1 be pressure, volume, and temperature for the initial condition, and let p, v, and T be these quantities for the final condition, then the two equations

$$pv = BT - Cp^n \quad \text{and} \quad p_1v_1 = BT_1 - Cp_1^n$$

at once give for $T = T_1$, by subtraction, the equation of the isothermals:

$$pv = p_1v_1 + C(p_1^n - p^n), \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where C must be in accordance with the data on p. 237, and we must substitute

$$n = \frac{\kappa - 1}{\kappa} = \frac{1}{4} \quad \text{and} \quad \kappa = \frac{4}{3}.$$

The heat quantity Q which must be supplied to the unit of weight of steam while it is passing under constant temperature T_1 from the pressure p_1 to the pressure p is found from the third of equations (50), p. 244:

$$Q = c_p \frac{\kappa - 1}{\kappa} T_1 \log_e \frac{p_1}{p}, \quad \ldots \ldots \quad (2)$$

and according to equation (56), p. 249, there is determined the increment of steam heat,

$$J - J_1 = \frac{A}{\kappa - 1}(pv - p_1 v_1), \quad \ldots \ldots \quad (3)$$

or, from the preceding equation (1),

$$J - J_1 = \frac{AC}{\kappa - 1}(p_1^n - p^n), \quad \ldots \ldots \quad (3a)$$

and then the external work L is found from the relation

$$Q = J - J_1 + AL. \quad \ldots \ldots \quad (4)$$

Accordingly with superheated steam a portion of the supplied heat is consumed in performing inner work; while with gases, for which $C = 0$, the whole supplied heat Q is converted into external work.

This result corresponds exactly to the atomistic conception that has been formed of vapors; during their isothermal expansion there must be overcome the forces of attraction with which the particles act on each other; while in the case of gases the particles have passed beyond the sphere of the mutual interactions of the forces.

Example. Let steam expand isothermally from a pressure of five atmospheres down to one atmosphere and let it be dry saturated at the beginning, then we must place $T_1 = 273 + 152.22°$ [765.396° F.] and must also substitute

$$n = \frac{\kappa - 1}{\kappa} = \frac{1}{4}, \quad c_p = 0.4805.$$

Then, using the above-given constants, we get, according to the given equation of condition for the initial volume, $v_1 = 0.3630$ [5.8149], and the final volume $v = 1.9084$ [30.5705].

The heat to be imparted according to equation (2) is

$$Q = 82.204 \text{ Cal. } [147.9672 \text{ B.t.u.}],$$

the change in the steam heat is

$$J - J_1 = 6.802 \text{ Cal. } [12.2436 \text{ B.t.u.}],$$

and the external work, measured in units of heat, is found to be

$$AL = 75.402 \text{ Cal. } [135.7236 \text{ B.t.u.}].$$

The last two values give for the work consumed in the interior $U - U_1 = 2884$ mkg. [20860 ft-lb.], and for the external work $L = 31970$ mkg. [231245 ft-lb.].

If we think of isothermal compression with superheated steam, then the state of saturation is reached at a certain pressure, i.e., the isothermal curve intersects the limit curve; with further compression under constant pressure condensation of the steam will take place. The pressure corresponding to the point of intersection is of course identical with the saturation pressure belonging to the temperature T_1.

With the isodynamic curve the inner work or energy U is constant; consequently with vapors the steam heat J will also be constant. Accordingly, with $J = J_1$, equation (3) becomes the equation of the isodynamics

$$pv = p_1 v_1, \quad \ldots \ldots \quad (5)$$

and hence the heat to be supplied is

$$Q = AL = Ap_1 v_1 \log_e \frac{p_1}{p}. \quad \ldots \ldots \quad (6)$$

Hence the curve is an equilateral hyperbola, as in gases; but it does not here coincide with the isothermal, the pressures sinking more rapidly during expansion along the isodynamic than along the isothermal. Moreover, there follows, from the equation of condition, because $pv = p_1 v_1$, the change of temperature accompanying the isodynamic change of state, and it is

$$T_1 - T = \frac{C}{B}(p_1^n - p^n). \quad \ldots \ldots \quad (7)$$

Example. Dry, saturated steam of $p_1 = 5$ atmospheres pressure expands isodynamically, for example by flowing into a vacuum chamber

till the terminal pressure amounts to $p=1$ atmosphere. Here, according to equation (5), the final volume at the end of the expansion is five times that of the initial volume, and the fall of temperature can be computed, according to equation (7), with the help of the table on p. 238, and is found to be

$$t_1-t=18.88^\circ\ [33.984^\circ\ \text{F}].$$

When the initial temperature amounts to $t_1=152.22^\circ$ [305.996° F.], the temperature at the end is $t=133.34^\circ$ [272.012°]. The steam is now superheated because its temperature at atmospheric pressure amounts to more than 100° [212°]; the superheating is accordingly 33.34° [60.012°].

The adiabatic curve of superheated steam was found on p. 231 to have a course agreeing with that of gases, only we must substitute $\kappa=1.333$ in place of $\kappa=1.410$.

Here it is evident from the first of equations (50), p. 244, that when $dQ=0$

$$pv^\kappa=p_1v_1^\kappa, \quad \ldots\ldots\ldots \quad (8)$$

and from this we find the work L, produced during adiabatic expansion, to be

$$L=\frac{1}{\kappa-1}(p_1v_1-pv). \quad \ldots\ldots\ldots \quad (9)$$

Example. Superheated steam of $p_1=5$ atmospheres pressure and of the temperature $t_1=300^\circ$ C. [572° F.] expands adiabatically down to the pressure of $p=1$ atmosphere. Here equation (8) gives

$$v=3.344\,v_1,$$

and from equation

$$\frac{T}{T_1}=\left(\frac{p}{p_1}\right)^{\frac{\kappa-1}{\kappa}}$$

there follows

$$T=384.2^\circ \quad \text{or} \quad t=110.2^\circ\ \text{C.}$$
$$[T=691.56^\circ \quad \text{or} \quad t=230.36^\circ\ \text{F.}].$$

The steam is still superheated at the end of the expansion and the superheat amounts to 10.2° C. [18.36° F.]. The work produced per unit of weight is found from equation (9) to be

$$L=26117\ \text{mkg.}\ [85687\ \text{ft-lb.}],$$

because, according to the equation of condition, it is found that $v_1=0.5087$ [8.1488] and $v=1.7008$ [27.2450].

If we compare the course of the adiabatic curve with that of the limit curve, it is seen that, approximately, both obey the same law.

For the limit curve there was found with great accuracy the relation

$$pv^{\nu}=D, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where $\nu=1.0646$ and $D=1.7049$ [32.670] when the pressure p was in atmospheres (each 10333 kg. [2116.3 lb.]).

Now because $\kappa=1.333$ is greater than ν, it follows that the adiabatic curve during expansion approaches the axis of abscissas more rapidly than the limit curve; both curves will therefore intersect at a particular point.

In Fig. 27 let *dd* represent the limit curve and let the state of the superheated steam be given by the point T_1 possessing the coordinates v_1 and p_1, then for the adiabatic to the point of intersection T_0 we have the equation

$$p_1 v_1^{\kappa}=p_0 v_0^{\kappa},$$

where p_0 and v_0 represent the coordinates of the point of intersection.

On the other hand, for the limit curve,

$$p_0 v_0^{\nu}=D,$$

and from the two equations we can now calculate the coordinates p_0 and v_0 of the point of intersection T_0.

Let us suppose that from the given values p_1 and v_1 the value E is determined from the relation

$$p_1 v_1^{\kappa}=E, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

then we find

$$\log v_0 = -0.8622906+3.7216 \log E, \quad . \quad . \quad . \quad (12a)$$

$$\log p_0 = +1.1496935-3.9620 \log E, \quad . \quad . \quad . \quad (12b)$$

and

$$\log (p_0 v_0) = +0.2874029-0.2404 \log E, \quad . \quad . \quad (12c)$$

where the logarithm is the B r i g g s logarithm and both the pressures p_1 and p_0 are taken in atmospheres (at 10333 kg. [2116.31 lb.]).

If in adiabatic expansion there are given the initial values p_1 and v_1, and also the final values p and v, then in the first place we calculate p_0 and v_0.

If $v < v_0$ or $p > p_0$, then, for the final condition, we have the relation $pv^\kappa = p_1 v_1^\kappa$, and the work of expansion is found from equation (9) to be

$$L = \frac{1}{\kappa - 1}(p_1 v_1 - pv),$$

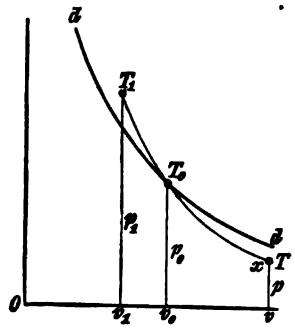

FIG. 27.

where p and p_1 are to be estimated in kg. per sq. m. [lb. per sq. ft.].

On the contrary, if we are to have $v > v_0$ or $p < p_0$, then this is a sign that the adiabatic is to cut through the limit curve, and its course T_0T within the limit curve will be different (Fig. 27), for from the point of intersection on it is subject to the law

$$pv^\mu = p_0 v_0^\mu, \quad \ldots\ldots\ldots \quad (13)$$

where we must substitute $\mu = 1.135$ (see p. 82). According to the latter equation p can be computed from the given value v or vice versa.

The whole work of expansion can be found from the equation

$$L = \frac{1}{\kappa - 1}(p_1 v_1 - p_0 v_0) + \frac{1}{\mu - 1}(p_0 v_0 - pv). \quad \ldots \quad (14)$$

In the second part of the expansion, moreover, a condensation of steam takes place, and the steam quality x at the end of expansion is found from the formula

$$v = xu + \sigma.$$

If the expansion in the cylinder of a steam engine working with superheated steam is regarded as adiabatic, then the case just treated will be the ordinary one.

Example. Let superheated steam of $p_1 = 5$ atmospheres pressure and of the temperature $t_1 = 200°$ [392°] expand adiabatically down to $p = 1$ atmosphere. In the first place we find from the equation of condition and with the help of the constants given on p. 237:

$$p_1 v_1 = BT_1 - Cp_1^n = 2.05043$$
$$[p_1 v_1 = BT_1 - Cp_1^n = 32.8456].$$

From this follows the initial volume

$$v_1 = 0.41009$$
$$[v_1 = 6.5692],$$

and then, according to equation (11),

$$\log E = 0.1828089, \quad E = 1.5233$$
$$[\log E = 1.7889865, \quad E = 61.5158].$$

Accordingly we have for the point of intersection, from equations (12),

$$v_0 = 0.6577, \quad p_0 = 2.6632 \text{ atm.}, \quad p_0 v_0 = 1.7517$$
$$[v_0 = 10.5356, \quad p_0 = 2.6632 \text{ atm.}, \quad p_0 v_0 = 28.0585].$$

From equation (13) we now get the final volume v for one atmosphere pressure:

$$v = 1.5589$$
$$[v = 24.972],$$

and because $u = 1.6494$ [26.3912] we have for the steam quality at the end of expansion

$$x = 0.9445,$$

and for the expansion ratio

$$\frac{v}{v_1} = 3.801.$$

Finally the work of expansion per unit of weight is, according to equation (14),

$$L = 24016 \text{ mkg.}$$
$$[L = 78793 \text{ ft-lb.}].$$

But if the steam is dry saturated at the beginning, then, according to the exact formulas, (66), p. 75, and (68), p. 76, the steam quality at the end is $x = 0.9090$ and the initial and final volumes are respectively $v_1 = 0.3636$ [5.82450] and $v = 1.5003$ [24.0332], and the expansion ratio is $\frac{v}{v_1} = 4.126$, and the work of expansion

$$L = 24210 \text{ mkg.}$$
$$[L = 79430].$$

For the same ratio of expansion as before, namely for $\frac{v}{v_1} = 3.801$, we cal-

culate the final tension from the equation $pv^{\mu}=p_1v_1^{\mu}$ and find it to be 1.098 atmospheres, and according to the equation

$$L=\frac{p_1v_1}{\mu-1}\left[1-\left(\frac{v_1}{v}\right)^{\mu-1}\right]$$

the work of expansion is $L=22952$ mkg. [166014 ft-lb.] when we substitute $\mu=1.135$.

From this we must conclude that with the same initial pressure and the same ratio of expansion the superheated steam develops a somewhat greater work during adiabatic expansion than saturated steam.

§ 36. GENERATION OF SUPERHEATED STEAM UNDER CONSTANT PRESSURE.

In steam engines which work with superheated steam the production of the steam occurs in two ways. In the first method the steam generated in the boiler is either wet or dry saturated, and on the way from the boiler to the steam cylinder is led through a superheating apparatus, where, under invariable pressure, it experiences a further increase of temperature and increment of volume on account of the heat imparted to it, ordinarily by the waste fire gases; in the second, mixed vapors

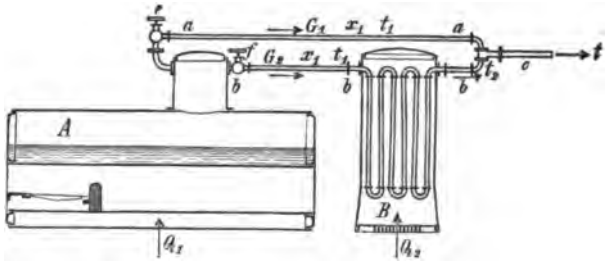

FIG. 28.

are employed. The steam is led from the boiler A (Fig. 28) to the steam cylinder through two pipes *aa* and *bb*, which unite into one pipe *c* in front of the cylinder.

Through one branch pipe *aa* the saturated or wet steam moves without change of state, but the other pipe leads to the superheating apparatus B, where the steam is strongly superheated, and then

flows to the place of the union of the two pipes, and there the mixture of the two steam jets now takes place. This steam, which is technically designated as "mixed steam," is of course nothing but simple superheated steam, whose temperature lies between the temperatures of the two steam jets before mixture and is dependent on the distribution of the total steam weight in the two branch pipes. By a suitable adjustment of the two valves *e* and *f* we have complete control of the temperature of mixture, i.e., of the steam entering the steam cylinder, and can maintain it at a particular prescribed height, and this constitutes the main advantage of the mixing method in contradistinction to the direct generation of superheated steam in which the total steam mass is led through the superheater; in the latter case the temperature of the superheating is dependent on the intensity of the firing, it is difficult to regulate, and it may easily become so high that it will decompose the lubricants and thus introduce disturbance into the running of the steam engine.

A further advantage of the given method lies in the ability to put the superheating apparatus entirely out of action by the closing of the valve *f* and then running the engine directly by the boiler steam. The employment of superheated steam for the running of engines has been advantageous, because a number of experiments show that with like or equal performance of an engine a very considerable saving in fuel has been effected. There exist very complete and early experiments by Hirn[1]; later it was maintained by others, and particularly by the American Wethered (and these views were also based on experiments), that a further saving of fuel was effected when the superheated steam was generated by the method of mixture; according to the experiments in question a decided advantage in favor of mixed vapors was manifest in one and the same engine even in those cases in which the mixed steam reached the cylinder of the engine with the same pressure and the same temperature as the directly generated superheated steam. The latter result was surprising even at that time and led to lively discussions among English engineers.

[1] Bulletin de la Société industrielle de Mulhouse, 1857.

The use of superheated steam for engine running was already widespread in 1854 in the United States of America, and it was the experimental results of W e t h e r e d in particular which caused the matter to receive much attention.[1] Great success was expected at the beginning from the use of superheated steam in steam engines, and indeed these have not been absent from the "hot-vapor motors"; a part of the success is to be ascribed to the fact that in time an even higher boiler pressure and expansion were employed; the fuller considerations of the occurrences taking place during admission to the engine cylinder show that even moderate superheating entails advantages.

It is therefore appropriate to establish the heat economy of superheating in a theoretical way and at the same time show whether W e t h e r e d was right in concluding that, other things being equal, the generation of superheated steam by mixture involved an additional advantage.[2]

Let us immediately take up the case of mixture and in so doing take as a base Fig. 28, which gives a schematic representation of the steam boiler with the superheater; here, in correspondence with practical arrangements, it may be assumed that the superheater B is heated by the products of combustion coming from the steam boiler A, or it may be assumed that the superheater is provided with a special heating arrangement.

Now let p_1 and t_1 be the pressure and temperature and let the steam leaving the boiler be wet and possess the steam quality x_1; the boiler is fed with water at the temperature t_0.

Let G_1 represent the weight of steam and water which in a unit of time leaves the boiler and flows directly through the pipe aa to the place of discharge, and let G_2 be the weight of the steam simultaneously flowing through the superheater.

[1] Compare D i n s e, "Über die Verwendung des überhitzen Dampfes in den Dampfmaschinen." Zeitschrift des Vereins deutscher Ingenieure, Vol. IX, 1865, pp. 573 and 665; also Vol. X, 1866, pp. 245 and 483. The article gives an historic review and contains the experimental results thus far attained, of course without any theoretical explanation whatsoever; at the same time it contains pictures of different arrangements of superheated apparatus.

[2] The first theoretical treatment of the question came from the author: "Über das Verhalten der überhitzen und der gemischten Wasserdämpfe." Zivilingenieur, Vol. XIII, 1867, p. 343.

We can now determine the heat quantity Q' which must be supplied to the boiler in order to generate the G_1 kg. [lb.] mixture of steam and water, and, because here the evaporation takes place under constant pressure, we find that

$$Q' = G_1(q_1 - q_0 + x_1 r_1),$$

or substituting $r_1 = \rho_1 + Ap_1u_1$ and inserting the value u_1 in place of the specific volume v_1 of dry saturated steam of the pressure p_1, we obtain

$$Q' = G_1(q_1 + \rho_1 + Ap_1v_1 - q_0 - (1 - x_1)r_1),$$

or, because $q_1 + \rho_1$ represents nothing but the steam heat J of dry saturated steam for which we can insert equation (56), p. 249, we find

$$Q' = G_1\left(J_0 + \frac{A\kappa}{\kappa - 1}p_1v_1 - q_0 - (1 - x_1)r_1\right). \quad . \quad . \quad . \quad (15)$$

The other part, G_2, of the total steam goes through the superheater and is here brought to the temperature t_2 with the corresponding volume v_2; the heat quantity Q'' which must be supplied in part to the boiler, in part to the superheater can be determined with the help of equation (57), p. 249, and is

$$Q'' = G_2\left(J_0 + \frac{A\kappa}{\kappa - 1}p_1v_2 - q_0\right). \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Let t be the temperature of the superheat after the mixture; the total steam weight $(G_1 + G_2)$ therefore enters the steam cylinder with the specific volume v; now disregarding any throttling and consequently assuming the pressure in the cylinder identical with the boiler pressure p_1, the steam will perform in the cylinder the admission work

$$L = (G_1 + G_2)p_1v. \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Of the total heat supply there finally remains behind in the steam the amount

$$Q' + Q'' - AL,$$

and this value is nothing else than the steam heat diminished by the heat quantity already possessed by the feed-water; it is therefore

$$(G_1+G_2)\left(J_0+\frac{Ap_1v}{\kappa-1}-q_0\right)=Q'+Q''-AL.$$

If we use here equations (15), (16), and (17), we get, after simple reduction,

$$(G_1+G_2)v=G_1v_1+G_2v_2-G_1(1-x_1)\frac{\kappa-1}{\kappa}\frac{r_1}{Ap_1}, \quad . \quad (18)$$

and this equation constitutes the first main result of the present investigation; it can be employed to determine the specific volume of the superheated steam in the cylinder which results from the mixture when the mixture ratio $G_1:G_2$ is given and when the pressure p_1, temperature t_1, and the steam quality x_1 of the boiler steam is known and when there is also given the temperature t_2 with which the steam leaves the superheating apparatus; for then there is given the specific volume v_2 of the superheated steam as well as v_1 of the dry saturated steam.

The value $(G_1+G_2)v$ of the left member of equation (18) moreover represents the total volume of the mixed steam at the end of the occurrence; the volume of the steam coming from the superheater is G_2v_2 b e f o r e mixture, and that of the wet steam is $G_1(x_1u_1+\sigma)$, or $G_1x_1v_1$, or we may neglect σ and replace u_1 by v_1.

Hence the total volume of the steam b e f o r e mixture is

$$G_1x_1v_1+G_2v_2,$$

and therefore the change of volume ΔV of the mixture under constant pressure is

$$\Delta V=(G_1+G_2)v-(G_1x_1v_1+G_2v_2),$$

or, utilizing equation (18),

$$\Delta V=G_1v_1(1-x_1)\left(1-\frac{\kappa-1}{\kappa}\frac{r_1}{Ap_1v_1}\right). \quad . \quad . \quad . \quad (19)$$

This expression for the vapor of water always leads to a negative value whenever a part of the mixing steam is wet, i.e., whenever $x_1 < 1$.

Consequently, if superheated steam is mixed under constant pressure with wet steam of the same pressure, a diminution of the total volume will take place.

On the other hand if the saturated steam is dry or if both steam jets are superheated but of different temperatures, then no changes of volume occur during the mixing under constant pressure.

But it is more important to determine the changes of total volume than to determine the changes of temperature connected with the mixing.

If we substitute, in the equation of condition for superheated steam,

$$pv = BT - Cp^n,$$

the pressure p_1 and the temperature T of the mixture, we obtain the specific volume v of the mixed steam; on the other hand if we substitute the absolute temperature T_2 with which the steam leaves the superheating apparatus and then later on the temperature T_1 of the saturated steam, we shall find the two values v_1 and v_2; utilizing these in equation (18) and considering the relation

$$AB = \frac{\kappa - 1}{\kappa} c_p,$$

we get

$$(G_1 + G_2)T = G_1T_1 + G_2T_2 - G_1(1 - x_1)\frac{r_1}{c_p},$$

or, if we replace the absolute temperature by the centigrade [Fahrenheit] reading we obtain

$$(G_1 + G_2)t = G_1t_1 + G_2t_2 - G_1(1 - x_1)\frac{r_1}{c_p}. \quad . \quad . \quad . \quad (20)$$

With the help of this equation we can easily calculate the temperature t of the mixture when there is known the weight G_1

of the saturated steam of the steam quality x_1, and also the weight G_2 which passes through the superheater to the place of mixing.

If the directly conducted steam is dry saturated or likewise superheated we have $x_1 = 1$, and therefore, according to equation (20), the mixing temperature becomes

$$t = \frac{G_1 t_1 + G_2 t_2}{G_1 + G_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (20a)$$

We therefore very correctly obtain the formula which is used in Physics when there is to be determined the temperature of the mixture of two quantities of fluid of the same kind but of different temperatures, provided the mixture takes place under constant pressure.

If we designate the total generated steam weight $G_1 + G_2$ by G and substitute $G_2 = G - G_1$ in equation (20) we get

$$\frac{G_1}{G} = \frac{t_2 - t}{t_2 - t_1 + \frac{r_1}{c_p}(1 - x_1)} \quad \text{and} \quad \frac{G_2}{G} = 1 - \frac{G_1}{G}. \quad . \quad . \quad (21)$$

Accordingly we can compute how the total steam must be distributed in the two branch pipes *aa* and *bb* (Fig. 28) in order to obtain a prescribed temperature of the mixture.

Example. A steam engine works with mixed vapors of $p_1 = 5$ atmospheres pressure; the temperature in the boiler is therefore $t_1 = 152.22°$ [305.996°]; let the part of the steam passing through the superheater be superheated to $t_2 = 250°$ [482°], and suppose that the mixed steam when it enters the steam cylinder has the temperature $t = 180°$ [356°].

Here $c_p = 0.4805$ and, according to Table 11 of the Appendix,

$$r_1 = \rho_1 + A p_1 u_1 = 499.186 \text{ [898.535]}.$$

First let us assume that the steam is dry, then that it contains 10 per cent of water, and finally that it contains 20 per cent of water, then we calculate from equation (21) for

$x_1 = 1$	0.90	0.80
$\frac{G_1}{G} = 0.7159,$	0.3471,	0.2291 (directly)
$\frac{G_2}{G} = 0.2841,$	0.6529,	0.7709 (led through the superheater).

We see from these numerical values how great an influence the water mixed with the boiler steam exerts on the results of the mixing, and we see that with particular prescribed temperatures the distribution of the total steam mass in the two branches of the steam pipe may be very different for boiler steam of different qualities.

By suitable adjustment of the proper valves e and f (Fig. 28) in the two branches of the steam pipe we can easily arrange practically for the correct distribution of the steam. From the equation of condition we can compute the specific volume v of the mixed steam; for $p_1=5$ atmospheres of pressure and $t=180°$ [356°] we find $v=0.3904$ cbm. [6.2538 cu. ft.], and hence the work of admission in the steam cylinder for 1 kg. [lb.] of steam is

$$L=p_1v=20170.4 \text{ mkg. [145895 ft-lb.]}.$$

Equation (19) would also enable us to easily find the change of volume due to the mixing in the three cases before us.

Now the determination of the heat quantities which must be supplied to the steam boiler and to the superheater for a prescribed distribution of the steam is also of importance. In the first place suppose the steam boiler is to generate under constant pressure the steam weight Gx_1 from G kg. [lb.] of feed water of the temperature t_0. Then the heat quantity Q_1 which must be introduced into the boiler is

$$Q_1=G(q_1-q_0+x_1r_1), \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and this heat quantity is the same however the steam weight may be distributed in the two branches of the steam pipe.

According to the notation just used G_2 kg. [lb.] of wet steam is led to the superheater; here, in the first place, there must be evaporated at constant pressure p the admixed water $G_2(1-x_1)$ kg. [lb.]; this requires the heat quantity $G_2(1-x_1)r_1$; then the steam weight G_2 must be superheated under constant pressure from the temperature t_1 to the temperature t_2 and this requires the heat quantity $c_pG_2(t_2-t_1)$. Consequently the whole heat quantity Q_2 which must be supplied to the superheater is

$$Q_2=G_2[(1-x_1)r_1+c_p(t_2-t_1)]. \quad . \quad . \quad . \quad . \quad (23)$$

Example. As in the preceding example, suppose the steam pressure to be 5 atmospheres and the temperature of the mixture to be $t=180°$

[356°]; suppose the temperature of the steam coming from the superheater again to be $t_2 = 250°$ [482°] and let the steam quality be $x_1 = 0.90$, in which case the weight of the mass passing through the superheater amounts to $G_2 = 0.6529\ G$. Now if the boiler is fed with water of temperature $t = 40°$ [104°] (condensing engine), according to Table 1 of the Appendix we may substitute $q_0 = 40.051$ [72.092], $q_1 = 153.741$ [276.734], and $r_1 = 499.186$ [898.535]; we then get from equation (22) the heat quantity which must be supplied to the boiler:

$$Q_1 = 562.99\ G \text{ Cal. } [1013.391 \text{ B.t.u.}],$$

and from equation (23) the heat quantity from the superheater:

$$Q_2 = 63.27\ G \text{ Cal. } [113.886 \text{ B.t.u.}].$$

If we substitute $G_1 + G_2 = G$ in equation (20) we obtain

$$G_2[(1-x_1)r_1 + c_p(t_2 - t_1)] = G[(1-x_1)r_1 + c_p(t - t_1)].$$

Here comparison with equation (23) shows that the left member is identical with the value of Q_2; consequently there follows for the heat quantity which must be supplied to the superheater

$$Q_2 = G[(1-x_1)r_1 + c_p(t - t_1)], \quad . \quad . \quad . \quad . \quad . \quad (24)$$

in which equation the mixture temperature t takes the place of the temperature t_2 and the total steam weight G takes the place of the weight of G_2. The expression, therefore, gives at the same time the heat quantity which must be supplied to the superheater when the total steam quantity is passed through it and is there brought to the mixture temperature t.

From this follows that with respect to the heat quantities required it is perfectly immaterial whether the superheated steam for an engine is generated directly or by mixture.

These results appear to contradict the results of Wethered's experiments mentioned above; Wethered found in all experiments that, with respect to fuel consumption, the use of mixed steam was to be preferred to the directly generated steam. But the matter can be easily explained. Our investigations show

that superheated steam at a particular temperature and particular pressure, when produced under constant pressure, requires the same quantity of heat whether it has been generated directly or by mixture; we may remark incidentally that it would contradict the mechanical principles of heat if the contrary were maintained. Accordingly we must assume that in Wethered's experiments the same quantity of heat was supplied to the superheating apparatus whether the whole steam mass coming from the boiler or only a part of it was led through the apparatus, provided that in both cases the steam entered the cylinder at the same pressure and was superheated to the same temperature.

But the question as to the utilization of the heat contained in the fire gases is very different in the two cases. For one and the same superheating apparatus, i.e., for the same heating surface, the heat transmission from the fire gases evidently depends on the steam weight passing through the apparatus and on the degree of the superheating to be there achieved. In the example just calcuated the whole steam weight G was directly generated by passing through the superheater and was there superheated from 152.22° to 180° [305.996° to 356°]; in the case of mixture only 0.6529 G kg. [lb.] of steam was led into the superheating apparatus, and there it was superheated to 250° [482°]; the required heat quantities were the same in the two cases, but the changes of temperature experienced by the fire gases are evidently different and therefore the degree of heating and the air supplied to the grate for the mass, and consequently the weight of fuel, would be different in the two cases. Wethered's experiments therefore by no means furnish a proof that mixed vapors are to be preferred to directly generated superheated vapors; it is rather a purely accidental matter that in the steam engines with which Wethered experimented the heating surface of the superheater was of a size which furnished results in favor of mixture. With another heating surface the results might have been in favor of the direct generation of the superheated vapors. What has been said also explains the fact that the experiments with different engines gave such surprisingly different results.

For practical purposes the use of mixed vapors is indeed to be recommended, but not for the reasons which have been derived from Wethered's experiments, but simply and solely because in the adjustment of the valves in the two branches of the steam pipes we have such a suitable and simple means of regulating the temperature of the superheated steam entering the cylinder, and for maintaining it at a constant height.

To these investigations we will now add the proof that the use of superheated steam in engines is advantageous with respect to the work developed.

By the addition of the two equations (22) and (24) we get the total heat quantity Q which must be supplied to the steam boiler and to the superheater in order to generate superheated steam of the temperature t, no matter whether the superheated steam is generated directly or by mixture, and we find that

$$Q=G[q_1-q_0+r_1+c_p(t-t_1)].$$

The work which is produced during admission into the cylinder under constant pressure p_1 is $L=Gp_1v$, or expressed in units of heat is

$$AL=GAp_1v,$$

and therefore we get by division of the equations

$$\frac{Q}{AL}=\frac{q_1-q_0+r_1+c_p(t-t_1)}{Ap_1v}. \quad \ldots\ldots \quad (25)$$

On the other hand if the engine works with wet boiler steam, then the heat supplied to the boiler must be

$$Q=G(q_1-q_0+r_1x_1),$$

and the admission work measured in units of heat is

$$AL=GAp_1(x_1u_1+\sigma),$$

and the ratio of the two

$$\frac{Q}{AL}=\frac{q_1-q_0+r_1x_1}{Ap_1(x_1u_1+\sigma)}. \quad \ldots\ldots \quad (26)$$

For the same boiler pressure and for the same feed water temperature t_0 these values are different; the smaller this ratio the more favorable is the utilization of heat under consideration.

Example. Suppose a steam engine to work with steam of $p_1 = 5$ atmospheres and the boiler to be fed with water of the temperature $t_0 = 40°$ [104°]. Utilizing the corresponding values of Table 1*b* and 11 of the Appendix, we get

$$q_0 = 40.051, \quad q_1 = 153.741, \quad r_1 = \rho_1 + Ap_1u_1 = 499.186$$
$$[q_0 = 72.092, \quad q_1 = 276.734, \quad r_1 = \rho_1 + Ap_1u_1 = 898.535].$$

If we use the given equation of condition of steam for the calculation of p_1v the preceding equations (26) and (25) give the following numerical results, if we successively assume for wet steam $x_1 = 0.80$, 0.90, and 1, and for superheated steam insert $t = 180°$ [356°] and 200° [392°]:

	Saturated Steam.			Superheated Steam.	
$x =$	0.80	0.90	1	$t =$ 180°	200°
$\frac{Q}{AL} =$	14.424	14.077	13.795	13.165	12.724

We see from this that for the same admission work the saving of heat is greater the dryer the saturated vapor and also the more the steam is superheated. If in addition we take into account the work of expansion we get a further saving with an equal expansion ratio; of course we here expressly consider the heat quantity which actually enters the boiler and the superheating apparatus; if it were a question of determining the actual saving of fuel, it would be necessary also to consider the conditions of the heating plant and particularly the temperature with which the fire gases leave.

§ 37. THE FLOW OF STEAM FROM ONE VESSEL TO ANOTHER AND THE THROTTLING OF THE STEAM.

The investigation of the flow of steam from one vessel into another is only of technical interest under the special hypothesis that the pressure in each of the two spaces is kept at constant height. Let us suppose the cylinder A (Fig. 29) to be con-

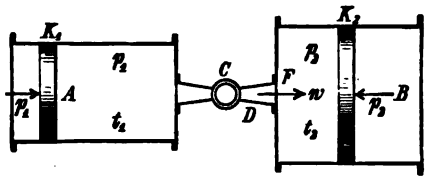

FIG. 29.

nected by a pipe D with a second cylinder B, both cylinders to be provided with moving pistons and the pipe to possess a valve C (throttle valve); let the steam flow from A to B; then by a suitable pushing forward of the piston K_1 the pressure p_1 of the steam cylinder A is kept at constant height, while in a similar manner the piston K_2 in cylinder B is pushed back with a constant pressure p_2. By a suitable load p_1 and p_2 on both pistons (reduced to the unit of area) and by an appropriate setting of the throttle valve C we can conceive a desired difference of pressure to be effected; assuming a comparatively slow motion of both pistons the steam will flow through the connecting pipe with a certain velocity, will then spread itself in the cylinder B, completely lose its velocity and pass again into the condition of equilibrium.

If we follow the unit of weight of steam during its transfer from A to B we will see that piston K_1 in cylinder A traverses a certain space under the constant pressure p_1; in like manner piston K_2 will progress in cylinder B, overcoming the constant pressure p_2. If the specific volume at the beginning is v_1, and is v_2 at the end, then the unit of weight of mass has received in the first cylinder the work p_1v_1 and the other has given off the

work p_2v_2, and consequently during the transfer there has been generated the heat quantity Ap_1v_1 and the heat quantity Ap_2v_2 has disappeared.

Furthermore if J_1 is the steam heat at the beginning (of the unit of weight) in cylinder A, and J_2 that at the end in cylinder B, we evidently have the relation

$$J_1 + Ap_1v_1 - Ap_2v_2 = J_2,$$

or

$$J_1 + Ap_1v_1 = J_2 + Ap_2v_2, \quad \ldots\ldots \quad (27)$$

and this equation is valid whatever the constitution of the steam at the beginning and end may be, provided heat is neither supplied to, nor withdrawn from, the mass during the transfer. Now we assume that the steam in cylinder A, at the start, is dry saturated or superheated and is likewise so at the end of the transfer; then the steam heat, according to equation (56), p. 249, is

$$J_1 = J_0 + \frac{A}{\kappa - 1}p_1v_1 \quad \text{and} \quad J_2 = J_0 + \frac{A}{\kappa - 1}p_2v_2.$$

By substitution in equation (27) we therefore get the simple result

$$p_2v_2 = p_1v_1, \quad \ldots\ldots\ldots \quad (28)$$

in accordance with which we can calculate for given pressures the specific volume v_2 after the spreading in the receiving vessel. It follows from the assumption made that the steam is always superheated at the end, even if it were dry saturated at the beginning. Furthermore, if T_1 is the temperature at the beginning and T_2 that at the end, we get from the equation of condition of superheated vapors

$$p_2v_2 = BT_2 - Cp_2^n \quad \text{and} \quad p_1v_1 = BT_1 - Cp_1^n.$$

With the help of equation (28) the difference between the two preceding equations gives for the corresponding fall of temperature

$$T_1 - T_2 = t_1 - t_2 = \frac{C}{B}(p_1^n - p_2^n). \quad \ldots\ldots \quad (29)$$

Example. If steam, dry saturated or superheated, is throttled from $p_1 = 10$ atmospheres down to one atmosphere, under the conditions discussed with Fig. 29, we get from equation (29), with the help of the auxiliary table on p. 238,

$$t_1 - t_2 = 67.764 - 38.106 = 29.658° \; [53.384°].$$

If the steam is initially saturated, then its temperature is $t_1 = 180.31°$ [356.56°] and therefore the final temperature is $t_2 = 150.65°$ [303.17]; hence it is superheated to the extent of 50.65° [91.17°].

Moreover there exist experiments which confirm the principle expressed by equation (29). Hirn[1] allowed the steam coming from a boiler to flow under different boiler pressures through a small efflux orifice into the open air, the steam then spreading out under atmospheric pressure; the pressure p_2 in all experiments therefore amounted to one atmosphere. In order to avoid the cooling influence of the atmospheric air on the issuing steam jet during its passage into the condition of equilibrium, Hirn allowed the steam to first flow into a wooden box which was concentrically surrounded by a second and larger wooden box. The steam then flowed, after leaving the orifice, through a wide opening out of the inner box into the second outer wooden box and then through another wide opening into the free atmosphere. The orifice of flow in the inner box was situated in the upper wall, and that of the outer box in that of the lower wall. Now by means of a thermometer the temperature t_2 of the steam in the inner box was observed directly, and in so doing it was incidentally found that its pressure p_2 was only appreciably greater than the external pressure observed by the barometer.

In experiments with saturated steam Hirn found the following values for the temperature t_2, and beside these we have placed the values computed from equation (29); in so doing we have only adduced those experiments which Hirn has characterized as the more reliable ones (in the second edition of his work, p. 179).

---

[1] Hirn, "Exposition analytique et expérimentale de la théorie mécanique de la chaleur." T. I. p. 290, 3d edition, Paris, 1865.

Boiler Steam.

Pressure p_1 in Atmospheres.	Temperature t_2 (Table 11 of the Appendix).	Hirn's Experiment. t_2	Calculated from Equation (29). t_2
13	192.08°	155.58°	157.83°
11	184.50	152.50	153.21
9	175.77	149.57	147.87
7	165.34	144.10	141.46
5	152.22	138.72	133.34
3	133.91	128.40	121.86

In the second series of experiments Hirn passed the boiler steam, before efflux, through the superheating apparatus.

Pressure p_1 in Atmospheres.	Observed by Hirn. Temperature t_1	Temperature t_2	Calculated from Equation (29) t_2
13	200°	166°	165.75°
13	205	171.5	170.75
13	210	177	175.75
10	208	183	178.36
10	242	223	212.36
8	242	229	216.02
7	244	233	220.12
6	246	237	224.47
5	246	238.5	227.12

There is a very satisfactory agreement between the results of calculation and of experiment at high steam pressure and comparatively little superheat; greater differences appear with small steam pressure in the boiler.

The differences which are particularly marked with great superheating are undoubtedly due to the fact that the limits within which our equation of condition and the assumed constants are valid have been nearly reached or passed; but perhaps Hirn's experimental results are not perfectly reliable; for he specially calls attention to the fact (ibid., p. 392) that the temperature t_2 observed by him in the first (inner) wooden box was probably too great, because there the steam was for a considerable distance in contact with the hot surface of the metallic discharge

pipe. It is very desirable that similar experiments should be repeated, and perhaps also with other kinds of vapor.

If, in place of steam, atmospheric air or another gas were throttled in the manner described and then discharged, we should have to substitute $C=0$ in equation (29); we should then get $t_2=t_1$ and hence the air, after its spreading in the receiving vessel, must show nearly the same temperature as that which it possessed in the discharging vessel itself. Joule's experiments have sufficiently confirmed this statement.

Let us now treat the same problem with Fig. 29 as a basis and under the assumption that the steam in the discharging vessel is wet and possesses the steam quality x_1; we will also assume that, after the spreading in the receiving vessel, liquid is present with the steam and that the steam quality there is x_2.

Here we must substitute in the fundamental equation (27), p. 294,

$$J_1=q_1+x_1\rho_1 \quad \text{and} \quad J_2=q_2+x_2\rho_2,$$

also

$$v_1=x_1u_1+\sigma \quad \text{and} \quad v_2=x_2u_2+\sigma;$$

we then get

$$x_2r_2=q_1-q_2+x_1r_1+A\sigma(p_1-p_2), \quad . \quad . \quad . \quad . \quad (30)$$

and from this we can compute x_2 when x_1 is known and the pressure at the beginning and end is given.

From this we always obtain the inequality $x_2<x_1$, i.e., throttling always involves evaporation provided the assumption made is realized, namely, that during the transfer no heat is supplied to the steam from the outside, and also that no cooling takes place. The latter supposition is not fulfilled in steam engines which work with wet steam; since the admission pressure is always smaller than the boiler pressure, we have here a sort of throttling, and therefore admission should be attended by a partial evaporation of the accompanying water; in reality however it is known that just the opposite takes place, because the cylinder walls exert a cooling action on the entering steam; this circumstance will be discussed later on.

If, when using equation (30), $x_2 = 1$ results, this is a sign that the steam, after its spreading in the receiving vessel, is just dry saturated; from equation (30) we can then determine the corresponding steam quality x_1 in the steam boiler.

Example. Let the boiler steam have a pressure of $p_1 = 10$ atmospheres, and let it be throttled in such a way that it will show only a pressure of $p_2 = 1$ atmosphere in the receiving vessel.

Here, according to Table 11 of the Appendix,

$$q_1 = 182.719, \quad q_2 = 100.500, \quad r_1 = 478.776. \quad r_2 = 536.500$$
$$[q_1 = 328.894, \quad q_2 = 180.900, \quad r_1 = 861.797, \quad r_2 = 965.7],$$

and besides we find

$$A\sigma(p_1 - p_2) = 0.219\ [0.3942].$$

From equation (30) then follows

$$x_2 = 0.1536 + 0.8924\, x_1.$$

Therefore if the boiler steam has $x_1 = 0.90$, i.e., contains 10% of water, we get for the final steam quality in the receiving vessel $x_2 = 0.9568$ and hence only $4\frac{3}{10}\%$ moisture.

On the other hand if the steam in the receiving vessel is to be just dry saturated at the end so that $x_2 = 1$, we will have the corresponding steam quality $x_1 = 0.9484$.

Finally if the boiler steam is dry saturated so that $x_1 = 1$, the preceding formula gives $x_2 = 1.046$, which is impossible and is a proof that the steam is superheated at the end; in this case equation (30) loses its validity and equation (29) takes its place; superheating occurs in the present case when the quality of the boiler steam lies between 0.9484 and 1.

If the steam in the boiler is wet, as indicated at the end of the preceding example, but superheated after spreading in the receiving vessel, then the occurrence can be judged neither by equation (29) nor (30), but equation (27) can be transformed for this case.

We must substitute for the boiler steam

$$J_1 = q_1 + x_1\rho = q_1 + \rho_1 - (1 - x_1)\rho_1;$$

but we have in general

$$J = q + \rho = J_0 + \frac{Apv}{\kappa - 1},$$

and therefore, for the boiler steam, the steam heat becomes

$$J_1 = J_0 + \frac{Ap_1v_1}{\kappa - 1} - (1 - x_1)\rho_1,$$

while for the superheated steam in the receiving vessel we must substitute

$$J_2 = J_0 + \frac{A p_2 v_2}{\kappa - 1};$$

we therefore obtain from equation (27)

$$(1 - x_1)\rho_1 = \frac{A\kappa}{\kappa - 1}(p_1 v_1 - p_2 v_2),$$

or, if we utilize the equation of condition of superheated steam and at the same time substitute

$$\frac{A\kappa}{\kappa - 1} = \frac{c_p}{B},$$

we get

$$(1 - x_1)\rho_1 = c_p\left[t_1 - t_2 - \frac{C}{B}(p_1^n - p_2^n)\right]. \quad . \quad . \quad . \quad (31)$$

The solution is now contained in this equation; for $x_1 = 1$, i.e., when the boiler is dry or superheated, it again leads to equation (29); on the other hand if x_1 is so small that the steam is still wet after spreading outside, then the equation gives a value for the temperature t_2 which is smaller than the temperature which corresponds to the pressure p_2 in the saturated condition, a sign that the equation in this case loses its significance and that equation (30) is the one to be employed.

Example. Let the boiler steam again have the pressure of $p_1 = 10$ atmospheres, and let the steam quality amount to $x_1 = 0.96$, so that the steam, by weight, carries 4% of water; moreover if, as in the preceding example and in the example on p. 298, the outside pressure after throttling is one atmosphere, then we get from equation (31) with the help of Table 11 of the Appendix and the tabulation on p. 238, because

$$\rho_1 = 432.775, \quad t_1 = 180.31°, \quad c_p = 0.4805$$
$$[\rho_1 = 778{,}995, \quad t_1 = 356.56],$$

and

$$\frac{C}{B}(p_1^n - p_2^n) = 29.658\ [-53.384],$$

the temperature in the receiving vessel becomes

$$t_2 = 114.62°\ [238.316°];$$

so that the steam is superheated 14.62° [26.316°].

Connected with the preceding investigations there is another technical question to be considered.

After the superheated steam has spread in cylinder B (Fig. 29) we get for the steam heat (or the heat contents per unit of weight of steam) as compared with a kg. [lb.] of water at 0° [32°] the relation

$$J_2 = J_0 + \frac{A}{\kappa - 1} A p_2 v_2,$$

and as this steam during admission to the cylinder must perform the work $p_2 v_2$, we get for the total heat λ

$$\lambda = J_2 + A p_2 v_2 = J + \frac{\kappa}{\kappa - 1} A p_2 v_2.$$

If t_2' is the temperature of the steam possessing the pressure p_2 in the saturated condition, we can substitute for this equation the equation (57*d*), p. 252, and can then write

$$\lambda = (606.5 + 0.305\, t_2') + c_p(t_2 - t_2')$$
$$[\lambda = 1091.7 + 0.305(t_2' - 32) + c_p(t_2 - t_2')].$$

If we suppose the place of cylinder A (Fig. 29) to be taken by a steam boiler in which there is generated steam at constant pressure p_1, and if we designate the feed-water temperature by t_0 and the corresponding heat of the liquid by q_0 (which in ordinary cases can be replaced by t_0 with sufficient accuracy), we find for the heat quantity Q which enters the boiler to generate one kg. [lb.] of steam

$$Q = (606.5 + 0.305\, t_2' - q_0) + c_p(t_2 - t_2') \quad . \quad . \quad . \quad . \quad (32)$$
$$[Q = 1091.7 + 0.305(t_2' - 32) - q_0 + c_p(t_2 - t_2')].$$

Now let the cross-section of the pipe D (Fig. 29), where it enters the cylinder B, be designated by F, let the temperature and the specific volume of the following steam at this place be t and v, and let w be the velocity of flow; the quantity of heat which corresponds to the unit of weight of steam must then be

$$A\frac{w^2}{2g},$$

and it is this heat quantity which is expended in heating the steam from t to t_2 at constant pressure p_2 while the jet spreads itself in the receiving vessel B. There follows the relation

$$A\frac{w^2}{2g}=c_p(t_2-t), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

and if we substitute the value t_2, derived from this equation, in equation (32), we get

$$Q=(606.5+0.305\,t_2'-q_0)+c_p(t-t_2')+A\frac{w^2}{2g}. \quad . \quad . \quad . \quad (32a)$$

$$\left[Q=1091.7+0.305(t_2'-32)-q_0+c_p(t-t_2')+A\frac{w^2}{2g}\right].$$

If G kg. [lb.] of water are evaporated per second in the steam boiler we have

$$Gv=Fw$$

or

$$Gp_2v=Fp_2w,$$

or, utilizing the equation of condition for superheated steam,

$$Fp_2w=G(BT-Cp^n), \quad . \quad . \quad . \quad . \quad . \quad (34)$$

from which equation the efflux velocity w can be calculated whose value may then be utilized in equation (32a).

The preceding development explains the experimental method employed by Lewicki[1] to determine the heat quantity consumed by a steam boiler to generate one kg. [lb.] of steam,—or, as the weight G of the feed water per second was observed, to determine the heat quantity actually introduced per second into the boiler.

Lewicki did not let the generated steam go to the engine but, after throttling it, allowed it to flow through a pipe 8 m. [26.24 ft.] long and 120 mm. [4.72 in.] in diameter directly into the open air. In so doing care was taken to make sure that there was atmospheric pressure at the orifice and that the steam jet was perfectly transparent from the orifice for a long distance, a proof

[1] Lewicki, "Untersuchung eines Pauckschen Flammrohrkessels." Zeitschrift des Vereins deutscher Ingenieure, Vol. 31, 1887, p. 974.

that it was either dry saturated or superheated. Since the steam flows into the open air the pressure $p_1 = 10333$ kg. [2116.3 lb.] (1 atmosphere) and the corresponding temperature of saturation is $t_2' = 100°$ [212°]; the feed water G kg. [lb.] per second and the temperature t of the steam jet in the orifice of the discharge pipe were observed.

Example. In an experiment with the Pauck boiler there was supplied in 8 hours = 480 minutes 21252 kg. [46852.6 lb.] of feed water at a temperature of $t_0 = 38.9°$ [73.22°] in the boiler; therefore the weight G of steam discharged per second was $G = 0.7379$ kg. [1.6268 lb.]. The diameter of the discharge pipe was 120 mm. [4.724 in.]; hence its cross-section was $F = 0.01131$ sq. m. [0.121742 sq. ft.]. The temperature observed in the orifice was $t = 122°$ [251.6°]; consequently, from equation (34), the efflux velocity would be

$$w = 114.76 \text{ m. [376.512 ft.]},$$

and accordingly

$$A\frac{w^2}{2g} = 1.58 \text{ Cal. [2.844 B t.u.]}.$$

Furthermore, because $q_0 = 39.10$ [70.38], we get from equation (32a)

$$Q = 649.15 \text{ [1168.47 B.t.u.]}$$

per kg. [lb.] of steam, or the heat quantity entering the boiler per second must be

$$GQ = 479.01 \text{ Cal. [1897 B.t.u.]}.$$

After the spreading of the steam jet outside of the orifice, the temperature t_2 according to equation (33) will be found to be $t_2 = 125.29°$ [257.522°], provided the external atmospheric air has not exerted a gradually cooling action on the steam; but for the purpose in view this question needs no further consideration in the experimental method discussed.

Moreover in the experiment the pressure in the steam boiler was $p_1 = 3.66$ atmospheres, which corresponds to the temperature $t_1' = 140.71°$ [285.278°]; if this steam is reduced to the pressure of one atmosphere, in the way shown in Fig. 29, and becomes dry saturated, then according to equation (29) the temperature t_2 at the end of the spreading will be $t_2 = 126.11°$ [258.998°], a value closely agreeing with that found above. We can conclude from this that the steam was nearly dry in the experimental boiler, and also that the temperature t of the steam jet in the orifice can be measured with sufficient accuracy for the present purpose by an ordinary thermometer; it is only necessary to employ a sufficiently large discharge pipe. The experimental method considered of course becomes useless when the discharging steam is wet at the orifice.

C. ATMOSPHERIC AIR AS LIQUID AND VAPOR.

§ 38. LINDE'S MACHINE FOR THE LIQUEFACTION OF AIR.

If the changes of state of a vapor, indicated by change of pressure and volume, occur within the area bounded by the limit curve (Fig. 22, p. 203), then partial liquefaction will take place. We can conceive of another sort of a limit curve being drawn, however, in which the specific volume is again the abscissa, but the ordinate is the saturation temperature instead of the pressure; here the same proposition holds and the curve has a similar course; but here the maximum ordinate is represented by the critical temperature. For all vapors for which tables have been calculated in the Appendix the limit curve can be represented graphically in one way or the other (so far as Regnault's observations extend); for the other series of vapors, and to these belong those formerly designated as gases, there is still almost a complete lack of data for the graphical presentation of the limit curve under consideration; that they will have a course similar to those of the discussed vapor is indubitable; but at least we know the value of the maximum ordinate for all gases and vapors, i.e., know the critical temperature (see table on p. 207). We can now say that with a vapor we get into the realm of liquefaction as soon as we deal with temperature values lying below the critical temperature.

For example, according to Olszewski the critical temperature of atmospheric air is $-140°$ C. [$-220°$ F.] and therefore liquefaction is not to be thought of so long as it is impossible, in any way, to bring it below said temperature by cooling with simultaneous compression. For the critical temperature no higher pressure is necessary for liquefaction than $p_k = 39$ atmospheres;

if still lower temperatures can be produced then the corresponding saturation pressure is also lower, for example at the boiling-point of air, $t_0 = -191°$ [$-311.8°$], liquefaction can take place even with atmospheric pressure.

The knowledge of these conditions is quite recent; it was therefore a question of finding ways and means of reducing certain bodies to the lowest temperatures, and then, with their help, to cool atmospheric air and other gases, formerly called " permanent," and lead them toward liquefaction. The purpose must be to finally obtain gas in the form of a liquid body. The one method of liquefaction was gradually developed by Cailletet, Olszewski and Dewar, and was employed by the latter with complete success.[1]

In the carbonic acid (cold-vapor) engines liquid carbonic acid is obtained by compression and cooling; it can then be made to evaporate at a lower pressure by drawing off with a pump the vapors developed and then again condensing them. During this evaporation a great cooling of the carbonic acid takes place and the corresponding cold vapor of carbonic acid can be led by a pipe to the pump which is surrounded by a second pipe, through which compressed vapor of ethylene is forced in the opposite direction, this vapor being now condensed at lower temperature.

The liquid ethylene is next evaporated in the same manner as the carbonic acid, and now the far colder ethylene vapor is led by a pipe to the corresponding pump which is also enclosed by a second pipe, through which compressed oxygen vapor is forced in the opposite direction, the latter being brought to liquefaction.

By a reevaporation of the liquid oxygen in the same way as was just described with carbonic acid and ethylene we now reach the lowest temperature useful here.

Here we have to do with the combination of three different cold-vapor engines and by their means we reach,

[1] See Linde's detailed presentation: "Maschine zur Erzielung niedrigster Temperaturen, zur Gasverflüssigung und zur mechanischen Trennung von Gasgemischen." Bayer, Industrie- und Gewerbeblatt, 1896, p. 361.

by stages, the lowest temperature. In the manner described important results were obtained. Now although Dewar effected some simplification in the experimental arrangement, nevertheless it is at once evident that the method discussed is, on account of its circumstantial and costly character, entirely unavailable for technical and industrial purposes. These demand the continuous production of larger masses of liquid air, and that the condition be observed that liquefaction take place in a purely mechanical way, by the performance of work without the employment and cooperation of other kinds of vapors.

Such a method of liquefaction of air does exist in a machine which Linde (toward the end of May 1895) exhibited in action to a circle of physicists, chemists, and technical men at the experimental laboratory of the company producing Linde's ice machines in Munich.

Here we have an example of the rarely occurring case of a machine completely thought out from the beginning, resulting from purely scientific, physical investigations, and offered to the technical world.

In a very clever manner Linde, the most prominent promoter of the refrigerating industry of to-day, utilized an observation, known for some time to physicists, which had thus far only given rise to a few theoretical investigations. Mention has already been made (Vol. I, p. 167 of this book) of an earlier observation of Joule's, namely, that highly compressed air enclosed in a vessel experiences no change of temperature when it expands from the first vessel into a second vessel out of which the air has been pumped, p ovided the temperature is observed after the equalization of pressure and after the passage into a state of rest, and that no heat has been imparted from the outside. As no outer work was performed during this change of state, the inner work or energy of the air at the end (U_2) was identical with that at the beginning (U_1); but for a gas that obeys the law of Mariotte and Gay-Lussac we have, according to equation (52), Vol. I, p. 132,

$$U = U_0 + \frac{pv}{\kappa - 1}; \quad \ldots\ldots\ldots \quad (1)$$

therefore for the present case we also have $p_1v_1 = p_2v_2$, and on account of the equation of condition of a perfect gas $pv = BT$ it also follows that $T_1 = T_2$, as was concluded from the older experiments of Joule.

The circumstances are similar when a gas is allowed to pass through a throttle valve from a vessel in which there is a constant pressure p_1 into a second vessel in which the pressure is maintained at the constant height p_2. Here the work p_1v_1 is produced in the first vessel and the work p_2v_2 is consumed in the second vessel; the relation

$$U_2 = U_1 + p_1v_1 - p_2v_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

therefore obtains, and if we now utilize the preceding equation (1) there also follows $p_2v_2 = p_1v_1$, so that $T_2 = T_1$; accordingly for the transfer under the assumed conditions, throttling under constant pressure, no change of temperature will occur.

This analytical result is, however, entirely due to the circumstance that the equation of condition, in the form $pv = BT$, is assumed as permissible; this was regarded as sufficiently exact for technical purposes, when investigating the behavior of gases in Vol. I. But in certain investigations this assumption is not permissible and equation (1), for the energy U of the gases, can also be regarded merely as an approximation.

In § 37, p. 293, it has been shown that when superheated steam flows under constant pressure from a discharging vessel into a receiving vessel, also at constant pressure, a fall of temperature occurs; such a fall is also to be expected with gases, but here it will be so slight in the ordinarily occurring cases that its presence may be neglected.

It is highly interesting to note that Linde, in his machine, utilized the slight temperature fall occurring here, in order to obtain the lowest temperatures.

Linde, in his reflections on this matter, started from the results obtained by W. Thomson and Joule,[1] whose

[1] W. Thomson and Joule, Philos. Transactions of the Royal Society of London, 1862, p. 579.

experiments have been mentioned in Vol. I, p. 236. These experiments with atmospheric air gave, for the cooling or lowering of temperature δ during the transfer just discussed, the relation

$$\delta = 0.276(p_1 - p_2)\left(\frac{273}{T}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\left[\delta = 0.496(p_1 - p_2)\left(\frac{491.4}{T}\right)^2\right].$$

Here the pressure difference $(p_1 - p_2)$ is to be taken in atmospheres, and T denotes the absolute temperature of the air at the entrance to the orifice. Designating the pressure difference by p, the formula may also be written

$$\delta = \alpha \frac{p}{T^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

where we must substitute $\alpha = 20570$ [119771].

For example, for $t = 10°$ C. [50° F.] equation (3*a*) gives $\delta = 0.257\, p$, and hence for $p = 1$, 10 or 40 atmospheres, we get, respectively,

$$\delta = 0.257°, \quad 2.57°, \quad 10.28°$$
$$[\delta = 0.4626, \quad 4.626, \quad 18.504]$$

Although the above formula is an empirical one, still Linde concludes from observations on his machine that it is confirmed even within wider limits, and that it is particularly confirmed as to the dependence of the cooling δ on temperature T at the entrance into the orifice. There is also agreement as regards the pressure difference p up to $p = 50$ atmospheres, and it is not until $p > 50$ that the proportionality between δ and p ceases to exist.

Now, according to the foregoing, a single efflux, even with a great difference of pressure, will effect a relatively small lowering of temperature; consequently Linde combines the action of any number of effluxes in such a way that each preceding one serves to effect a preliminary cooling of the gas for the succeeding

one. Fig. 30 is a schematic representation of Linde's machine.[1]

The compressor C draws the air through the suction pipe BBB under the pressure p_2, compresses it to the pressure p_1 toward the high-pressure pipe AAA, and in so doing passes it through a cooler K, and brings it to the temperature t_1, the heat of compression being withdrawn. The air in this system of pipes therefore describes a complete circuit; there is a second compressor

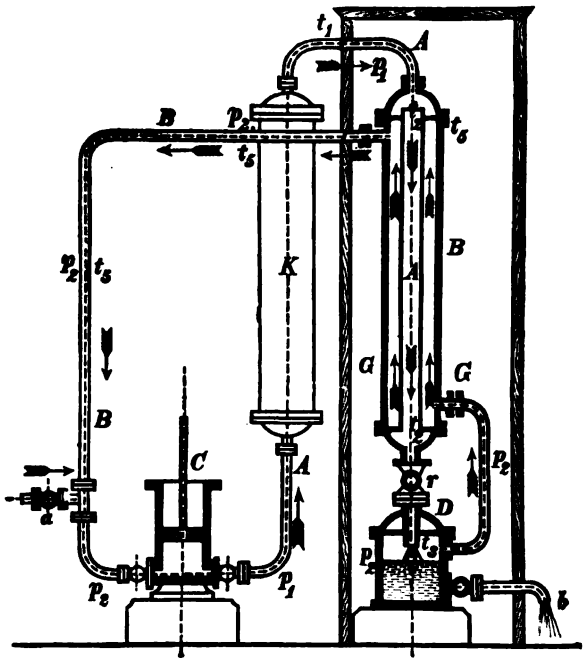

FIG. 30.

at a for the maintenance of the pressures p_1 and p_2 during the starting of the machine and during liquefaction, and thus suitable quantities of air are introduced into the circuit. In the experimental machine the pressure p_1 in the delivery pipe amounts to 65 atmospheres, and that in the suction pipe to $p_2=22$ atmospheres. The part of the pipe system within the casing GG con-

[1] Linde, "Erzielung niedrigster Temperaturen. Gasverflüssigung." Annalen der Physik und Chemie, 1896, Vol. 57, p. 328.

stitutes a counter-current apparatus; in the inner pipe A the air, compressed up to p_1, possesses the initial temperature t_1 and flows down to a regulating valve r, through which it enters the collecting vessel D in which the pressure p_2 prevails; from here on the air is led upward through the point B and sucked back to the compressor. The fall of temperature occurs when it enters the collecting vessel; let the temperature there, at a given instant, be t_3; with this initial temperature the air rises upwards in the pipe B and cools the air flowing down the delivery pipe A from t_1 to t_2; it reaches the upper part of the flow with the temperature t_5, which is nearly the same as that of the initial temperature t_1. While these temperature values remain nearly unchanged during the continuous circulation of the air, the values t_2 and t_3 sink continually; the latter value sinks more rapidly than the former, till, finally, t_3 falls below the value of the critical temperature and liquefaction of the air begins in the collecting vessel D and continues from now on. The procedure up to the beginning of the normal and permanent condition, in which the temperatures b_2 and b_3 remain unchanged, we designated before as the starting up.

The liquid air can then be conducted through the discharge pipe b to a vessel outside.

The portion of the piping lying within the casting G, the counter-current apparatus, in reality consists of two telescoped pipes, 100 m. [328 ft.] long and 10 and 4 cm. [3.9937 and 0.1575 in.] in diameter, which are wound spirally; the whole is carefully protected against inward radiation.

The liquid air, led to the outer vessel, has a light blue color and evaporates under the external atmospheric pressure and is thus cooled still more. In so doing, the nitrogen evaporates first and there is thus gradually formed a mixture even richer in oxygen. In recent observations Linde has shown how the mixture ratio of nitrogen and oxygen in liquid air gradually varies,[1] during evaporation under constant atmospheric pressure; moreover, by an appropriate change in the arrangement of his machine, he has

[1] Linde, "Über Vorgänge bei Verbrennung in flüssiger Luft." Sitzungsberichte der mathem.-phys. Klasse der königl. bayer. Akademie der Wissenschaften, 1899, Vol. XXIX, Book 1.

directly effected the separation of the two constituents in the gaseous form and has thus developed a mechanical method of obtaining oxygen; in the machine itself the liquefied air has the same composition as the external atmospheric air. In the article just quoted Linde also shows that a mixture of liquid air with pulverized coal will produce a very effective explosive. At this time we cannot foresee the great consequences which will result from the applications of liquid air in chemical and scientific investigations and particularly in the application to chemical industries. Interesting suggestions in this direction have already been given by W. Hempel.[1]

A fuller consideration of the occurrences in his machine has already led Linde[2] to the important law of the invariability of the specific heat c_p of air under constant pressure to which reference has already been made in Vol. I, p. 150. In the counter-current apparatus (Fig. 30) the air current flowing down the delivery pipe A is cooled under the constant pressure p_1 from t_1 to t_2. The heat quantity W, which in so doing is withdrawn from the unit of heat, is

$$W = c_p(t_1 - t_2), \quad \ldots\ldots\ldots \quad (4)$$

where c_p represents a mean value of the specific heat. During the entrance into the collecting vessel D the air is cooled still further down to t_3, and indeed according to W. Thomson and Joule's equation (3a) there obtains the relation

$$t_2 - t_3 = \frac{\alpha(p_1 - p_2)}{T_2^2} \cdot \quad \ldots\ldots\ldots \quad (5)$$

The air now flows upward in the suction pipe B of the counter-current apparatus under the constant pressure p_2 and with the initial temperature t_3. If, in so doing, we suppose the unit of

[1] Walther Hempel, "Über Prozesse bei welchen die von Linde erfundene Maschine zur Erzeugung flüssiger Luft Anwendung finden kann." Chemische Industrie, Year 1899, No. 1.

[2] Linde, "Über die Veränderlichkeit der spezifischen Wärme des Gases." Sitzungsberichte der mathem.-phys. Klasse der königl. bayer. Akademie der Wissenschaften, 1897, Vol. XXVII, No. 3.

weight of air to again receive the heat quantity W the air will be heated to t_4 and the mean value of the specific heat will be c_{p_0}; hence there follows

$$W = c_{p_0}(t_4 - t_3) = c_p(t_1 - t_2). \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Evidently the temperature t_4 must be the same as that which would obtain if the air had fallen directly (without passing through the counter-current apparatus) during efflux from the pressure p_1 to the pressure p_2 with the efflux temperature t_1; it follows therefore that

$$t_1 - t_4 = \frac{\alpha(p_1 - p_2)}{T_1^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Subtracting this equation from equation (5) gives

$$(t_4 - t_3) - (t_1 - t_2) = \alpha(p_1 - p_2)\left[\frac{1}{T_2^2} - \frac{1}{T_1^2}\right].$$

As the expression on the right hand is positive there follows

$$(t_4 - t_3) > (t_1 - t_2)$$

and hence, according to equation (6),

$$c_p > c_{p_0}.$$

Since c_p corresponds to the higher pressure p_1 it follows indubitably that the specific heat c_p of the air grows with the pressure.

Equation (6) can also be written

$$W = c_{p_0}(t_4 - t_2 + t_2 - t_3) = c_p(t_1 - t_2).$$

If we designate, as in equation (3*a*), the fall of temperature $(t_2 - t_3)$ during efflux, by δ, we get, on passing to the differential,

$$c_{p_0}(dt - d\delta) = c_p dt,$$

or

$$c_p = c_{p_0}\left(1 - \frac{d\delta}{dt}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and this is the general form of the equation as established by Linde; it involves no assumption whatever with respect to the law of temperature variation δ.

Now if, in pursuing the problem further, we assume the expression of W. Thomson and Joule as a basis we get, for an infinitesimal difference of pressure and replacing δ by dT, according to equation (3*a*),

$$T^2 dT = \alpha dp,$$

and from this follows, by integration, if we temporarily designate the temperature after efflux by T_0,

$$T^3 - T_0^3 = 3\alpha p;$$

hence the fall of pressure is

$$\delta = T - T_0 = T - (T^3 - 3\alpha p)^{\frac{1}{3}}, \quad \ldots\ldots \quad (8a)$$

and with this we get

$$\frac{d\delta}{dt} = 1 - \left(1 - \frac{3\alpha p}{T^3}\right)^{-\frac{1}{3}},$$

and thus we get from equation (8) for the specific heat c_p of air under constant pressure

$$c_p = c_{p_0}\left(1 - \frac{3\alpha p}{T^3}\right)^{-\frac{1}{3}}. \quad \ldots\ldots \quad (9)$$

For the factor c_{p_0} Linde assumes, according to Witkowski, the value $c_{p_0} = 0.237$, which is valid for $p = 0$; we have assumed that $\alpha = 20570$ [119771] when p is measured in atmospheres. In Vol. I, p. 150, there has already been calculated, from the preceding formula, the specific heat c_p for a series of pressure and temperature values.

If at $p = 1$ atmosphere the air is just dry saturated, then because the corresponding temperature is shown by experiments to be $t = -191°$ [$-311.8°$] or $T = 82°$ [147.6°] (see p. 205), we find for this point $c_p = 0.2431$.

At the critical point $c_p = \infty$ must obtain; because here the

pressure $p_k=30$ atmospheres we find from equation (9), by means of the formula

$$T_k=3\alpha p_k,$$

that $T_k=134°$ [241.2°] and hence the critical temperature is $t_k=-139°$ [−218.2°], while on p. 205 it was stated that the experiments gave $t_k=-140°$ [−220°].

§ 39. EXTENDED EQUATION OF CONDITION FOR ATMOSPHERIC AIR.

The perfectly general heat equations (Vol. I, p. 143) are valid under the supposition that the pressure p and the volume v are the independent variables; on the other hand if, to simplify the formulas, we take the pressure p and the temperature t as independent variables, we must substitute the value $1:\left(\frac{dv}{dt}\right)_p$ in place of $\left(\frac{dt}{dv}\right)_p$, for in the first case we have

$$dt=\left(\frac{dt}{dp}\right)_v dp+\left(\frac{dt}{dv}\right)_p dv,$$

in the other case

$$dv=\left(\frac{dv}{dt}\right)_p dt+\left(\frac{dv}{dp}\right)_t dp.$$

If p is constant so that $dp=0$, the comparison will furnish the expressed relation.

Accordingly there follows from the third of the three identical equations (IIIb), Vol. I, p. 143,

$$dQ=c_p dt-AT\left(\frac{dv}{dt}\right)_p dp. \quad \ldots\ldots \quad (10)$$

From this there follows for the entropy P

$$dP=\frac{dQ}{AT}=\frac{c_p}{AT}dt-\left(\frac{dv}{dt}\right)_p dp,$$

and as this is a complete differential we have

$$\frac{d}{dp}\left(\frac{c_p}{AT}\right)_t = \frac{d}{dt}\left(\frac{dv}{dt}\right)_p,$$

or

$$\left(\frac{dc_p}{dp}\right)_t = AT\left(\frac{d^2v}{dt^2}\right)_p. \quad \ldots\ldots \quad (11)$$

Now let the equation of condition be written in the form

$$pv = BT - R, \quad \ldots\ldots \quad (12)$$

which is valid for all bodies in which B is a constant factor and R must be regarded as a function of p and t.

From the differentiation of equation (12) with p taken as a constant we then get

$$\left(\frac{dv}{dt}\right)_p = \frac{1}{p}\left[B - \left(\frac{dR}{dt}\right)_p\right], \quad \ldots\ldots \quad (13)$$

also

$$\left(\frac{d^2v}{dt^2}\right)_p = -\frac{1}{p}\left(\frac{d^2R}{dt^2}\right)_p, \quad \ldots\ldots \quad (14)$$

and hence there follows from equation (11)

$$\left(\frac{dc_p}{dp}\right)_t = -\frac{AT}{p}\left(\frac{d^2R}{dt^2}\right)_p. \quad \ldots\ldots \quad (15)$$

Since $R=0$ for a perfect gas we must conclude from this equation that for this gas the specific heat c_p is independent of p and must therefore be a function of the temperature only, a conclusion which we reached in Vol. I, p. 145. Furthermore, if we use equation (13) for the transformation of equation (10) there follows

$$dQ = c_p dt - A\left[BT - T\left(\frac{dR}{dt}\right)_p\right]\frac{dp}{p},$$

and if we here substitute BT from equation (12) we get

$$dQ = c_p dt - Avdp + A\left[R - T\left(\frac{dR}{dT}\right)_p\right]\frac{dp}{p},$$

or

$$dQ = c_p dt - Avdp + \frac{AT^2}{p}\frac{d}{dt}\left(\frac{R}{T}\right)_p dp. \quad \ldots \quad (16)$$

We can now pass to the utilization of the efflux experiments of W. Thomson and Joule.

From equation (2), p. 306, there follows

$$U_2 - U_1 + p_2 v_2 - p_1 v_1 = 0,$$

or

$$dU + d(pv) = 0,$$
$$AdU + Apdv + Avdp = 0.$$

The first two terms represent dQ and hence we get from equation (16)

$$c_p \frac{dt}{dp} = -\frac{AT^2}{p} \frac{d}{dt}\left(\frac{R}{T}\right)_p \quad \ldots\ldots \quad (17)$$

as a perfectly general, final result.

To the difference of pressure dp corresponds therefore a fall of temperature dt; for a perfect gas (for $R=0$) the temperature before and after the efflux has the same value.

If the form of the function R were known the preceding formula would enable us to test the statement of W. Thomson and Joule; conversely we will here assume their statement in order to determine the form of the function R.

If we use equation (3a) in the form $\frac{dt}{dp} = \frac{\alpha}{T^2}$ in the preceding equation, we get

$$\frac{d}{dt}\left(\frac{R}{T}\right)_p = -\frac{c_p}{A}\frac{\alpha p}{T^4} = \frac{c_p}{9A}\frac{d}{dt}\left(\frac{3\alpha p}{T^3}\right)_p.$$

If, for simplicity, we temporarily take

$$\frac{3\alpha p}{T^3} = x,$$

then, according to Linde,[1] the specific heat (equation 9) is

$$c_p = c_{p_0}(1-x)^{-\frac{1}{3}},$$

[1] Starting from the above equations (10) and (11), Planck, in his "Vorlesungen über Thermodynamik," Leipsic, 1897, p. 117, derives for the specific heat c_p of the air a formula which is in complete agreement with Linde's equation.

hence

$$d\left(\frac{R}{T}\right)=\frac{c_{p_0}}{9A}(1-x)^{-\frac{1}{3}}dx,$$

and by integration

$$\frac{R}{T}=-\frac{c_{p_0}}{3A}\sqrt[3]{1-\frac{3\alpha p}{T^3}}+K,$$

where K signifies an integration constant which, for the present, is to be regarded as a function of p. But now we may assume that infinitely rarefied air, when p approaches zero, behaves like a perfect gas so that we may take $R=0$; it follows therefore that

$$0=-\frac{c_{p_0}}{3A}+K,$$

and with this we get

$$\frac{R}{T}=\frac{c_{p_0}}{3A}\left(1-\sqrt[3]{1-\frac{3\alpha p}{T^3}}\right),$$

or

$$R=\frac{c_{p_0}}{3A}[T-\sqrt[3]{T^3-3\alpha p}]. \quad . \quad . \quad . \quad . \quad . \quad (18)$$

According to equation (8*a*), p. 312, the bracketed quantity is only an expression for the fall of pressure in the experiments of W. Thomson and Joule.

We can therefore write

$$R=\frac{c_{p_0}}{3A}\delta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18a)$$

and from this follows for the equation of condition of atmospheric air

$$pv=BT-\frac{c_{p_0}}{3A}\delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

If, for the present, we substitute here for δ the bracketed quantity of equation (18) there will follow the equation of condition for air:

$$\frac{pv}{T}=B-\frac{c_{p_0}}{3A}\left[1-\sqrt[3]{1-\frac{3\alpha p}{T^3}}\right], \quad . \quad . \quad . \quad . \quad (20)$$

an equation which Planck has also developed.

Because for $t=0$ and $p=1$ atmosphere, we have according to Vol. I, p. 103, the relation $pv=29.269\ T$ [53.349], we find for the constant B of equation (20)

$$B=29.303\ [53.411].$$

The equation has of course a limited validity like the formula (3*a*) of W. Thomson and Joule, and should be available only for low temperatures and for comparatively small pressures.

For dry saturated air of $p=1$ atmosphere pressure and for $t=-191°$ [$-311.8°$] or $T=82°$ [147.6°], equation (20) gives, with probably sufficient accuracy, the specific volume of the air as $v=0.222$ cbm. [3.5562 cu. ft.].

On the other hand at the critical point, for which we found (p. 312) $3\alpha p=T^3$, we get a negative value for the critical volume v_k, a proof that at the temperature $t_k=-140°$ [$-220°$] and $p_k=39$ atmospheres the formula is already useless.

But if we substitute in equations (18*a*) and (19) the value given by W. Thomson and Joule for δ, according to equation (3*a*), there follows

$$R=\frac{c_{p_0}}{3A}\frac{\alpha p}{T^2} \qquad (21)$$

and when we use the notation

$$\frac{c_{p_0}\alpha}{3A}=C$$

we get for the equation of condition

$$pv=BT-C\frac{p}{T^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where p must be expressed in kg. per sq. m. [lb. per sq. ft.]; then

$$B=29.303 \quad \text{and} \quad C=66.681$$
$$[B=53.411 \quad \text{and} \quad C=3460.83].$$

This equation has already been given by Clausius[1] and

[1] Clausius, "Die mechanische Wärmetheorie." Brunswick, 1887, Vol. 1 3d Edition, p. 236.

also by Kirchhoff.[1] Here Clausius assumes beforehand that c_p is constant, and Kirchhoff assumes only insignificant changes for c_p.

For dry saturated air of 1 atmosphere pressure and of the corresponding temperature $t=-191°$ [$-311.8°$] this equation gives the specific volume $v=0.2226$ cbm. [3.5658 cu. ft.], almost the same as before. For the critical point, $p_k=39$ atmospheres and $t_k=-140°$ [$-220°$], the formula gives the critical volume $v_k=0.0059$ cbm. [0.094511 cu. ft.], which, to be sure, is to be regarded as a first approximation.

Assuming, as Kirchhoff did, that equation (22) is still serviceable for slight variations of c_p, we can add a further remark Differentiation gives

$$\left(\frac{dt}{dv}\right)_p=\frac{p}{B\left(1+\frac{2C}{B}\frac{p}{T^3}\right)} \quad \text{and} \quad \left(\frac{dt}{dp}\right)_v=\frac{T}{p\left(1+\frac{2C}{B}\frac{p}{T^3}\right)}.$$

Now according to equation (II*b*), Vol. I, p. 143, we have

$$(c_p-c_v)\left(\frac{dt}{dv}\right)_p\left(\frac{dt}{dp}\right)_v=AT,$$

hence follows

$$c_p-c_v=AB\left(1+\frac{2C}{B}\frac{p}{T^3}\right)^2.$$

If we substitute $\frac{2C}{B}=\beta$, we get

$$c_p-c_v=AB\left(1+\frac{\beta p^2}{T^3}\right), \quad \ldots\ldots \quad (23)$$

where, when p is measured in atmospheres, we must take $\beta=47027$ and $B=29.303$. As Linde has given c_p for different values of p and T, we can calculate from this formula the specific heat c_v at constant volume for every individual case.

[1] Kirchhoff, "Vorlesungen über mathematische Physik. Theorie der Wärme." Leipsig, 1894.

For example we get the following numerical results and include here the values of c_p as adduced by Linde:

		$t=-100°$	$-50°$	$0°$	$+100°$
$p=$ 1 atm.	$c_p=$	0.2389	0.2379	0.2375	0.2372
	$c_v=$	0.1685	0.1682	0.1681	0.1680
$p=$ 5 "	$c_p=$	0.2468	0.2415	0.2394	0.2379
"	$c_v=$	0.1713	0.1694	0.1687	0.1685
$p=10$ "	$c_p=$	0.2579	0.2462	0.2419	0.2389
	$c_v=$	0.1757	0.1711	0.1696	0.1687

For technical investigations there results the remarkable outcome that with moderate pressures and from 0° [32°] up to the highest temperatures the specific heat c_v, under constant volume can be regarded as nearly constant.

In this case we can determine the quantity p from the equation $pv=BT$ and substitute it in the second term of the right member of equation (22) which represents the supplementary term R; we then get

$$pv=BT-\frac{BC}{Tv},$$

a form of the equation which was proposed by Rankine for superheated steam (see p. 228) and one which is also obtained if we assume $\alpha=0$ and $\beta=0$ in the first equation of condition given by Clausius (p. 215).

From the preceding equations it seems likely that with closer observations of the Linde machine it may be possible to derive, for atmospheric air, the more exact form of the equation of condition.

D. ON THE BEHAVIOR OF MIXTURES.

a) Mixtures of Atmospheric Air and Steam.

§ 40. THE GENERAL BEHAVIOR OF THE MIXTURE OF AIR AND WET STEAM.

The thermodynamic investigation of the behavior of a mixture of gas and vapor, and particularly of atmospheric air and vapor of water, is of great importance in meteorology, but cases do occur in engineering which justify a fuller discussion here of this sort of phenomena. The production of high-pressure air from low-pressure mostly takes place in compressors, with the injection of water. The idea has also frequently been suggested that steam engines should not be run by pure steam, but by a combination of steam and compressed air. The question is, however, of special importance in the theory of the open, cold-air engine; the vapor of water, ordinarily mixed with atmospheric air, has an appreciable influence on the occurrences in air engines; for instance the formation of snow during expansion is a serious evil which hitherto, in practical constructions, has only been partially removed and then only with sacrifices, namely, the reduction of the efficiency of the engine.

In the following investigations we will only consider mixtures of atmospheric air with the vapor of water and will distinguish between two cases, according as saturated or superheated steam is mixed with the air; the first case always exists when water in the liquid condition, as well as steam, is on hand, and this case we will discuss first; but we will say in advance that the quantity of water present will be assumed as inconsiderable throughout, in

order that we may leave out of the calculations that quantity of air which is absorbed by the water.

Let us now imagine the space V to contain G_1 kg. [lb.] of air and G_2 kg. [lb.] of water and steam; let us suppose that the common temperature is T, the pressure of the air p' and that of steam p'', then the total pressure of the mixture is

$$p = p' + p''. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

According to the equation of condition of air we have

$$G_1 B_1 T = V p', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where B_1 represents now, and in the following, the constant $B = 29.269$ [53.349]; for the mixture of steam and water present, or, as we will say for the sake of brevity, for the wet steam, the relation

$$G_2(xu + \sigma) = V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

obtains, where x represents the steam quality; we will also assume that space V contains $G_2 x$ kg. [lb.] of steam and $G_2(1-x)$ kg. [lb.] of water; the value of u corresponds to the temperature t or to the pressure p''.

For an infinitesimal change of state the heat quantity converted into work, in the case of the air, becomes

$$AdL' = Ap'dV = G_1 AB_1 T \frac{dV}{V}, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

when we consider equations (2) and (3), and for the wet steam it becomes

$$AdL'' = G_2 Ap''d(xu),$$

or

$$AdL'' = G_2 \left[d(Ap''ux) - Axu \frac{dp''}{dt} dt \right].$$

If we here substitute $Ap''u = r - \rho$ and utilize in the second term Clapeyron's equation (p. 29), we get, after simple transformation,

$$AdL'' = G_2 \left[Td\left(\frac{xr}{T}\right) - d(x\rho) \right]. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The sum of equations (4) and (5) then gives for the whole mixture the heat converted into work:

$$AdL = G_1 A B_1 T \frac{dV}{V} + G_2 \left[T d\left(\frac{xr}{T}\right) - d(x\rho) \right]. \quad . \quad . \quad (6)$$

On the other hand the change of inner work, measured in units of heat, becomes, for the air (Vol. I, p. 132),

$$AdU' = c_v G_1 dT, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where c_v is the specific heat of the air at constant volume ($c_v = 0.1685$). Furthermore, for the wet steam (p. 54) we have

$$AdU'' = G_2 [dq + d(x\rho)]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

By adding equations (7) and (8) we get the change dU of the inner work for the whole mixture:

$$AdU = G_1 c_v dT + G_2 [dq + d(x\rho)], \quad . \quad . \quad . \quad . \quad (9)$$

an expression that can be integrated.

Again, if we add equations (6) and (9) and at the same time make use of the relation $AB = c_v(\kappa - 1)$ (Vol. I, p. 123), we get the heat quantity dQ, which must be supplied to the whole mixture for this change of state:

$$dQ = c_v G_1 \left[dT + (\kappa - 1) T \frac{dV}{V} \right] + G_2 \left[dq + T d\left(\frac{xr}{T}\right) \right]. \quad . \quad (10)$$

The first term of the right member is the quantity of heat demanded by the air present (see Vol. I, p. 134, equation 53*d*); the second term gives the quantity of heat which is absorbed by the wet steam (see p. 59). Both terms can be converted into other forms, as shown in the references, but these forms will be omitted here; we repeat, however, that the preceding formulas are valid only so long as $x \leq 1$ during the changes of condition; if the calculations for the final condition lead to a value $x > 1$, then this must be taken as a sign that the water originally present has disappeared and that the steam has passed into the superheated condition; a case which will be discussed later on. If we divide both members of equation (10) by AT we get an expression that can be integrated;

the integration gives the entropy P of the mixture, and if we designate the initial values of all the variable quantities by the subscript 1 (see Vol. I, p. 135, and this volume, p. 61) we shall get

$$A(P-P_1)=c_v G_1 \log_e \frac{T}{T_1}\left(\frac{V}{V_1}\right)^{\kappa-1}+G_2\left[\tau-\tau_1+\frac{xr}{T}-\frac{x_1 r_1}{T_1}\right]. \quad (11)$$

If we treat the specific heat c of the water as constant on the average, which is permissible within comparatively narrow limits of temperature, and if we remember the significance of τ, p. 59, we get

$$\left.\begin{aligned} &A(P-P_1)\\ &=(c_v G_1+cG_2)\log_e\frac{T}{T_1}+G_1 A B_1 \log_e\frac{V}{V_1}+G_2\left[\frac{xr}{T}-\frac{x_1 r_1}{T_1}\right]. \end{aligned}\right\} \quad (11a)$$

These equations can also be brought into different forms, but it will be best to undertake the corresponding transformations when treating particular problems.

§ 41. ISOTHERMAL CHANGES OF STATE OF THE MIXTURE OF AIR AND WET STEAM.

Here the temperature T, at every instant of the change of state, is equal to the initial temperature T_1 and therefore the pressure p'' of the steam present is constant; hence $p''=p_1''$, $u=u_1$, etc.

On the other hand the pressure of the air is found from equation (2) by means of

$$Vp'=B_1 G_1 T_1=V_1 p_1', \quad \ldots\ldots \quad (12)$$

and therefore the whole pressure of the mixture is

$$p=\frac{G_1 B_1 T_1}{V}+p_1''=\frac{V_1 p_1'}{V}+p_1''. \quad \ldots\ldots \quad (13)$$

If in Fig. 31*a* we lay off the volume V as abscissa and the pressure $p=p''+p'$ as ordinate, we get, for the course of the isothermal, an equilateral hyperbola; one of its

asymptotes is the axis of ordinates, the other is a straight line through O_1, parallel to the axis of abscissas.

If x_1 is the steam quality at the volume V_1 we get the corresponding value x, at the volume V, from equation (3), or

$$G_2(xu_1+\sigma)=V, \quad \ldots\ldots \quad (14)$$

or from

$$\frac{xu_1+\sigma}{x_1u_1+\sigma}=\frac{V}{V_1}, \quad \ldots\ldots \quad (14a)$$

or finally, as we can neglect in most cases the specific volume τ of the water, from

$$x=\frac{V}{V_1}x_1. \quad \ldots\ldots \quad (14b)$$

Isothermal expansion, therefore, involves evaporation; conversely, compression involves condensation of the steam.

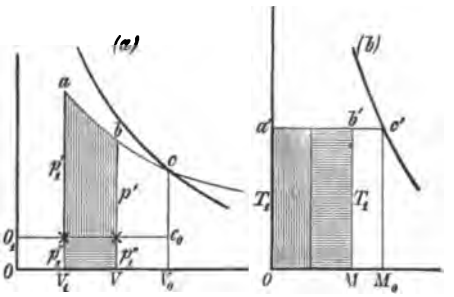

FIG. 31.

At a certain value $V=V_0$ isothermal expansion causes its curve to reach the point c (Fig. 31*a*), at which $x=1$, and with further expansion the steam passes into the superheated condition; the point c is therefore a point of the limit curve; according to equation (14) its abscissa is

$$V_0=G_2(u_1+\sigma), \quad \ldots\ldots \quad (15)$$

and the ordinate consists of two parts: the lower part corresponds to the saturation pressure p_1'' of the steam at the temperature T_1, and can therefore be taken directly from the steam table; the

upper part, c_0c, corresponds to the pressure p_0' belonging to the air present and can be found from equation (2):

$$p_0' = \frac{G_1 B_1 T_1}{V_0}. \quad \ldots\ldots\ldots \quad (16)$$

Consequently the ordinate p_0 of the point c of the pressure curve is

$$p_0 = \frac{G_1}{G_2} \frac{B_1 T_1}{(u_1 + \sigma)} + p_1''. \quad \ldots\ldots\ldots \quad (17)$$

Therefore a particular mixture ratio,

$$m = \frac{G_2}{G_1} = \frac{\text{weight of steam and water}}{\text{weight of air}},$$

is accompanied by a particular limit curve. If no air is present, $G_1 = 0$, the limit curve passes through the point c_0 and, under the supposition that $G_2 = 1$, is identical with the limit curve which was fully discussed when investigating the vapor of water.

There are therefore, for the assumed mixtures, an infinite number of limit curves, and the limit curve corresponding to a particular mixture ratio can easily be drawn with the help of the preceding statements.

If the curve is represented (drawn with a heavy line in Fig. 31*a*) for a given mixture ratio, then all changes of state must take place within the course of the limit curve if the above developed equations are to preserve their validity. The limit curve will be pushed farther upward the more air the mixture contains.

Returning to the consideration of the isothermal change of state, we can now determine the work L performed during expansion along the curve ab (Fig. 31*a*); it is determined from equation (6), because the quantities r and ρ corresponding to the temperature $T = T_1$ are also constant, and, after some reduction, we get

$$L = V_1 p_1' \log_e \frac{V}{V_1} + p_1''(V - V_1), \quad \ldots\ldots \quad (18)$$

which equation may also be derived directly from Fig. 31*a*. The two hatched areas of the figure represent the two terms of the right member of the equation.

The heat quantity Q, which is herewith supplied from the outside, is found from equation (10):

$$Q = c_v G_1 (\kappa - 1) T_1 \log_e \frac{V}{V_1} + G_2 r_1 (x - x_1), \quad . \quad . \quad . \quad (19)$$

or

$$Q = A V_1 p_1' \log_e \frac{V}{V_1} + \frac{r_1}{u_1} (V - V_1). \quad . \quad . \quad . \quad . \quad . \quad (19a)$$

Finally we get the entropy, according to equation (11), from

$$A(P - P_1) = c_v (\kappa - 1) G_1 \log_e \frac{V}{V_1} + G_2 \frac{r_1}{T_1} (x - x_1), \quad . \quad (20)$$

or, after simple reduction, with the help of equation (3), and making $\sigma = 0$, which is permissible, particularly for the low temperatures obtaining in meteorological applications, we get

$$P - P_1 = B G_1 \log_e \frac{x}{x_1} + G_2 \frac{r_1}{A T_1} (x - x_1). \quad . \quad . \quad . \quad (20a)$$

If, in Fig. 31*b*, we lay off the value $OM = P - P_1$ as abscissa and lay off T_1 as the ordinate, the horizontal $a'b'$ represents the transformation of the isothermal curve ab, and according to the earlier presentations the hatched area represents the value $Q:A$, i.e., the heat supply measured in units of work. The indicated division of the hatched area is intended to represent the two terms of the right member of equation (20).

If, in equation (20*a*), we make $x = 1$, we get in Fig. 31*b* the value OM_0 as abscissa of the transformation of the intersection c' of the isothermal curve with the corresponding limit curve.

The curve passing through c' is intended to represent the transformation of the corresponding limit curve; in order to compare the course of the limit curve for different mixture ratios $m = \frac{G_2}{G_1}$ we will reduce, with the help of this ratio and the relation $G_1 + G_2 = 1$, the curve to the u n i t o f w e i g h t o f m i x t u r e ;

we then get respectively for the corresponding weight of air and for the weight of water and steam

$$G_1 = \frac{1}{1+m} \quad \text{and} \quad G_2 = \frac{m}{1+m}, \quad \ldots \quad (21)$$

and then, representing as before by p and s the pressure and the specific volume of dry saturated steam at the saturation temperature T, and on the other hand representing by p_0 and v_0 the coordinates of the contemplated point on the limit curve for the mixture ratio m, we get, according to equations (17) and (15),

$$p_0 = \frac{B_1 T}{ms} + p \quad \text{and} \quad v_0 = \frac{ms}{1+m}. \quad \ldots \quad (22)$$

If the limit curve, therefore, is drawn for dry saturated steam and its equation is approximately given on p. 37 by $ps^\nu = D$, then we can with ease represent graphically the limit curves corresponding to other mixture ratios m, and will return to this later. If we utilize the relations (21) in the equations (1) to (20) developed above, these will be valid for the unit of weight of mixture; on the other hand if we wish to reduce them to the unit of weight of air we must substitute $G_1 = 1$ and $G_2 = m$.

Example. Suppose given a cubic meter of mixture ($V_1 = 1$) possessing the temperature $t_1 = 20°$ C. [58° F.] and that it is subject to the average atmospheric pressure $p_1 = 760$ millimeters [29.922 inches] of mercury. The mixture is to experience the compression $\frac{V}{V_1} = \frac{1}{5}$ at constant temperature, and there are to be determined the final state, the work L to be expended, and the heat quantity Q to be withdrawn.

Assumption 1. Let the air at the beginning be just saturated, that is, let no water be present. Here, according to the steam table, we get, for $t_1 = 20$ [58], the steam pressure $p_1'' = 17.39$ [0.685 in.], and the air pressure is $p_1' = p_1 - p_1'' = 742.61$ [29.237 in.], or, estimated in kilograms per square meter [lb. per sq. ft.],

$$p_1 = 10{,}333 \text{ kg.}, \quad p_1'' = 236.4 \text{ kg.}, \quad p_1' = 10096.6 \text{ kg.}$$
$$[p_1 = 2116.3 \text{ lb.}, \quad p_1'' = 48.4 \text{ lb.}, \quad p_1' = 2067.9 \text{ lb.}].$$

Because $B_1 = 29.269$ [53.349] we get from equation (2), for the weight G_1 of air present,

$$G_1 B_1 T = V_1 p_1', \quad G_1 = 1.1773 \text{ kg. } [2.5955 \text{ lb.}],$$

and for the weight of the steam present we get, from equation (3), and because, according to Table 1*b* of the Appendix, $u_1 = 58.720$, the quantity

$$G_2 = \frac{V_1}{x_1 u_1 + \sigma}, \quad G_2 = 0.0170 \text{ kg. } [0.03748 \text{ lb.}],$$

and therefore the mixture ratio

$$m = \frac{G_2}{G_1} = 0.01446.$$

Now the steam quality x at the end of the compression is $x = 0.2$, according to equation (14*b*). The work of compression according to equation (18) is

$$L = 16249.7 + 189.1 = 16438.8 \text{ mkg. } [= 118903 \text{ ft-lb.}],$$

and the heat to be withdrawn, as given by equation (19*a*), is

$$Q = 38.771 + 8.072 = 46.843 \text{ Cal. } [= 185.89 \text{ B.t.u.}].$$

Assumption 2. On the other hand suppose that at the beginning air, steam, and water are present (water being injected before the compression); and let us assume the mixture ratio to be $m = 1$, then, as before, the weight of the air is $G_1 = 1.1773$ kg. [2.5955 lb.]; the steam quality at the beginning follows from the relation $G_2 = G_1 = \frac{V_1}{x_1 u_1 + \sigma}$ and is

$$x_1 = 0.01446 \text{ kg. } [0.0319 \text{ lb.}],$$

and hence, according to equation (14*b*), the corresponding value at the end is

$$x = 0.00289 \text{ kg. } [0.00637 \text{ lb.}].$$

The work of compression and the heat to be withdrawn are found from equations (18) and (19*a*) exactly as in the preceding case.

In both cases the pressure at the end of the compression is found from equation (13) and is

$$p = 5\,p' + p_1'' = 3730.44 \text{ mm.} = 4.908 \text{ atmospheres.}$$

For dry saturated steam of the temperature $t_1 = 20°$ [58°], we have, according to Table 1*b* of the Appendix, for the coordinates of the corresponding point of the limit curve,

$p = 236.4$ kg. and $v_0 = s = 58.721$
[$p = 48.42$ lb. per sq. ft. and $v_0 = s = 940.65$].

Under assumption 1 (for the mixture ratio $m = 0.01446$) we get from equation (22) for the coordinates of the corresponding point of the limit curve and for the unit of weight of mixture:

$p_0 = 10336.3$ kg. and $v_0 = 0.8370$
[$p_0 = 2117.2$ lb. and $v_0 = 13.408$].

Under assumption 2 ($m=1$), on the other hand,

$$p_0 = 382.4 \quad \text{and} \quad v_0 = 29.361$$
$$[p_0 = 78.320 \text{ lb. per sq. ft.} \quad \text{and} \quad v_0 = 470.33].$$

Assumption 3. Suppose the air at the beginning to be dry, without any admixture of steam and wa'er. Here we have, according to the propositions concerning the isothermal of gases (Vol. I, p. 136) and $p=5$ atmospheres, the terminal value of the compression, the work

$$L = V_1 p_1 \log_e \frac{V}{V_1} = 16630.3 \text{ mkg. } [120288 \text{ ft-lb.}],$$

and for the heat to be withdrawn,

$$Q = LA = 39.222 \text{ cal. } [155.645 \text{ B.t.u.}].$$

§ 42. ISOTHERMAL CHANGE OF STATE FOR THE MIXTURE OF AIR, STEAM, WATER, AND ICE.

One important case of isothermal change of state which deserves to be more especially considered arises at the temperature of 0° [32°], i.e., at the freezing-point of water; here there may be present, with the liquid water, some solid water in the form of ice. Let us therefore consider a mixture of G_1 kg. [lb.] of air with G_2 kg. [lb.] of steam, water, and ice; then for every change of state (expansion or compression) we shall have the constant temperature of 0° [32°]. So long as ice is present with the water, changes of temperature in one or the other direction will not take place till all the ice is melted or all the water is frozen. Now, at a certain instant of the change of state, let x be the specific steam quantity (quality) and y the specific ice quantity; then the mixture contains by weight, G_2x steam, G_2y ice, and $G_2(1-x-y)$ water.

Now, if we assume expansion, we have for the heat quantity which must be imparted to the air at constant temperature

$$G_1 A B_1 T \frac{dV}{V},$$

when the same notation is employed as in § 41.

The heat quantity necessary for evaporation is

$$G_2 r dx,$$

and the heat quantity for ice formation is

$$-G_2 r_e dy,$$

where r_e represents the latent heat or heat of fusion of ice, i.e., $r_e = 79$ cal. [142.2 B.t.u.].

The magnitudes T and r are constant and here $T = 273$ [491.4] and $r = 606.5$ [1091.7].

The volume V is determined with sufficient accuracy from the relation

$$V = G_2 x u, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where $u = 210.66$ [3374.55] (Table 1*b* of the Appendix). Accordingly the heat to be imparted to the mixture is

$$dQ = G_1 A B_1 T \frac{dx}{x} + G_2 (r dx - r_e dy), \quad . \quad . \quad . \quad . \quad (24)$$

and therefore, by integration, when the initial values are indicated by the subscript 1, we find

$$Q = G_1 A B_1 T \log_e \frac{x}{x_1} + G_2 [r(x - x_1) - r_e(y - y_1)]. \quad . \quad . \quad (25)$$

Dividing both members of equation (24) by AT, we obtain the differential of the heat weight or of the entropy, and, by integration, get

$$P - P_1 = G_1 A B_1 \log_e \frac{x}{x_1} + G_2 \left[\frac{r}{T}(x - x_1) - \frac{r_e}{T}(y - y_1) \right] . \quad (26)$$

If, in a particular case, the specific steam weight x_1 at the beginning is known, and, likewise, the specific ice weight y_1, and if the heat quantity Q which must be supplied is known as a function of x or V or of p, then we can calculate from equation (25), for any arbitrarily assumed steam weight x, the specific ice weight y at the end.

The initial volume V_1 is determined from equation (23):

$$V_1 = G_2 x_1 u, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23a)$$

and the final volume from

$$\frac{V}{V_1}=\frac{x}{x_1}. \quad \ldots\ldots\ldots \quad (27)$$

The initial pressure of the air is determined, according to equation (2), from

$$V_1 p_1' = G_1 B_1 T,$$

and its final pressure p' by

$$\frac{p'}{p_1'}=\frac{V_1}{V}-\frac{x_1}{x}. \quad \ldots\ldots\ldots \quad (28)$$

The steam pressure is constant:

$$p''=p_1''=4.600 \text{ mm. [0.1811 in.] of mercury,}$$

or 62.5 kg. per square meter [12.801 lb. per sq. ft.].

The total pressure is therefore at the beginning $p_1'+p_1''=p_1$ and at the end it is $p'+p_1''=p$; the expansion curve can be represented accordingly; its transformation can be found when we arbitrarily assume P_1 and take the value P, determined by equation (26), as abscissa and the temperature $T=273$ [491.4] as ordinate; it therefore appears as a particular, limited, horizontal, straight line.

The problem plays an important part in meteorological questions (hail formation); only the matter is simplified there by the assumption of adiabatic change of state. We then substitute $Q=0$ in equation (25) and $P=P_1$ in equation (26); the equations then become identical and we get from equation (26)

$$G_1 A B_1 \log_e \frac{x}{x_1}=G_2\left[\frac{r_e}{T}(y-y_1)-\frac{r}{T}(x-x_1)\right]. \quad . \quad (26a)$$

The corresponding equations (23), (23*a*), (27), and (28) remain unchanged.

As adiabatic-isothermal change of state obtains here, we have for the transformation of the pressure curve both abscissa and

ordinate constant and the transformation therefore reduces itself to one point.

The combination of the equations referred to furnishes moreover, in place of equation (26*a*),

$$G_1 A B_1 T \log_e \frac{p_1'}{p'} = G_2 \left[r_e (y - y_1) - r x_1 \left(\frac{p_1'}{p'} - 1 \right) \right]. \quad . \quad (26b)$$

With the help of this equation we can determine, from the change of the pressure of the air, the increment $G_2(y - y_1)$ in the ice present.

If we substitute in equation (26*a*) values of T, r_e, and r corresponding to the temperature $t = 0°$ [32°], we also find

$$0.2385\, G_1 \log_e \frac{x}{x_1} = G_2 \left[(y - y_1) - 7.677\, x_1 \left(\frac{x_1}{x} - 1 \right) \right]. \quad (26c)$$

Example. Let us assume a mixture of air, steam, and water of 0° [32°] temperature and mean atmospheric pressure $p_1 = 10333$ kg. [2116.3 lb.]; at the beginning there is no ice present, and accordingly $y_1 = 0$. The steam pressure is, therefore, $p_1'' = 62.5$ kg. [12.801 lb.], and the air pressure is $p_1' = p_1 - p_1'' = 10270.5$ kg. [2103.5 lb.].

If x_1 is the steam quality at the beginning, then the initial volume, according to equations (3) and (2), is

$$V_1 = G_2 x_1 u \quad \text{and} \quad V_1 (p_1 - p_1'') = G_1 B_1 T,$$

and therefore, by equating, there follows the mixture ratio

$$m = \frac{G_2}{G_1} = \frac{BT}{x_1 u (p_1 - p_1'')}.$$

If we substitute for the temperature of 0° [32°] the corresponding values $T = 273°$ [491.4° F.] and $u = 210.66$ [3374.55] (Table 1*b* of the Appendix), and also substitute the pressures just given, we find

$$m = \frac{G_2}{G_1} = \frac{0.00369}{x_1}.$$

If no water was present at the beginning, but only steam in the state of saturation, then we have $x_1 = 1$, and the mixture ratio $m = 0.00369$.

Now let us suppose that in the present case the initial steam quality is $x_1 = 0.100$, so that we have 0.1 G_1 kg. [lb.] of steam and 0.9 G_2 kg. [lb.] of water, and let us assume that the expansion is continued till all the water present is converted into ice. Then the final quantity of water is

$$G_2 (1 - x - y) = 0,$$

or the ice quantity $G_2 y = G_2(1-x)$, and, according to the above-given values of m, the assumed mixture ratio $m = 0.0369$. Under these suppositions, and because $y_1 = 0$, we get by trial, from equation (26c), for the steam quality at the end, $x = 0.129$, and for the specific ice weight $y = 0.871$.

The expansion ratio is as follows:

$$\frac{V}{V_1} = \frac{x}{x_1} = 1.29;$$

the air pressure at the end is determined from

$$\frac{p'}{p_1'} = \frac{V_1}{V}, \quad \text{or} \quad p' = 7961.6 \text{ kg.} = 585.6 \text{ mm.}$$
$$[p' = 1630.6 \text{ lb.} = 23.056 \text{ in.}].$$

Accordingly the total pressure, which at the beginning amounted to 760 mm. [29.922 in.], is at the end 590.2 mm. [23.236 in.] of mercury.

§ 43. ADIABATIC CHANGE OF STATE OF THE MIXTURE OF AIR AND WET STEAM.

The adiabatic expansion of moist air, for the special purpose of investigating meteorological questions, has often been made the subject of discussion by others.[1]

Let us assume, as in § 39,

	At the Beginning.	At the End.
the pressure of the air	p_1',	p'
the pressure of the steam	p_1'',	p''
the pressure of the mixture	$p_1 = p_1' + p_1''$,	$p = p' + p''$
the volume	V_1,	V
the steam quality	x_1,	x
the temperature	t_1,	t

[1] The first investigation of the sort, with the help of equations of thermodynamics, was instituted by Th. Reye, "Über vertikale Luftströme in der Atmosphäre." Zeitschrift für Mathematik und Physik, 9th year, 1864, p. 250.

Later this question was specially taken up by Hertz, "Graphische Methode zur Bestimmung der adiabatischen Zustandsänderungen feuchter Luft." Meteorologische Zeitschrift von Köppen, 1st year, 1884; also by von Bezold, "Zur Thermodynamik der Atmosphäre." Transactions of the Royal Prussian Academy of Sciences.

then the combination of equations (2) and (3), p. 321, and neglecting the very small value σ, gives the mixing ratio

$$m=\frac{G_2}{G_1}=\frac{B_1T_1}{p_1'x_1u_1}, \quad \ldots\ldots \quad (29)$$

or also

$$m=\frac{G_2}{G_1}=\frac{B_1T_1}{p'xu}, \quad \ldots\ldots \quad (29a)$$

and after equating we get

$$\frac{xu}{x_1u_1}=\frac{V}{V_1}=\frac{p_1'}{p'}\frac{T}{T_1}. \quad \ldots\ldots \quad (30)$$

For adiabatic change of state the heat weight is constant; therefore if we substitute $P=P_1$ in equation (11), p. 323, we get as a fundamental equation

$$0=c_vG_1\log_e\frac{T}{T_1}\left(\frac{xu}{x_1u_1}\right)^{\kappa-1}+G_2\left[\tau-\tau_1+\frac{xr}{T}-\frac{x_1r_1}{T_1}\right]. \quad (31)$$

Here the calculation becomes more convenient if we consider the relation $c_v(\kappa-1)=AB$, utilize equation (30), and insert the mixing ratio m; then we have

$$0=c_v\log_e\frac{T}{T_1}+AB_1\log_e\frac{V}{V_1}+m\left[\tau-\tau_1+x_1u_1\left(\frac{V}{V_1}\frac{r}{uT}-\frac{r}{u_1T_1}\right)\right]. \quad (31a)$$

With the help of this transcendental equation, with known initial values and for any arbitrarily chosen final temperature T, we can find the volume ratio $V:V_1$ and can then get, from equation (29) and the rest of the equations just given, the steam quality x and the total pressure p at the end. In this way we can determine the course of the adiabatic curve and reproduce it graphically.

In adiabatic change of state we substitute $dQ=0$ in the fundamental equation

$$dQ=A(dU+dL),$$

and then we get the corresponding work of expansion or of compression L in accordance with equation (9), p. 322, and find that it is

$$AL = G_1 c_v (T_1 - T) + G_2 [q_1 - q + x_1 \rho_1 - x\rho], \quad . \quad . \quad . \quad (32)$$

which equation is capable of different transformations.

Of the possible transformations of the two preceding fundamental equations we emphasize only the following.

If we may assume the average specific heat c of the water as constant, then according to equation (54a), p. 59, we may take

$$\tau - \tau_1 = c \log_e \frac{T}{T_1}.$$

If, moreover, we utilize equations (30) and (29) and consider the relation $c_p = c_v + AB_1$, we get in place of equation (31a), as is easily seen, the equation

$$0 = \frac{c_p + mc}{AB_1} \log_e \frac{T}{T_1} + \log_e \frac{p_1'}{p'} + \left(\frac{r}{Aup'} - \frac{r_1}{Au_1 p_1'} \right), \quad . \quad (31b)$$

from which can be computed for any arbitrary final temperature T the final pressure p' of the air; then, because the steam table will give the steam pressure p'' corresponding to the temperature t, we can also determine the final pressure $p = p' + p''$ of the whole mass. With the help of equation (30) we can next calculate V and x for the final condition.

Following the same method of transformation we furthermore get from equation (32) the work produced or consumed:

$$L = B_1 G_1 \left\{ \frac{c_v + mc}{AB_1} (T_1 - T) + \left(\frac{\rho_1 T_1}{A p_1' u_1} - \frac{\rho T}{A p' u} \right) \right\}, \quad . \quad (32a)$$

where we must imagine the final pressure p' of the air determined by equation (31b).

In meteorological investigations we usually have to do with relatively narrow limits of temperature and pressure and may therefore assume that saturated steam behaves like a gas; desig-

nating the constant B of the equation of condition for steam by B_2 (generally we make $B_2 = 47.061$ [85.778]) and using the above notation,

$$p_1'' u_1 = B_2 T_1 \quad \text{and} \quad p'' u = B_2 T;$$

the preceding equations (31a) and (32) may then be written as follows:

$$0 = (c_p + mc) \log_e \frac{T}{T_1} + AB_1 \log_e \frac{p_1'}{p'} + \frac{B_1}{B_2}\left[\frac{r}{T}\frac{p''}{p'} - \frac{r_1}{T_1}\frac{p_1''}{p''}\right] \quad (31c)$$

and

$$AL = G_1\left\{(c_v + mc)(T_1 - T) + \frac{B_1}{B_2}\left(\rho_1 \frac{p_1''}{p_1'} - \rho \frac{p''}{p'}\right)\right\}. \quad (32b)$$

Equation (31b) is also given by Hertz (ibid.).

Example 1. Let a cylinder contain air, steam, and water of the atmospheric pressure $p_1 = 10333$ kg. [2116.3 lb.], and of the temperature $t_1 = 20°$ [58°] or $T_1 = 293°$ [527.4°]. After the water has been injected suppose that, measured by weight, there is just as much air present as steam and water, i.e., suppose $G_2 = G_1$ and consequently the mixture ratio $m = 1$. Since the steam pressure $p_1'' = 236.4$ kg. [48.42 lb.], the initial steam pressure is $p_1' = 10096.6$ kg. [2067.9 lb.]. The initial steam quality, according to equation (29), is $x_1 = 0.01446$.

Now let the whole mass be compressed adiabatically till the temperature has risen from $t_1 = 20°$ [58°] to $t = 50°$ [122°].

Substituting in equation (31) the known constants and the given quantities, with the help of Tables 1a and 1b of the Appendix, we find

$$\log_e \frac{V_1}{V} - 1.80664 \frac{V}{V_1} = 1.22640,$$

and from this we easily get by trial the corresponding ratio of compression,

$$\frac{V_1}{V} = 4.926.$$

Consequently when we have at the beginning $V_2 = 1$ cbm. [1 cu. ft.] of mass, the final volume $V = 0.2030$ cbm. [cu. ft.].

From equation (30) then follows the steam quality at the end, namely, $x = 0.01429$, and the air pressure at the end, $p' = 54824.5$ kg. [11218.7 lb.]. The steam on hand at the temperature 50° [122°] has a pressure of 91.98 mm. [3.6213 in.], or $p'' = 1250.5$ kg. [256.18 lb.]; accordingly the final pressure of the whole mass is $p = 56075$ kg. [11474.9 lb.] or 5.427 atmospheres.

The work of compression is found from equation (32) through

$$AL = 34.698\ G_1$$
$$[AL = 62.456\ G_1].$$

If, at the beginning, $V_1 = 1$ cbm. [1 cu. ft.], then G_1 is found from the relation $G_1 B_1 T_1 = V_1 p_1$, and so $G_1 = 1.1773$ kg. [0.0333 lb.] is the weight of the air present; therefore the work of compression is

$$AL = 40.850 \text{ Cal.} \quad \text{or} \quad L = 17320 \text{ mkg.}$$
$$[AL = 2.082 \text{ B.t.u.} \quad \text{or} \quad L = 1609 \text{ ft-lb.}].$$

We get very different results if we compress 1 cbm. [cu. ft.] of dry air of 1 atmosphere pressure and of the temperature $t_1 = 20°$ [58°] to the same final volume $V = 0.2030$ cbm. [cu. ft.]. Equations (60), (61), and (62*a*), Vol. I, pp. 139 and 140, then give the final temperature $t = 270.35°$ [486.63°], the terminal pressure 9.471 atmospheres, and the compression work $L = 19315$ mkg. [1800 ft-lb.].

We see from this how greatly the injection of water prevents superheating in air-compressors; the given formulas (30) and (32) constitute the bases for the theory of such compressors.

Example 2. Let a cylinder contain air and saturated steam without water under the atmospheric pressure $p_1 = 10333$ kg. [2116.3 lb.] and of the temperature $t_1 = 20°$ [58°]. Here $x_1 = 1$, and because the steam pressure $p_1'' = 236.4$ kg. [48.42 lb.], the pressure of the air is $p_1' = 10096.6$ kg. [2067.9 lb.], and hence, according to equation (29), the mixing ratio becomes

$$m = \frac{G_2}{G_1} = 0.01446.$$

Now let the mixture expand adiabatically till the temperature at the end is $t = 0°$ [32°], so that $T = 273°$ [491.4°]. Here, utilizing known constants and the values of the corresponding temperature functions, Tables 1*a* and 1*b*, we obtain

$$\log_e \frac{V}{V_1} + 0.12972 \frac{V}{V_1} = 0.61103,$$

and from this, by trial, the expansion ratio

$$\frac{V}{V_1} = 1.514,$$

and then, according to equation (30), the steam quality at the end is $x = 0.4220$, because at the beginning it was assumed that $x_1 = 1$. The pressure of the air at the end is $p' = 456.88$ mm. [17.988 in.], and accordingly the final pressure of the mixture is $p = 461.48$ mm. [18.169 in.], while the initial pressure amounts to $p_1 = 760$ mm. [29.922 in.] of mercury.

With further adiabatic expansion the temperature remains constant at 0° [32°] till all the water is converted into ice, and the further calculation will have to be made as in the example on p. 332.

Moreover, in the present example, after introducing the corresponding quantities in equation (32) for the work of expansion, we find

$$AL = 4.9497\, G_1$$
$$[AL = 8.9095\, G_1],$$

or if at the beginning we had on hand one cubic meter [cubic foot], this work will become

$$AL = 5.8273 \text{ Cal.} \quad \text{or} \quad L = 2470.8 \text{ mkg.}$$
$$[AL = 0.297 \text{ B.t.u.} \quad \text{or} \quad L = 229.5 \text{ ft-lb.}],$$

because, from the relation $G_1 B_1 T = V_1 p_1'$, when $V_1 = 1$, we find the weight of the air to be $G_1 = 1.1773$ kg. [0.0333 lb.].

§ 44. BEHAVIOR OF THE MIXTURE OF AIR AND UNSATURATED VAPOR.

At ordinary atmospheric temperatures the pressure of the steam mixed with the air is so slight, even when saturated, that we may unhesitatingly proceed from the assumption that the steam present behaves like a gas and is therefore, in the saturated as well as in the unsaturated state, subject to the equation of condition of gases. This assumption is general in meteorology, but it is also thoroughly permissible when investigating engines and machines, i.e., in blowers, compressors, cold-air engines, etc., provided we only consider the vapor contents of ordinary atmospheric air without the simultaneous presence of liquid water. Here, therefore, the propositions derived in Vol. I, p. 107, § 20, on "Gas Mixtures," will hold. Nevertheless these propositions require further examination, for, applied to air and steam, they possess only a limited validity and are useless as soon as the changes of state are accompanied by condensation of steam; it is particularly important to determine the range of their applicability.

If the volume V, as before, includes G_1 kg. [lb.] of air and G_2 kg. [lb.] of steam, and if T is the temperature, p' the pressure of

the air, p'' that of the steam, and p the total pressure, then, according to the preceding assumptions, we have the relations

$$Vp' = G_1 B_1 T \quad \text{and} \quad Vp'' = G_2 B_2 T. \quad . \quad . \quad . \quad (33)$$

Here we must let $B_1 = 29.269$ [53.349] for air, and for steam $B_2 = 49.061$ [89.424], when the pressures are given in kilograms per square meter [pounds per square foot]. The ratio

$$\varepsilon = \frac{B_1}{B_2} = 0.622 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

is the weight of the steam relatively to air, possessing the same pressure and the same temperature.

If the pressure is given in millimeters [inches] of mercury, then we must substitute $B_1 = 2.1528$ [0.75428], and $B_2 = 3.4614$ [1.2128]. Dividing one of the two equations (33) by the other, there follows

$$\frac{p''}{p'} = \frac{G_2}{G_1} \frac{B_2}{B_1},$$

and from this we get

$$\frac{p''}{p'} = \frac{m}{\varepsilon} = \frac{p''}{p - p''}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

where m represents the mixing ratio or ratio of steam weight to air weight. Hence there follows, because $p = p' + p''$,

$$p' = \frac{\varepsilon}{\varepsilon + m} p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

and

$$p'' = \frac{m}{\varepsilon + m} p. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

On the contrary, if we add the two equations (33) and assume a unit of weight of mixture, i.e., $G_1 + G_2 = 1$, we shall find

$$pv = \frac{\varepsilon + m}{1 + m} B_2 T = B_m T \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

as the equation of condition of the mixture considered, but this will only be valid so long as dry steam, saturated or unsaturated, is mixed with the air.

If in the present case, which presupposes unsaturated or superheated steam, the steam were just saturated for the same pressure p and the same temperature T, then we should take from the steam table the pressure corresponding to this temperature and designate it by p_0'' and according to equation (35) we should get

$$m_0 = \frac{\epsilon p_0''}{p - p_0''}, \quad \ldots \ldots \quad (39)$$

where m_0 represents the corresponding mixture ratio. For example, this equation gives, for a pressure $p = 760$ mm. [29.922 in.] of the mixture, the ratio m_0 for a mixture of saturated steam and air at the following temperatures:

$t =$	$-20°$	$-10°$	$0°$	$+10°$	$+20°$	$+30°$
$m_0 =$	0.00076	0.00172	0.00379	0.00759	0.01457	0.02694

If the air is not saturated with vapor, then $m < m_0$.

From equation (35) we find for unsaturated steam the mixture ratio

$$m = \frac{p''\epsilon}{p - p''}, \quad \ldots \ldots \quad (39a)$$

where p'' is the "vapor pressure" of meteorology; the ratio $m : m_0$, designated by x, can now be found from

$$x = \frac{m_0}{m} = \frac{p''(p - p_0'')}{p_0''(p - p'')},$$

which can be written sufficiently accurately

$$x = \frac{p''}{p_0''},$$

which value is called the "degree of moisture" or the "relative moisture of the air." [1]

Equation (38) now gives for a known mixture ratio m the relation between pressure, volume, and temperature of the unit of weight of mixture; when the change of state is arbitrary it is only valid so long as no condensation of steam takes place.

Suppose that for a certain mixture ratio m the initial condition is given by v_1, p_1, and T_1, then a certain limit curve DD will belong to the given mixture ratio m (point m of Fig. 32), and this limit curve must not be overstepped by the change of condition, as this would involve condensation of the steam present. The course of this limit curve can be fixed with sufficient accuracy for the ordinary temperatures of the atmosphere. For pure steam (without admixture of air) there was found earlier (p. 37) an approximate equation of the limit curve

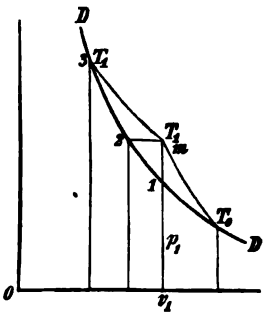

FIG. 32.

$$pv^{\nu}=D, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

where ν and D are constants which have elsewhere been given for the higher pressures occurring in steam engines.

For the present case of ordinary atmospheric temperatures we could specially determine the constants ν and D, for the purpose of determining more accurately the course of the limit curve; this could be done with the help of the values of p and $u=v$, given in Tables 1*a* and 1*b* of the Appendix for temperatures ranging from $-20°$ to $+30°$ [$-4°$ to $+86°$ F.]. The calculation of u on the basis of the experimental results is, however, hardly reliable enough for the given limits of temperatures, i.e., the values Apu

[1] Here examine the calculations in Vol. I, pp. 264 to 263, which are conducted as is customary in Physics and Meteorology. For the thermodynamic study of the occurrences in the atmosphere it is clearer and more correct to pursue the path taken above in the text; here there have been introduced in the calculations the temperature T and the common pressure p, also the mixture ratio m,—the ratio of the steam weight G_2 to the air weight G_1.

and u adduced in the table should hardly be used for this purpose.

It would therefore be better to hold fast to the hypothesis that steam for these temperature values is subjected to the equation of condition of gases.

If we compute, according to equation $pv=pu=B_2T$, for several temperature values, the corresponding specific volumes v, and in so doing substitute $B_2=3.4641$ [1.21373], under the hypothesis that the pressure p is given in millimeters [inches] of mercury, then we get the following results:

$t=$	$-20°$	$-10°$	$0°$	$+10°$	$+20°$
$v=u=$	944.7	435.0	205.4	106.9	58.3
$t=$	$-4°$	$+14°$	$32°$	$50°$	$68°$
$v=u=$	15133.0	6968.0	3290.0	1712.0	933.9

The values for the temperatures below the freezing-point differ not inconsiderably from those of Table 1*b* of the Appendix, but may nevertheless be regarded as reliable. With the help of these values and of the corresponding pressures p given in Table 1*a*, we can now calculate the constants of the equation of the limit curve,

$$pv^\nu=D, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

and find them to be

$$\nu=1.0525 \quad \text{and} \quad D=1254 \quad . \quad . \quad . \quad . \quad (40a)$$
$$[\nu=1.0525 \quad \text{and} \quad D=914.83],$$

under the hypothesis that the pressure p is given in millimeters [inches] of mercury.

This equation gives for

$t=$	$-20°$	$-10°$	$0°$	$+10°$	$+20°$
$v=$	944.1	435.5	206.1	107.1	58.2
$t=$	$-4°$	$+14°$	$32°$	$50°$	$68°$
$v=$	15124	6976	3302	1716	932.5

which agree sufficiently well with the values just given so that this equation, under the assumptions made, can be made the basis of further investigations.

In order to fix the course of the limit curve for a mixture of air and steam and indeed for any mixture ratio m whatsoever, let us start from the point m (Fig. 32) with any change of state T_1T_0 and continue it till the limit curve DD is met at T_0; if T_0 is the corresponding temperature of saturation, then, according to equation (38), we have for this point

$$pv = \frac{\varepsilon + m}{1 + m} B_2 T_0.$$

On the other hand equation

$$p_0 v_0 = B_2 T_0$$

also obtains when p_0 is the steam pressure, and v_0 is the specific volume of saturated steam of the temperature T_0.

The division of both equations gives

$$\frac{p_0 v_0}{pv} = \frac{1 + m}{\varepsilon + m},$$

and because equation (37) is

$$p_0 = \frac{m}{\varepsilon + m} p,$$

we get from the combination of the two formulas the specific volume v_0 of the steam at the point of intersection T_0 (Fig. 32), as follows:

$$v_0 = \frac{1 + m}{m} v.$$

If we now substitute the two values of p_0 and v_0 in place of p and v in equation (40), we get an equation for the limit curve corresponding to the mixture ratio m:

$$pv^\nu = \frac{\varepsilon + m}{m} \left(\frac{m}{1 + m} \right)^\nu D; \quad \ldots \ldots \quad (41)$$

here, when p is expressed in millimeters [inches], we must use the constants

$$\epsilon = 0.622, \quad \nu = 1.0525, \quad D = 1254\,[=914.83]. \quad . \quad . \quad . \quad (41a)$$

If we designate the value of the right member of equation (41) by D_m, and if we prefer to introduce the reciprocal value of m, i.e., $z = \frac{1}{m}$, there follows

$$pv^{\nu} = D_m. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

where

$$D_m = \frac{1 + \epsilon z}{(1+z)^{\nu}} D. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42a)$$

We see from this that an infinite number of limit curves differ only by the constant D_m which can easily be found for every mixture ratio from equation (42*a*).

As regards the general properties of these curves and their graphical representation we can apply the propositions for the polytropic curve, derived in Vol. I, §§ 29 to 31, p. 151.

Example. For the mixture ratio $m = 0.01$ or for $z = 100$ there follows, from equations (42) and (42*a*),

$$pv^{1.0525} = 615.84 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42b)$$
$$[pv^{1.0515} = 449.27].$$

The constant B_m of the equation of condition of the mixture is found from equation (38) to be

$$B_m = \frac{\epsilon + m}{1 + m} B_s = 2.1659\ [0.75887],$$

provided the pressure p is given in millimeters [inches] of mercury.

If the initial pressure of the mixture is $p_1 = 760$ mm. [29.922 in.] and the initial temperature is taken $t_1 = 20°$ [58°], then we can compute from the equation of condition $p_1 v_1 = B_m T_1$, the specific volume of the mixture of the beginning, namely, $v_1 = 0.8350$ [13.3758]. We will now consider the following changes of state.

Case 1. The mixture is cooled at constant volume till point 1 (Fig. 32) of the limit curve is reached, i.e., till the steam present has passed into the saturated condition.

If we substitute in equation (42*b*) $v = v_1 = 0.8350$, then the corresponding pressure is $p = 744.54$ mm. [29.313 in.], and we get, from the equation of

condition $pv = B_m T$, the temperature $T = 287.04°$ [516.676°], or by Celsius $t = 14.04°$ [57.276°]. The steam pressure at the instant, according to equation (37), is

$$p'' = \frac{m}{\varepsilon + m} p = 11.78 \text{ mm. } [0.46378],$$

in excellent agreement with Regnault's experiments on the relation between temperature and pressure of saturated steam; these experiments give $p'' = 11.91$ mm. [0.46890 in.] for $t = 14°$ [57.2°].

Case 2. The mixture is compressed at constant pressure and cooled till point 2 (Fig. 32) of the limit curve is crossed.

If we here substitute $p = p_1 = 760$ [29.922] in equation (42*b*), there follows for the point of intersection $v = 0.8188$ [13.1163], and from the equation of condition we get the temperature $T = 287.33°$ [517.194°] or $t = 14.33°$ [57.794°]; according to equation (37) the corresponding steam pressure is $p'' = 12.02$ mm. [0.47323 in.].

Case 3. The mixture is compressed at constant temperature T_1 up to the limit curve.

Here the combination of equation (42*b*) with the equation of condition, gives for the intersection 3 (Fig. 32) the specific volume $v = 0.5759$ [9.2253] and the pressure $p = 1102$ mm. [43.347 in.]. The corresponding steam pressure according to equation (37) is $p'' = 17.43$ mm. [0.68623 in.]. (According to Regnault we have, corresponding to the temperature 20° [68°], the steam pressure 17.39 mm. [0.68465 in.].) In all three cases further compression or cooling will bring about a condensation of the steam.

Under the hypothesis that in the initial condition ($t_1 = 20°$ [68°], $p_1 = 760$ mm. [29.922 in.]) of the mixture the air is just saturated with vapor, we find from the tabulation on p. 340 that the mixture ratio $m_0 = 0.01457$. The ratio $m : m_0$ is therefore for the assumed mixture equal to 0.686 and the relative moisture amounts to 68.6%. (More exactly, we have, according to equation (29), p. 334, for $x = 1$, the mixture ratio $m_0 = 0.01446$, for this equation is not based on the approximate assumption that the steam is subject to the equation of condition of gases.)

As regards the heat equations for mixtures of air and unsaturated steam, the propositions hold which were given in Vol. I, p. 171, § 34.

Let us designate the two values of the specific heat for air by c_p' and c_v', for steam by c_p'' and c_v'', and for the mixture by c_p and c_v, then according to the formulas (13), (14), and (15), developed in Vol. I, p. 172, we have

$$c_p = \frac{c_p' + m c_p''}{1 + m}, \quad \ldots \ldots \quad (43)$$

$$c_v = \frac{c_v' + mc_v''}{1+m}, \quad \ldots\ldots\ldots \quad (44)$$

and the ratio of the two values

$$\kappa = \frac{c_p}{c_v} = \frac{c_p' + mc_p''}{c_v' + mc_v''}, \quad \ldots\ldots\ldots \quad (45)$$

where m, as above, means the mixture ratio.

Here we must substitute $c_p' = 0.2375$, $c_v' = 0.1685$, $c_p'' = 0.4805$, and $c_v'' = c_p - AB_2 = 0.3695$.

For the unit of weight of mixture we then have for the change of inner work

$$AdU = c_v dT, \quad \ldots\ldots\ldots \quad (46)$$

and for the heat imparted (see Vol. I, p. 134)

$$\left.\begin{aligned} dQ &= \frac{A}{\kappa - 1}(vdp + \kappa pdv) \\ &= c_v dT + AB_m T\frac{dv}{v} \\ &= c_p dT - AB_m T\frac{dp}{p}. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (47)$$

With unsaturated as with saturated air, of ordinary atmospheric temperature, the mixture ratio m is so small that, according to equation (45), we obtain values of κ which hardly differ from the value $\kappa = 1.410$ for dry air; in such a case we can therefore unhesitatingly utilize this value for moist air.

For the calculation of the entropy, equations (55), Vol. I, p. 135, will serve.

For the important case of adiabatic change of state the first of equations (47) will give the equation of the adiabatic curve:

$$pv^\kappa = p_1 v_1^\kappa, \quad \ldots\ldots\ldots \quad (48)$$

and the work L is determined from equation (46) by

$$AL = c_v(T - T_1). \quad \ldots\ldots\ldots \quad (49)$$

Naturally these equations also hold for the region beyond the limit curve. In adiabatic expansion or compression we can easily determine the point of intersection T_0 (Fig. 32, p. 341) with the limit curve.

Let the initial state be represented by p_1 and v_1, and calculate from it $C = p_1 v_1^{\kappa}$; let p_0 and v_0 be the coordinates of the point of intersection, then one curve gives

$$p_0 v_0^{\kappa} = C,$$

and the other curve, according to equation (42), gives

$$p_0 v_0^{\nu} = D_m, \quad \ldots \ldots \quad (50)$$

and from them we determine v_0 from the relation

$$v_0^{\kappa - \nu} = \frac{C}{D_m}. \quad \ldots \ldots \quad (51)$$

Then p_0 follows from equation (50), and the corresponding temperature T_0 from the equation $p_0 v_0 = B_m T_0$.

Example. A mixture of atmospheric air and unsaturated steam has the temperature $t_1 = 20°$ [58°], and the pressure is $p_1 = 760$ mm. [29.922 in.], the mixture ratio is $m = 0.01$.

Here we get, as in the example on p. 344, $B_m = 2.1659$ [0.75887], $v_1 = 0.8350$ [13.3758], and the constant of the limit curve $D_m = 615.84$ [449.272]. On the other hand we get for the constant of the adiabatic curve

$$C = p_1 v_1^{\kappa} = 589.38 \text{ [1149.4]},$$

and according to equation (44) $c_v = 0.1705$. From equation (45) $\kappa = 1.407$, and therefore it is very nearly equal to the value $\kappa = 1.410$.

For the point of intersection with the limit curve we get from equations (51) and (50)

$$v_0 = 0.8844 \quad \text{and} \quad p_0 = 700.85 \text{ mm.}$$
$$[v_0 = 14.167 \quad \text{and} \quad p_0 = 27.593 \text{ in.}],$$

and from the equation of condition the corresponding temperature,

$$T_0 = 286.17° \quad \text{or} \quad t_0 = 13.17°$$
$$[T_0 = 515.106° \quad \text{or} \quad t_0 = 55.706°].$$

The corresponding steam pressure is found according to equation (37):

$$p'' = \frac{m}{\varepsilon + m} p_0 = 11.09 \text{ mm. } [= 0.43662 \text{ in.}].$$

The work of expansion L is given by equation (49):

$$L = 566.05 \text{ mkg. } [4094.3 \text{ ft-lb.}].$$

If the expansion is continued further, the condensation of steam takes place.

Assume, as before, a unit of weight of air and saturated steam; suppose given the mixture ratio m, the pressure p_1, the temperature T_1, and also the specific volume v_1; suppose the course of the corresponding limit curve DD (Fig. 33) to be determined by equations (42) and (42*a*), then, with prolonged adiabatic expansion, the limit curve is crossed at the point T_0, and condensation of steam takes place along the path T_0T_2 till the freezing point is reached at $T_2 = 273°$ [491.4° F.].

With a further prolongation of the adiabatic expansion along the path T_0T_2 (Fig. 33), the temperature remains constant, till, at the point T_2 possessing the coordinates p_3 and v_3, there are finally present only air, steam, and ice (without water).

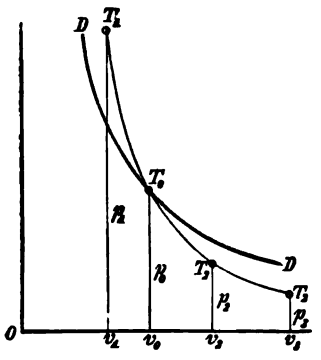

FIG. 33.

The occurrences during the expansion of the volume from v_1 to v_0 were just investigated; on the further occurrences during the expansion from v_0 to v_2 light is shed by the formulas (31) and (32), pp. 334 and 335, in § 43; in these formulas we have only to interchange subscripts 1 and 0 and to substitute $x_1 = 1$.

If we treat the average specific heat c of the water as constant, and also $c_v(\kappa - 1) = AB$, there follows from equation (31) for the present case

$$0 = (c_v + mc) \log_e \frac{T}{T_0} + AB_1 \log_e \frac{xu}{u_0} + m\left(\frac{xr}{T} - \frac{r_0}{T_0}\right). \quad . \quad (52)$$

According to this equation we can calculate for any arbitrarily chosen temperature T lying between T_0 and T_2 the steam quality x; then the corresponding specific volume $v = xu$, and, according

to equation (29a), the air pressure at the instant is

$$p' = \frac{B_1 T}{mv}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

The total pressure is $p = p' + p''$, when p'' represents the steam pressure corresponding to the temperature.

Equation (52) is valid only up to the point (p_2, v_2), Fig. 33, at which the temperature $T_2 = 273°$ [491.4° F.]. Let the values corresponding to this temperature be taken from the steam table and be marked by the subscript 2, then there will follow, according to equation (52),

$$0 = (c_v + mc) \log_e \frac{T_2}{T_0} + AB_1 \log_e \frac{x_2 u_2}{u_0} + m\left(\frac{x_2 r_2}{T_2} + \frac{r_0}{T_0}\right). \quad (52a)$$

From this we can find the steam quality x_2 at the freezing-point, and then, in the manner indicated, the corresponding volume v_2 and the pressure p_2.

The work of expansion along the path $T_0 T_2$ for the increase of volume from v_0 to v_2 results from equation (32), p. 335, and is

$$AL = G_1[c_v(T_0 - T_2) + m(q_0 + \rho_0 - x_2\rho_2)]. \quad . \quad . \quad . \quad (54)$$

With continued adiabatic expansion from the volume v_2 to the volume v_3 (Fig. 33) under constant temperature $T_2 = 273°$ [491.4°] ice formation takes place

For this occurrence equation (26), p. 330, is valid, and, because there is here an adiabatic change, we have $P = P_1$; moreover, because no ice is present at the beginning, we must substitute $y_1 = 0$. Therefore for the present case, and for any point whatever of the change of state, we get

$$0 = AB_1 \log_e \frac{x}{x_2} + m\left[\frac{r_2}{T_2}(x - x_2) - \frac{r_e}{T_1} y,\right.$$

where x is the steam quality, y the specific ice weight, and $(1 - x - y)$ the specific water weight.

For the point (p_3, v_3), Fig. 33, in which all water is converted into ice, and, because $(1-x-y)=0$, the preceding equation gives

$$0=AB_1 \log_e \frac{x_3}{x_2}+\frac{m}{T_2}[(r_2+r_e)x_3-r_2x_2-r_e], \quad . \quad . \quad (55)$$

where x_3 represents the steam quality at the end.

From this we can compute x_3 and the specific ice weight $(1-x_3)$ at the end; in so doing we must substitute

$T_2=273$, $r_2=606.5$, $r_e=79$, and $AB_1=0.0690$
[$T_2=4914.4$, $r_2=1091.7$, $r_e=142.2$, and $AB_1=0.0690$].

If we continue the expansion still farther, then the steam will be precipitated as snow, an occurrence which will be explained by the following investigations.

§ 45. BEHAVIOR OF THE MIXTURE OF AIR, STEAM, AND ICE.

Let us first assume a unit of weight of pure ice surrounded by an elastic envelope which exerts upon it a pressure identical with the steam pressure, corresponding to the melting temperature; furthermore let T_1 be the temperature of the ice and σ_e its specific volume, then the heat quantity which must be imparted to the ice in order to bring it to the melting temperature is

$$q_e=c_e(T-T_1),$$

where c_e represents the specific heat of the ice, which we will take to be $c_e=0.5$ and constant.

Now, in order to convert the ice under constant pressure completely into water of the same temperature, there is necessary the heat of fusion $r_e=79$ Cal. [142.4 B.t.u.]; the total heat λ_e to be imparted is therefore

$$\lambda_e=c_e(T-T_1)+r_e.$$

But during the conversion of ice into water a change of volume $\sigma-\sigma_e$ occurs, where, as before, σ represents the specific volume of

the mixture; consequently the heat quantity converted into work is

$$AL = Ap(\sigma - \sigma_0),$$

and this we will designate as the external latent heat of fusion; from this follows

$$\rho_0 = r_0 - Ap(\sigma - \sigma_0),$$

which we will call the inner latent heat of fusion.

Now it is well known that the specific volume σ_0 of the ice is greater than that of the water, and, accordingly, contraction takes place during melting. The observations show that at the atmospheric melting-point of 0° [32°] we have $\sigma_0 = 0.001087$ [0.017413] and $\sigma = 0.001$ [0.016]; accordingly the external heat of water is negative, but it is of such slight magnitude that we can assume the inner latent heat of fusion ρ_0 as identical with the latent heat r_0

Let the water produced be completely converted into steam under the constant pressure p; the necessary heat quantity as before is r, and the heat quantity which is converted into work is Apu; the corresponding inner latent heat is $\rho = r - Apu$.

We now obtain for the steam heat, or the heat contained in a unit of weight of steam over and above that in a kilogram [pound] of ice of the temperature T_1,

$$J = c_0(T - T_1) + \rho_0 + \rho.$$

At a certain instant of the evaporation of the ice, let the steam quality be x and $1 - x$ be the specific ice weight, then the excess of heat in the mixture of the ice and steam over the heat contained in a unit of weight of ice is

$$A(U - U_1) = [c_0(T - T_1) + \rho_0 + \rho]x + c_0(T - T_1)(1 - x),$$

or

$$A(U - U_1) = c_0(T - T_1) + (\rho_0 + \rho)x,$$

and therefore

$$AdU = c_0 dT + d[(\rho_0 + \rho)x]. \quad . \quad . \quad . \quad . \quad . \quad (56)$$

This equation takes the place of equation (41), p. 54. We then get, for any reversible change of state of any mixture of ice and

steam, for the heat imparted

$$dQ = c_e dT + d[(\rho_e + \rho)x] + Apdv. \quad . \quad . \quad . \quad . \quad (57)$$

According to the development upon p. 29, it is evident that, for the present case, Clapeyron's equation takes the form

$$\frac{r + r_e}{u} = AT\frac{dp}{dt},$$

and that, therefore, the preceding heat equation, when transformed in the way given on p. 58 and p. 60, becomes

$$dQ = c_e dT + Td\left[\frac{x(r + r_e)}{T}\right]. \quad . \quad . \quad . \quad . \quad . \quad (58)$$

The combination of this equation with equation (56), in conjunction with the integration of the expression

$$AdL = dQ - AdU,$$

gives the corresponding work L.

The preceding formulas enable us to follow any change of state of a mixture of ice and steam in the same way as was illustrated above for the mixture of water and steam by a whole series of examples.

We can now examine the behavior of a mixture of air, steam, and ice.

Let G_1 be the weight of the air present, G_2 that of the ice and steam, then we get, in the same way as that followed in establishing equation (10), p. 322, and taking account of the relation $V = G_2 xu$,

$$dQ = c_v G_1\left[dT + (\kappa - 1)T\frac{d(xu)}{xu}\right] + G_2\left[c_e dT + Td\left(\frac{x(r + r_e)}{T}\right)\right], \quad (59)$$

or we get the change of entropy $A(P - P_1)$ if we designate the mixture ratio $G_2 : G_1$ by m:

$$\frac{A(P-P_1)}{G_1} = (c_\varepsilon + mc_\varepsilon)\log_\varepsilon \frac{T}{T_1} + AB_1 \log_\varepsilon \frac{xu}{x_1 u_1} + m\left[\frac{x(r+r_\varepsilon)}{T} - \frac{x_1(r_1+r_\varepsilon)}{T}\right]. \quad (60)$$

For the ordinary case of adiabatic expansion the left member of this equation becomes 0; we can then, for any arbitrary temperature T, compute the steam quality x, and therefore, by means of $G_1(x-x_1)$, the steam weight which has been converted into snow or ice, provided we know the initial quality x_1 and the initial temperature T_1. The volume and pressure of the mixture are easily found in the manner given above.

By the presentations in the preceding articles we have considered all the possible cases of investigation; the transcendent form of all equations of course renders the special investigations extremely difficult; but the equations permit of many transformations, and for meteorological purposes, which are, however, foreign to the purpose of this work, may be replaced by simple approximate expressions.

b) Mixtures of Different Kinds of Vapors.

§ 46. GENERAL REMARKS ON MIXTURES OF DIFFERENT KINDS OF LIQUIDS AND THEIR VAPORS.

In attempting to examine more closely the behavior of mixtures of liquids and of their resulting vapors, we encounter difficulties which up to this time have not been overcome; this is due in part to the lack of experimental investigations bearing on the special questions that are to be answered, and in part to the fact that thermodynamics has thus far not been so applied to chemical actions as to be suitable for technical use. To be sure there are some penetrating investigations; the older classical works of Kirchhoff have been followed by investigations by von Helmholtz, Duhem, and others, which were inspired by the theoretical discussions of Horstmann and Gibbs, and these have led to considerable results and permit us to

hope that the problems involved here are approaching solution; just the problems that are important in technical circles, and which we will soon specify more particularly, are the very ones which have been furthered least, so that at the present time it is not possible to represent the cycle of certain machines in a theoretically satisfactory manner. Carré's ice machine (absorption machine) belongs here, and also Honigmann's machine, which works with a mixture of water and soda-lye.

In the following it will only be a question of reporting the existing experimental results on liquid mixtures, stating the conclusions that can be drawn therefrom and thus indicating the bases necessary for the solution of particular problems.

Let us first suppose that two liquids A and B are brought together and mixed; let G_1 kg. [lb.] of the one and G_2 kg. [lb.] of the other be given and let their mixture ratio $G_1 : G_2$ be expressed by y; now if a part of the mixture is converted into steam with an overcoming of the external pressure, then the questions arise, what heat quantity must herewith be supplied, what is the pressure of the steam mixture at the given temperature, what mixture ratio obtains at this particular instant in the remaining liquid, what mixture ratio exists in the steam mixture present, and finally what is the quality of the steam of each of the several liquids, that is, whether each steam, at the temperature under consideration, is in the saturated state or not?

Another series of questions relates to the mixing process proper. If two substances of the same temperature are at first separate, both in the liquid condition or both in the state of vapor, or one of them in a liquid condition and the other in the condition of vapor, and if they are now brought together under prescribed changes of pressure or of volume, then the question arises as to the state of the two bodies at the end of the mixing; the question particularly arises as to the changes of temperature which thus occur, or as to the heat quantities which must be withdrawn or supplied after the mixing in order to restore the initial temperature, Only a few of the questions indicated have been thus far subjected to theoretical and experimental investigation; for a series of mixtures the relation between temperature

and pressure of the mixed vapors has been fixed by experiment.

It was formerly assumed that the pressure of the steam mixture standing over a mixture of two liquids was the sum of the saturation pressures belonging to the two kinds of vapor at the corresponding temperature; thus if p' was the saturation pressure of the one steam, at the temperature t, provided that the steam existed alone, and if p'' was the pressure of the other steam, then the pressure p of the mixture was simply taken as $p=p'+p''$; it was assumed, therefore, that the two kinds of vapors were present in the saturated state after their mixture.

But Magnus[1] was the first to show that this assumption is confirmed by observation only when the two liquids mingle like oil and water, i.e., when they do not mix; on the other hand that with mixable liquids there always obtains the relation $p<(p'+p'')$ and consequently that for the temperatures under consideration the pressure of the steam mixture cannot be directly determined from the saturation pressures of the several kinds of vapor. It follows from this that of the two kinds of vapors one of them, and probably both, exist in the superheated condition.

Later Regnault took up the question in a thorough fashion and confirmed the proposition that in the commingling of two liquids which do not mix the pressure of the mixed vapor is equal to the sum of the saturation pressures of the separate vapors. The experiments in question[2] related to a commingling of water and bisulphide of carbon, of water and chloride of carbon, and also of water and benzine. Regnault designates such mixtures as mixtures of the first type, but distinguishes two other kinds; one of them includes those cases in which a liquid mixes with only a limited quantity of the other, as water and ether; the other, which Regnault characterizes as mixtures of the third type, are mixtures which can be effected for all ratios of the constituent liquids. Those occurring most often are of the third type. For

[1] Magnus, "Über das Sieden von Gemengen zweier Flüssigkeiten." Poggend. Ann., Vol. 38, 1836, p. 481.

[2] Rel. II. 720 and 721.

such Regnault instituted experiments for different mixture ratios of ether and bisulphide of carbon, of chloride of carbon and bisulphide of carbon, and of alcohol and benzine.[1] Further experiments were conducted by Wüllner[2] and were with mixtures of water and alcohol and also of ether and alcohol.

The steam pressure p of the mixture is here dependent upon the temperature t and the mixture ratio y and is always smaller than the sum of the saturation pressures p' and p'' of the separate vapors for the same temperature; the ratio

$$\mu = \frac{p}{p' + p''}$$

therefore appears as a proper fraction. If the mixture consists of equal parts (by weight) of the two liquids, i.e., if $y = 1$, then it is found that μ, for different temperatures, is a constant quantity; according to Wüllner on the average $\mu = 0.596$ with water and alcohol, while with ether and alcohol $\mu = 0.630$, and according to Regnault with ether and bisulphide of carbon $\mu = 0.590$. With other mixture ratios the value of μ is greater or smaller and also changes slowly with the temperature; the value μ is larger when the more volatile liquid predominates in the mixture, and it decreases with the temperature; the opposite takes place when, measured by weight, there is less of the volatile element in the mixture than of the other.

The existing experiments are, however, inadequate for the perception of the underlying laws and for drawing further conclusions from the experimental results, and are particularly insufficient for determining the partial pressures of these kinds of vapors in the mixture and for deducing from them their degree of superheat. More important conclusions would be given by experiments furnishing the pressure curve which exists when a particular mixture is converted into vapor at constant temperature. For a simple liquid the vapor pressure is constant as long as liquid is

[1] Rel II, 724 to 728.

[2] Wüllner, "Über die Spannkraft der Dämpfe von Flüssigkeitsgemischen." Poggend. Ann., Vol. 129, 1866, p. 353.

on hand, but it is not so with mixtures of liquids. If here y is the mixture ratio of two liquids before steam formation, then after a portion is converted into a vapor the mixture ratio of the liquid remaining behind is different, i.e., will be y'; on the other hand that of the steam mixture will be y''; both values will be different from and dependent on the total ratio y, and will depend upon the temperature t and the total volume of the unit of weight of the mixture of liquid and vapor.

§ 47. RECENT RESEARCHES ON THE BEHAVIOR OF "SATURATED AND SUPERHEATED STEAM."

The investigations of the present second volume are based exclusively on the grand and wonderful experiments recorded by V. Regnault in his great work, "Relation des expériences, pour déterminer les principales lois et les données numériques qui entrent dans le calcul des 'machines à vapeur.'"

The experiments began in 1840; the book appeared in Paris in 1847 as the first of a series of three volumes (Vol. II in 1862, Vol. III in 1870), the latter volumes being devoted to gases and to other vapors than those of steam, and intended to determine their physical and thermal properties.

For the development of thermodynamics the experimental results of the first volume were of the deepest influence, for from them there were obtained the first reliable data concerning the behavior of "saturated" steam. In all the years that have elapsed since the appearance of Regnault's works but few experiments in the same direction by others have become known, and these have mostly shown slight differences, and where greater differences did appear they did not cast doubt on Regnault's statements.

Finally, and quite recently, several works have become known which will now be discussed more in detail on account of their importance; the investigations of Battelli and Tumlirz have already been mentioned on pp. 48 and 50.

Of the experiments to be discussed we will first call attention to the communications from the laboratory for Technical Physics

belonging to the Royal Technical High School of Munich entitled "Die thermischen Eigenschaften des gesättigten und überhitzen Wasserdampfes zwischen 100° and 180° C.," whose first part contains the report on the density of steam by Osk. Knoblauch, Rich. Linde, and H. Klebe.[1]

The second part, possessing the same title, embraces the special work of Rich. Linde, which, at the same time, gives the theoretical presentation with reference to the older investigations and institutes a comparison with them.

Specific Volume of Vapors.

The author of the present work was the first to calculate the specific volume of "saturated" steam, in the Zivilingenieur, Vol. V, 1859; this was on the basis of Regnault's experiments and was calculated with the help of Clapeyron's equation

$$\frac{r}{u}=AT\frac{dp}{dT},$$

where $u=s-\sigma$ represents the difference between the specific volume s of the saturated steam and $\sigma=0.0010$ [0.016] of the liquid water, and r represents the heat of evaporation belonging to the temperature T according to Regnault's experimental determinations. The article mentioned is the forerunner of the more sharply determined values of s in the preceding editions of this book, in which determinations account has been taken of the more exact relations between the pressure p and the temperature T the values of s in question have been very generally employed up to the present time. The preceding formula is, of course, only valid for saturated vapor.

The experiments made in Munich constitute a decided step forward, for within certain limits they were extended to "superheated" vapors. In these experiments the specific volume s is not determined by calculation, but is found directly, just as was attempted by Battelli and others, though on a smaller scale.

[1] Mitteilungen über Forschungsarbeiten aus dem Gebiete des Ingenieur wesens, published by the Verein deutscher Ingenieure. Berlin, 1905.

The experimental apparatus in question was conceived and constructed by Prof. C. v. Linde, and was placed at the disposal of said experimenters in Munich. In an iron pot of 320 mm. [12.60 in.] inner diameter and of the same height there was inserted a glass globe containing an accurately weighed quantity ofwater; steam was admitted to the pot from a gas-heated steam boiler, and this steam flowed around the glass globe and warmed it. In this way the water introduced was evaporated till finally only superheated steam was present, whose temperature was determined in a suitable fashion, as was also the temperature of the heating steam.

Conversely, if the globe is filled with superheated steam, we can, by a gradual diminution of the pressure of the heating steam, reduce the temperature in the globe by stages, and thus obtain a series of separate values of the temperature and pressure of the superheated steam at constant volume, for the capacity of the globe remains unchanged. As the weight of the water supplied and the cubic capacity of the globe are given, there is easily determined the volume of a unit of weight of the steam, i.e., its specific volume. The various inserted glass globes (balloons) had moreover capacities of 1.7 to 3 l. [104 to 183 cu. in.], and the water supplied in the different experiments varied between 15 and 12 g. [0.0331 and 0.0265 lb.].

In all, 32 series of experiments were made for 32 values of the specific volume s of saturated steam, for which we will substitute v; these values lie between 1.598 and 0.1817 cbm. per kg. [25.598 and 2.884 cu. ft. per lb.]. If we lay off the temperature t as abscissa and the pressure p as ordinate, we get, for the specific volume of saturated steam considered, the corresponding point of the saturation curve. If, on the basis of these experiments, we lay off as coordinates the temperature and pressure of the superheated steam, corresponding to a particular volume v, a curve will branch off from the point in the saturation curve, which curve has been designated as an isochore, and with steam it has been found to run rectilinearly; at the higher pressures it is more strongly inclined to the axis of abscissas than at the lower pressures, the pressure p increasing proportionally to the temperature T.

With the ideal gas, obeying $pv=BT$, we have, for a constant value of v and $p=0$, the isochore passing at $t=-273°$ [$-459.4°$] through the temperature curve, but it is not so with the vapor of water; here within the range of the experiment the isochores are also represented by straight lines, but their prolongation does not pass through the same point, $-273°$ ($-459.4°$), of the temperature curve.

Another result of the experiments in Munich is of special importance, namely, that close up to the saturation limit the superheated steam agrees with the law for the perfect gas in showing proportionality between pressure and temperature, thus confirming what was stated by Thiesen, that the vicinity of condensation does not cause any specially marked deviation from the law of gases. To be sure, according to Battelli's experiments, more marked deviations do seem to exist, but there is reason for assuming that the differences are so insignificant that they are not directly appreciable in the experiments.

From the experiments discussed here in such a general fashion, Rich. Linde, in a very clear and clever manner, has drawn a series of theoretical consequences which we will now briefly examine.

Equation of Condition.

The different values of the specific volume of steam at different temperatures and different pressures furnish us the means of testing the various equations of condition that are known and of introducing suitable corrections in them.

In the first place R. Linde discusses the author's equation of condition for saturated steam, which up to the present time has been the one most often used in technical calculations. We have already discussed, on p. 208, various equations of condition for steam, and have emphasized those forms proposed for technical purposes. On p. 224, in particular, emphasis was laid on the requirements which these approximate equations must fill, and it was then shown that the equation

$$pv=BT-Cp^{\frac{1}{4}}, \quad \ldots\ldots\ldots \quad (1)$$

with B and C as constant quantities, satisfied these requirements very well. In so doing we made $B=50.933$ [0.644592] and $C=192.50$ [22.581875], when p is expressed in kg. per sq. m. [lb. per sq. in.] (p. 237).

The general form of the equation is given as

$$pv=BT-R, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where R is a function of p and v or T; but it is shown that this function, for the present, can be represented by the supplementary term

$$R=Cp^{\frac{1}{2}};$$

now R. Linde finds that the assumption that R is only a function of p is not fully confirmed, which of course could be foreseen.

According to the experiments in Munich we should write

$$pv=BT-p(1+ap)\left[C\left(\frac{373}{T}\right)^3-D\right], \quad . \quad . \quad . \quad (3)$$

which formula gives values for the specific volume v which agree very well with the experiments.

In using it we must take p in kg. per sq. m. [lb. per sq. in.], v in cbm. per kg. [cu. ft. per lb.], and for the constants

$$B=47.10; \quad a=0.000002; \quad C=0.031; \quad D=0.0052$$
$$[B=0.5962; \quad a=0.001406; \quad C=21.7955; \quad D=3.6560].$$

For most practical applications, however, the form

$$pv=BT-Cp. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

will suffice, with $B=47.10$ [0.5962] and $C=0.016$ [0.25633]. The form of this equation agrees with that given by Tumlirz (see p. 241).

The following numerical table gives an insight into the results of the experiments in Munich and a comparison with my numerical results according to Clapeyron's formula.

1.	2.	3.	Superheated Steam. v			
Temperature $t°$ C.	Saturated Steam according to the Clapeyron Equation Computed. v	Saturated Steam according to the Munich Experiments. v	$p=$ 2 kg./sq. cm.	4 kg.	6 kg.	8 kg.
100°	1.6508[1]	1.674				
105	1.3989	1.420				
110	1.1914	1.211				
115	1.0195	1.037				
120	0.8763	0.8922	0.9033			
125	0.7566	0.7707	0.9162			
130	0.6559	0.6690	0.9288			
135	0.5709	0.5822	0.9416			
140	0.4987	0.5091	0.9543			
145	0.4373	0.4466	0.9669	0.4741		
150	0.3849	0.3921	0.9794	0.4807		
155	0.3398	0.3470	0.9920	0.4875		
160	0.3011	0.3073	1.0045	0.4941	0.3236	
165	0.2675	0.2729	1.0171	0.5007	0.3283	
170	0.2385	0.2430	1.0294	0.5073	0.3329	0.2454
175	0.2132	0.2170	1.0418	0.5138	0.3375	0.2490
180	0.1911	0.1943	1.0542	0.5203	0.3421	0.2527

The values in Col. 3 are calculated from the preceding formula (3); we now clearly see the differences from the values formerly calculated by me according to Clapeyron's formula. The latter values are throughout somewhat smaller, but otherwise have a perfectly similar course with growing temperature.

If we write Clapeyron's formula, from which Col. 2 was calculated, in the form

$$v=u+\sigma=\frac{r}{AT\frac{dp}{dt}}+0.0010, \quad \ldots\ldots \quad (5)$$

we see that the value of v depends above all upon the magnitude of the heat of evaporation r and on the value of the differential coefficient, and these values have been determined by Regnault

[1] The values of Col. 2 are taken from Table 1b, Col. 8, of the Appendix of this book and were determined from

$$v=u+\sigma \quad \text{with} \quad \sigma=0.0010.$$

and have been used here. But the value of A, the heat equivalent of the unit of work or the reciprocal of the mechanical equivalent of heat, here plays a part, and for this value we have taken the number 424 throughout the present work. Recently and on various sides it has frequently been said that at least 427 mkg. ought to be substituted for 424; this would make the values of v in Col. 2 of the preceding table approach more closely to the Munich values of Col. 3, particularly as the Munich observers maintain that the values of $\frac{dp}{dt}$, derived by me from Regnault's pressure experiments, are taken somewhat too large. If in equation (5) we also take account of the influence of r we see that with correct choice of the quantities mentioned the results of equation (5) could readily be brought into harmony with the Munich data, assuming that the latter are perfectly reliable.

It is not suitable here, however, to effect an agreement by trial, for we do not know the degree in which the separate values contributed to the differences; here the matter can only be cleared up by experiments determining their correct values. On the whole, the comparison of Cols. 2 and 3 of the preceding table shows an agreement which again makes Regnault's experiments seem wonderful.

R. Linde finally uses Clapeyron's equation (5) in order to inversely determine from it the heat of evaporation r by employing the Munich values v given in Col. 3 of the preceding tabulation. In general the results thus found for r are somewhat greater than those of Table 1b of the Appendix, which was to be expected from the comparison of the values of v in Cols. 2 and 3, but nothing else remarkable was developed. It would be better to try to directly determine r experimentally, as was done by Dieterici,[1] who found that $r=596.8$ [1074.24] for 0° C. [32° F.], while the constants of Regnault's formula were derived from experiments lying between 63° [145.4°] and 185° [365°]. In any case Clapeyron's formula (5) does not seem

[1] Wiedemann's Annalen, Vol. 37, 1889, p. 494.

suited for drawing conclusions, in the way indicated, as to the reliable values of r.

Specific Heat of Superheated Steam.

The investigation of the value of the specific heat c of steam under constant pressure, which we are especially keeping in view, has recently occupied a whole series of writers; R. Linde also goes fully into the question.

Throughout the present book we have held fast to the value $c_p=0.4805$ given by Regnault, and the value has been regarded as constant, judgment being suspended as to whether it is not variable with the temperature as in gases. The use of high pressure and superheated steam at high temperature in engineering led to changed views in this matter, and the first suggestion in this direction was probably due to the results found by Bach[1] when he tried to determine the heat of generation of superheated steam by condensation in two spiral coils washed by cooling water. Bach concluded that, under the conditions existing in his experiments, the specific heat of the steam must be greater than that given by Regnault, and that about $c_p=0.6$ ought to be taken. In the meantime a large number of investigations, inspired by theoretical considerations, have become known, whose main propositions may be grouped as follows:

(*a*) Regnault's mean value $c_p=0.4805$ is only valid for steam under atmospheric pressure, at which his experiments were made.

(*b*) The specific heat c_p is a function of pressure and temperature.

(*c*) The value c_p grows with the pressure and diminishes with increasing temperature.

These propositions are all confirmed by the more recent investigations, but the separate data differ so much among themselves that it is difficult to choose between them.

Most recently H. Lorenz has conducted special experiments on the specific heat of steam, with the help of the Verein

[1] Bach, Zur Frage des Wärmewertes des überhitzen Wasserdampfes. Zeitschr. des Ver. deutscher Ingenieure, 1902, Vol. 46, p. 729.

deutscher Ingenieure, which are reported in the Zeitschrift des Vereins, 1904, Vol. 48, p. 700. The experimental results are as follows:

	1.	2.	3.	4.	5.	6.
Steam temperature $t°$ C. . . .	207.6°	279.2°	321.0°	222.4°	262.0°	313.8°
Absolute steam pressure p kg./sq. cm.	6.82	6.82	6.95	8.97	8.98	8.98
Specific heat c_p observed. . . .	0.622	0.568	0.555	0.698	0.640	0.570
Specific heat calculated according to equation (6). . .	0.657	0.575	0.547	0.696	0.640	0.590

	7.	8.	9.	10.	11.
Steam temperature $t°$ C. . . .	205.4°	295.0°	338.9°	207.6°	294.0°
Absolute steam pressure p kg./sq. cm.	4.31	4.05	3.94	1.93	1.89
Specific heat c_p observed. . . .	0.583	0.484	0.478	0.531	0.492
Specific heat calculated according to equation (6). . .	0.572	0.510	0.492	0.493	0.467

According to Lorenz the values of c_p can be satisfactorily represented by the empirical equation

$$c_p = 0.43 + 3600000\frac{p}{T^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\left[c_p = 0.43 + 1476000\frac{p}{T^3}\right],$$

where the absolute pressure p is in kg. per sq. cm. [lb. per sq. in.] and T represents the absolute steam temperature.

It is evident from the foregoing that the specific heat of superheated steam does increase with the pressure; on the other hand, starting from the region of saturation, it diminishes with increase of temperature.

For low pressures the values decidedly approach the value 0.4805, found by Regnault for atmospheric pressure between 108° [226.4°] and 217° C. [422.6° F.], while for the higher pressures they satisfactorily confirm Bach's assumption of 0.60.

On the Liquid Heat of Water according to Dieterici's Investigations.

Attention was called on p. 25 to the way in which Regnault found the heat of the liquid in the case of water and how he obtained its specific heat, but at the same time it was mentioned that doubts had been expressed by various persons as to the reliability of the several observations. These doubts do not seem to be well justified for investigations occurring within the temperature limits of steam engineering; at least, the results of the recent experiments by Dieterici in this field are worthy of attention, for he succeeded in accurately determining the heat of liquid water up to 300° C. [572° F.]. A carefully weighed mass of water was confined in a quartz tube which had been previously weighed and completely emptied of air, and after it was filled it was hermetically sealed and heated to any high temperature t. After the experimental tube and its contents had been thoroughly heated it was allowed to suddenly fall into a Bunsen ice calorimeter, and the heat quantity given off was observed. A parallel experiment with an empty quartz tube permits the heat to be determined which was absorbed by the vessel; the difference gives the heat quantity given off by the enclosed water when cooled from $t°$ to 0° [32°].

In so doing a further correction must be applied. The hollow space of the quartz tube must be somewhat greater than the volume of the enclosed liquid, otherwise the great liquid pressure would burst the experimental tube; so there is some saturated vapor constantly present with the liquid, and the total observed heat is composed of the heat of the liquid and of the heat carried by the steam; the latter part, however, as Dieterici shows,[1] can be neglected because it is so small.

The experiments were conducted with different quartz tubes having 1 to 2.5 ccm. [0.0610 to 0.1526 cu. in.] capacity with the

[1] C. Dieterici, "Die kalorischen Eigenschaften des Wassers und seines Dampfes bei hohen Temperaturen." Zeitschrift d. Ver. deutscher Ing., 1905. Die Energie-Isothermen des Wassers bei hohen Temperaturen. Annalen der Physik, Fourth Series Vol. 16, 1905.

different fillings and were constructed with different thicknesses of walls. Quartz was chosen because this material can endure such sudden jumps of temperature as occur when falling into the ice calorimeter; in the second place because the water at high temperature dissolves alkali from glass, which is not possible with quartz; and finally because quartz is the most homogeneous material that we possess.

Now Dieterici found that the specific heat c of the water between 40° and 300° [104° to 572°] is reproduced sufficiently well by the formula

$$c=0.9983-0.0001037\,t+0.000002073\,t^2$$
$$[c=0.9983-0.0000576(t-32)+0.00000064(t-32°)^2].$$

The heat of the liquid can therefore be calculated according to the formula

$$q=\int_0^t c\,dt=0.9983\,t-0.00005185\,t^2+0.000000691\,t^3$$

$$\left[q=\int_0^t c\,dt=0.9983(t-32)-0.0000288(t-32°)^2+0.000000213(t-32°)^3\right].$$

At low temperatures below 40° [104°] the dependence of the heat of the liquid of the water on temperature is an irregular one and cannot be represented by a simple formula, contrary to the representations of Regnault.

The preceding formulas give:

for $t=150°$	$q=150.9$ Cal.
$=200°$	$=203.1$
$=250°$	$=257.2$
$=300°$	$=313.5$

From Table 1b of the Appendix we have, according to Regnault,* $q=151.64$ [272.952] for $t=150°$ [302°] and $q=203.20$ [365.76] for $t=200°$ [392°]; for temperatures extending to 200° [392°] the values of this table may unhesitatingly be employed as heretofore.

APPLICATIONS.

TECHNICAL PART.

Theory of Steam Engines.

§ 48. PRELIMINARY REMARKS.

In steam as in air engines, we must distinguish between hot-vapor and cold-vapor engines. In the former, which we will call "steam engines," as is customary, the purpose is the production of work; the work produced is to be utilized to overcome resistance in the most widely different sorts of work processes. Steam engines, therefore, like the hot-air and internal-combustion engines, are motors or prime movers and must be counted among the heat engines. In the cycle described by steam engines heat is withdrawn by the mediating body (during a part of the process) from a body of higher temperature, the heating body (products of combustion), and during another part heat is given off by the mediating body to a body of lower temperature, to the cooling body in the condenser. The difference of the two heat quantities corresponds to the work produced.

The cold-vapor engines describe just the opposite cycle; in one part of it heat is withdrawn from a body of low temperature (a saline solution or pure water which is to be converted into ice), and in another part heat is given off to a body (water) of higher temperature.

The difference of the heat quantities here corresponds to the work necessary for running the machine.

The cold-vapor engine is therefore a driven machine for doing work; its purpose is to continuously withdraw heat from certain bodies—in short, its purpose is the production of cold.

The cold-vapor engine bears to the hot-vapor engine the same relation that the scoop wheel bears to the water wheel, the centrifugal pump to the turbine, the cylinder pump to the water-pressure engine; each of the prime movers mentioned can be converted into an operating machine for doing work by a reversal of its cycle. In the hydraulic motors mentioned here the water sinks from an upper to a lower level; in the operating machine the water is lifted from a lower level to a higher one. In both kinds of steam engines mentioned heat weight or entropy takes the place of the water weight, and the difference of temperature takes the place of the difference of level.

In the hot-vapor engine the external atmospheric temperature is theoretically the lower limit of temperature attainable, while in the cold-vapor engine the atmospheric temperature represents the upper limit. If a hot-vapor engine drives a cold-vapor engine, then, comparatively speaking, the circumstances are as if a pump plant were driven by an hydraulic motor, the water lifted by the pump being discharged at the level of the tail-race of the motor.

The repeated reference to the analogy existing between heat engines and hydraulic motors has, moreover, the sole purpose of facilitating the insight into the occurrences of the machines in question, and the analogy exists only when the heat engines work with a perfectly reversible cycle.

Certain other differences manifest themselves when it is a question of practically constructing hot-vapor and cold-vapor engines. In the former engines the mediating body is a liquid with its vapor, in which as high temperatures as possible occur, accompanied by pressures which are not too high. This requirement is best fulfilled by water with its steam, and it possesses besides the inestimable advantage that it can be had easily and free of cost; this permits the construction of open engines in which the steam is discharged during every cycle and is replaced by a new charge of the mediating body, the water.

On the other hand in the cold-vapor engine a liquid with its vapor is used, which already carries comparatively high steam pressure at low temperature, like carbonic acid, ammonia, sulphurous acid, etc.; these engines are always closed and describe the cycle continuously with the same fluid mass without replacement from the outside.

It is of special importance to note that the comparisons just instituted, and the fundamental equations, developed in the first section of Vol. I, are valid only under the hypothesis that no chemical changes occur during change of state of the mediating body. When investigating internal-combustion engines (Vol. I, p. 399, etc.) such deviating cycles did occur; in one part of the cycle described by the mediating body (an ignitible mixture of gas) a chemical change (combustion) occurred, and consequently the body could not be brought back again to the starting point, i.e., to the initial condition. Although in so doing the pressure and volume at the end of the cycle may be brought back to the initial values, nevertheless the rest of the suppositions which were made when the fundamental formulas were derived are not fulfilled, for in the chemical sense a different body exists at the end of the cycle than at the beginning; the propositions which were derived concerning cycles and their reversal here lose, in consequence, their general validity.

Non-reversible cycles, accompanied by simultaneously occurring changes of state, were subjected to investigation when treating internal-combustion engines (Vol. I); it was pointed out there that the thermodynamics of the present day is not yet able to present the process in question with perfect clearness, and that this is not solely due to lack of the necessary experimental bases.

A. STEAM ENGINES (HOT-VAPOR ENGINES).

a) Engines for Saturated Vapors.

§ 49. THE CARNOT CYCLE OF THE STEAM ENGINE.

In the following discussion we must distinguish whether the steam used in the engine is saturated, wet (as a mixture of liquid and vapor), or whether the steam is used in a superheated condition. We will now fully examine the first case; let us consider a piston steam engine which works with saturated vapor of water, which draws the steam from a steam boiler and, after the expansion is completed, delivers it to the condenser. We will consider here a condensing steam engine with one cylinder. The following considerations can then be easily transferred to open engines, i.e., to non-condensing engines in which the cylinder steam is directly discharged into the open air. The investigations can also be easily transferred to multiple-cylinder engines, and likewise to engines driven by some other vapor than that of water—for example, by the vapors of ether, carbonic acid, ammonia, or sulphurous acid.

The last-mentioned difference has been completely ignored in the general presentation, because it is not affected by the kind of steam employed, for the fundamental formulas are the same throughout and the difference is first manifested in the numerical calculations.

The generation of steam in the steam boiler and its transfer to the steam cylinder always takes place under constant pressure. The dimensions of the boiler, particularly the thickness of

the boiler shell, prescribe for a particular boiler a certain steam pressure which must not be exceeded; but as it is proper to run the engine at as high a pressure as possible, we will assume in the following that the highest possible pressure exists in the boiler; this pressure, however, is accompanied by a particular temperature, which we will assume as the upper temperature limit for the cycle described by the engine.

There is also nearly constant pressure in the condenser of the steam engine and consequently constant temperature. As the pressure in the condenser is kept as low as possible in order to reduce the hurtful back pressure on the steam piston to the minimum, we must regard the condenser temperature as the lower temperature limit in the actual working cycle of the steam engine.

Accordingly steam formation in the boiler takes place with heat supply at the upper constant temperature limit, and condensation, or heat withdrawal in the condenser, occurs at constant lowest temperature, and thus are fulfilled two of the conditions which must be realized in the Carnot cycle in order to obtain the most advantageous working process, i.e., obtain the maximum work. If, corresponding to other requirements, expansion takes place from the highest pressure to the lowest, in adiabatic fashion, and if compression follows the same law, then, neglecting hurtful resistances, it is possible to construct a steam engine as a theoretically perfect machine. It must be expressly emphasized, however, that this possibility occurs only with wet steam. If steam is used in the superheated condition the circumstances are different and will be discussed more fully later on.

For the present case and for the introduction of the question we will assume that the whole cycle can be conducted in the steam cylinder itself, and for the present will assume that the cylinder walls take no part whatever in the cycle, i.e., that there is no heat exchange between cylinder walls and steam.

Let the cylinder to the left of piston K (Fig. 35a) contain a unit of weight of water of the temperature $t_1(T_1)$ and subject to the steam pressure p_1 corresponding to this temperature. If we assume the piston cross-section to be equal to the unit of area,

then the volume, represented by OA, is identical with the specific volume σ of the water.

Now suppose the water is supplied from without with the heat quantity Q_1 under constant pressure, i.e., at constant temperature T_1, till we have steam of the quality x_1 at the point b.

Pass through the points a and b the two adiabatics ad and bc, and let us suppose the mixture to expand adiabatically from the

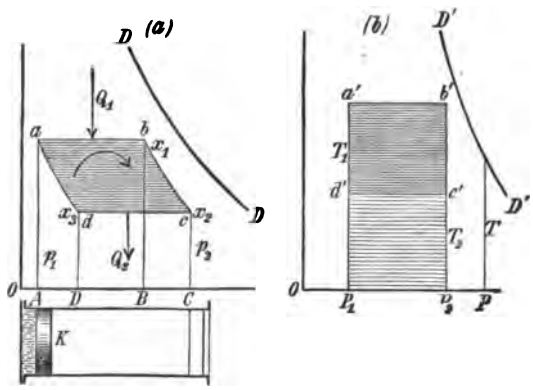

FIG. 35.

point b to the point c; in so doing the pressure sinks from p_1 to p_2, the temperature diminishes from T_1 to T_2, and the steam quality changes from the value x_1 to x_2.

Suppose the mass is compressed along the path cd under constant pressure p_2 and constant temperature T_2 till the point d on the first adiabatic is reached. In so doing condensation takes place; the heat quantity Q_2 must be withdrawn, and the steam quality diminishes from the value x_2 to the value x_3.

Finally, in the fourth part of the cycle, there is adiabatic compression till the initial condition is again reached at the point a. Along this path da the steam quality x_3 is reduced to zero, and then the adiabatic compression effects complete condensation.

The curve DD represents the limit curve; the circumstance that the whole diagram lies inside the limit curve shows that there is no superheating during the whole course of the cycle.

Let r_1 and r_2 represent the latent heats corresponding to the

temperatures T_1 and T_2 respectively, then the heat quantity Q_1, which must be supplied along the path *ab*, is

$$Q_1 = r_1 x_1. \quad \ldots \ldots \quad (1)$$

The heat quantity Q_2, which must be withdrawn along the path *cd*, is

$$Q_2 = r_2(x_2 - x_3), \quad \ldots \ldots \quad (2)$$

and therefore the work L_m which is performed is determined from

$$AL_m = Q_1 - Q_2, \quad \ldots \ldots \quad (3)$$

where the index m indicates that the work L_m is a maximum because we are dealing with the Carnot cycle.

Now for any point whatever of an arbitrary pressure curve we have, according to § 9, p. 61, for the entropy of a unit of weight of mixture,

$$P = \int \frac{dQ}{AT} = \frac{1}{A}\left(\tau + \frac{xr}{T}\right), \quad \ldots \ldots \quad (4)$$

and with the adiabatic curve we have P a constant quantity because $dQ = 0$; it follows therefore that for the adiabatic *bc* we have

$$AP_2 = \tau_1 + \frac{x_1 r_1}{T_1} = \tau_2 + \frac{x_2 r_2}{T_2}, \quad \ldots \ldots \quad (5)$$

and for the adiabatic *ad*

$$AP_1 = \tau_1 = \tau_2 + \frac{x_3 r_2}{T_2} \quad \ldots \ldots \quad (6)$$

because $x = 0$ at the point *a*.

If with given, limiting temperatures the steam quality x_1 is known, the values x_2 and x_3 can be computed from equations (5) and (6), and then equation $v = xu + \sigma$ will determine the specific volume corresponding to each of the four vertices of the diagram.

Subtraction of equation (6) from (5) furnishes the relation

$$r_1 x_1 = \frac{T_1}{T_2} r_2 (x_2 - x_3).$$

and, considering equations (1) and (2),

$$\frac{Q_2}{Q_1}=\frac{T_2}{T_1}, \quad \ldots\ldots\ldots \quad (7)$$

and then from equation (3)

$$L_m=\frac{Q_1}{AT_1}(T_1-T_2). \quad \ldots\ldots\ldots \quad (8)$$

The last two equations were found earlier for the Carnot cycle. In Fig. 35*b* we see the transformation; the two values P_1 and P_2 of equations (5) and (6) are laid off as abscissas; the ordinates represent the absolute temperatures T_1 and T_2.

The closely hatched rectangle $a'b'c'd'$ then also represents the work L_m while the whole area and the widely hatched area respectively represent, in units of work, the heat quantities Q_1 and Q_2, i.e., represent the values $\frac{Q_1}{A}$ and $\frac{Q_2}{A}$.

The curve $D'D'$ is a transformation of the limit curve DD; as $x=1$ for the latter, its abscissa P is to be calculated from equation

$$AP=\tau+\frac{r}{T},$$

and the ordinates correspond to different values of T.

Before we subject the equations just obtained to a closer discussion we will point out that the cycle indicated cannot be practically realized because its four portions cannot be carried on in one and the same space (cylinder); but it is possible to conceive of the Carnot cycle being described by a steam engine if we choose an arrangement for it like that which has already been presented as the fundamental form for heat engines, as was done when considering hot-air and cold-air engines (see Fig. 57, Vol. I, p. 377).

In Fig. 36 the figure just mentioned has been reproduced, but we will discuss it here as a perfect hot-vapor engine.

The steam boiler AA, or evaporator, is represented by a tubular boiler; it is filled with water and steam, and its tubes

are surrounded by a vessel B in which there is a fluid of high temperature which continually delivers heat to the water in the boiler and converts it into steam. This fluid we will call the heating fluid, and in ordinary steam engines it consists of the fire gases, i.e., of the products of combustion coming from the furnace at high temperature.

A second tubular boiler EE constitutes the cooling apparatus or condenser; it also contains steam and water, but of lower temperature. The tubes are also enclosed by a casing through which flows a current of cooling water; it enters the casing at m and leaves it at n and maintains the water and steam in

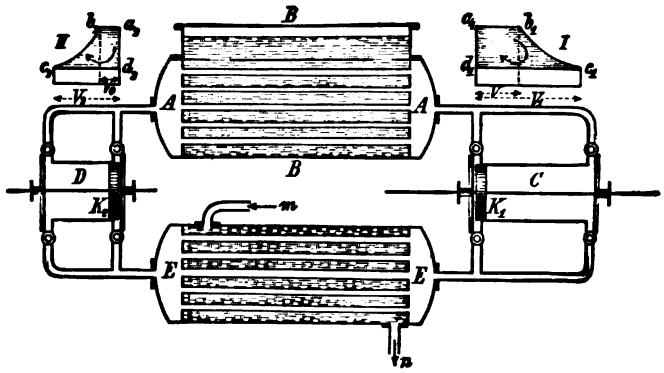

FIG. 36.

the condenser, which is here treated as a surface condenser, at constant lower temperature.

The two cylinders C and D, in which the pistons K_1 and K_2 move, are in communication with the steam boiler and condenser by conduits provided with valve gear, the character of the connection being evident from Fig. 36.

Cylinder C represents the steam cylinder proper and can also be designated as the power or expansion cylinder. The second cylinder D we may, for reasons to be given later, call the feed or compression cylinder, and may say in advance that it does not occur in actually constructed steam engines.

In the steam boiler let us suppose the constant pressure p_1 and the temperature T_1 to prevail, in the condenser the pressure p_2

and the temperature T_2; the capacity of the two spaces relatively to the contents of the cylinder is, however, to be so great that appreciable fluctuations of pressure will not occur in these spaces.

The working of the engine is as follows: The piston K_1 is at the left end of the cylinder C and, while the piston traverses the space $a_1b_1 = V$, the G kg. [lb.] mixture of liquid and steam leaves the boiler under the constant pressure p_1 and at the upper temperature limit T_1 enters the steam cylinder; the steam quality of the mixture is x_1, the steam weight admitted per stroke is therefore Gx_1 kg. [lb.], and the water weight is $G(1-x_1)$ kg. [lb.].

At the end of the piston stroke V the connection with the boiler is shut off and the steam now expands adiabatically over the portion b_1c_1 of the piston stroke, at the end of which it possesses the lower pressure p_2 and reaches the lower temperature limit T_2; in so doing the steam quality changes to the value x_2.

Now the space to the left of the piston is put into communication with the condenser E and the mass is pushed out of the space V_1 into the condenser, i.e., is forced along the path c_1d_1 against the constant pressure p_2, whereupon piston K_1 begins the repetition of the series of operations. The hatched area, shown in diagram I (Fig. 36), represents the work L_1 which has been performed in the steam cylinder; of course the indicator diagram, here representing the indicated work, is valid only under the express assumption that the cylinder possesses no clearance space, that the expansion takes place adiabatically, and that there is no compression, consequently that the cylinder is completely emptied during the return stroke in the cylinder.

During the return stroke of the steam piston K_1 we will suppose the piston K_2 to have moved in the feed cylinder D over the path $d_2c_2 = V_2$ under the constant pressure p_2 of condensation and thus to have removed from the condenser the G mixture of liquid and vapor possessing the steam quality x_3, so that only the steam weight $G(x_2-x_3)$ has been condensed in the condenser.

At the end of the piston stroke in the feed cylinder D the connection with the condenser is shut off and during the return stroke of the piston K_2 the mass is compressed adiabatically along the path c_2b_2 till at the point b_2 there is reached the

boiler temperature T_1 and the boiler pressure p_1, so that complete liquefaction is realized.

Finally, connection with the boiler is established and the water weight G is forced into the boiler along the path b_2a_2, overcoming the constant pressure p_1.

The indicator diagram II gives the work L_2 consumed in the feed cylinder per stroke.

The here assumed feed cylinder D, we must expressly note, is not to be confused with the feed pump occurring with actually constructed steam engines, which pump sucks in water without steam and forces it into the boiler; in the case before us the duty of the feed cylinder is to take water and steam of a prescribed mixture ratio x_3 from the condenser, to liquefy this mixture solely by adiabatic compression and to bring it to the boiler pressure p_1 and to the boiler temperature T_1. This arrangement alone will permit an ordinary condensing steam engine to be converted into a theoretically perfect one. Whether or not in existing steam engines the ordinary feed pump should be replaced by the feed cylinder discussed here is a question which must be considered more fully in what follows.

Reversing the indicator diagram II (Fig. 36) and superimposing it on diagram I, we get for the work performed $L_m = L_1 - L_2$ and at the same time we get the curved quadrilateral of the Carnot cycle shown in Fig. 35*a*, p. 373.

We will now follow analytically the operations mentioned. During the entrance of the steam into the steam cylinder we must supply per stroke the heat quantity

$$Q_1 = Gr_1x_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

to the steam boiler and must withdraw the heat quantity

$$Q_2 = Gr_2(x_2 - x_3) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

from the condenser. Furthermore we have for the pressure curves b_1c_1 and c_2b_2 the two equations adduced under (5) and (6):

$$\tau_1 + \frac{x_1r_1}{T_1} = \tau_2 + \frac{x_2r_2}{T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and

$$\tau_1 = \tau_2 + \frac{x_3 r_2}{T_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The two work quantities L_1 and L_2 are determined as follows: The admission volume V in the steam cylinder C is found from

$$V = G(x_1 u_1 + \sigma), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and the whole cylinder volume from

$$V_1 = G(x_2 u_2 + \sigma). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Consequently the admission work is $Gp_1(x_1 u_1 + \sigma)$ and the back-pressure work expended in pushing the whole mass into the condenser is $Gp_2(x_2 u_2 + \sigma)$. The work of expansion along the curve $b_1 c_1$ is found from the fundamental equation

$$dQ = G[dq + d(x\rho)] + A dL, \quad . \quad . \quad . \quad . \quad . \quad (15)$$

when we make $dQ = 0$, integrate, and temporarily designate the work of expansion by L'; it is then

$$AL' = G[q_1 - q_2 + x_1\rho_1 - x_2\rho_2]. \quad . \quad . \quad . \quad . \quad (15a)$$

If we suitably unite the three quantities of work mentioned and remember the relation $r = \rho + Apu$, we obtain for the calculation of the indicated work L_1 of the steam in the steam cylinder the equation

$$AL_1 = G[q_1 - q_2 + x_1 r_1 - x_2 r_2 + A\sigma(p_1 - p_2)], \quad . \quad . \quad (16)$$

or, utilizing equation (11),

$$AL_1 = G\left[\frac{x_1 r_1}{T_1}(T_1 - T_2) + q_1 - q_2 - T_2(\tau_1 - \tau_2) + A\sigma(p_1 - p_2)\right]. \quad (16a)$$

In the feed cylinder, however, the circumstances are as follows: The volume V_2 of the cylinder is

$$V_2 = G(x_3 u_2 + \sigma), \quad . \quad . \quad . \quad . \quad . \quad . \quad (14a)$$

and the volume V_0 at the end of the compression is

$$V_0 = G\sigma.$$

While the mass is sucked in from the condenser there is consequently produced the work $Gp_2(x_3u_2+\sigma)$, and while the water is being forced into the steam boiler the work $Gp_1\sigma$ is consumed.

The work of compression L'' along the curve c_2b_2 (Fig. 36) is determined by integration from equation (15) and is

$$AL'' = G(q_1 - q_2 - x_3\rho_2).$$

If here also we properly unite the three quantities of work we get, for the calculation of the indicated work L_2 of the steam in the feed cylinder, the equation[1]

$$AL_2 = G[q_1 - q_2 - x_3r_2 + A\sigma(p_1 - p_2)], \quad . \ . \ . \quad (17)$$

or, utilizing equation (12),

$$AL_2 = G[q_1 - q_2 - T_2(\tau_1 - \tau_2) + A\sigma(p_1 - p_2)]. \quad . \ . \quad (17a)$$

The difference of equations (16*a*) and (17*a*) finally gives for the engine's work per stroke

$$AL_m = G\frac{x_1r_1}{T_1}(T_1 - T_2), \quad . \ . \ . \ . \ . \quad (18)$$

or, utilizing equation (9),

$$L_m = \frac{Q_1}{AT_1}(T_1 - T_2), \quad . \ . \ . \ . \ . \ . \quad (I)$$

[1] It deserves to be noted that the two equations (16*a*) and (17*a*) are respectively identical with the efflux formulas for wet vapors and highly heated liquids. Compare equation (16*a*) with equation (I*d*), p. 156, and equation (17*a*) with equation (8), p. 160. There the kinetic energy of the mass in the orifice of efflux takes the place of the work quantities L_1 and L_2; the transformations of the equations given there could, if it were necessary, be easily transferred to the formulas adduced above.

while combination of equations (9) to (12) gives the relation

$$\frac{Q_2}{Q_1}=\frac{T_2}{T_1}. \quad \ldots\ldots\ldots \quad \text{(II)}$$

With this engine arrangement, therefore, there reappear equations (7) and (8), p. 375, which correspond to the Carnot cycle.

§ 50. INVESTIGATION OF THE PERFECT STEAM ENGINE AND CALCULATION OF ITS DIMENSIONS.

In what follows a perfect steam engine will be regarded as one that describes the Carnot cycle, and we shall expressly assume that the steam is used in the saturated, or wet, condition, but not in the superheated state. We shall leave till later the consideration of the difference between these two kinds of engines and the discussion of the views which have recently made themselves felt from the practical standpoint in this connection.

Formula (I), developed in the foregoing, for the work L_m of the perfect steam engine shows, in the first place, that for the same temperature limits T_1 and T_2, i.e., for the same temperatures in boiler and condenser, the work L_m produced is directly proportional to the heat quantity Q_1 which is supplied to the steam boiler; but the formula contains no quantity depending upon a special kind of vapor; with the same temperatures, therefore, the same work is obtained from the same heat quantity, whether the vapor of water, carbonic acid, ether, or any other, is employed. So far as the performance of work is concerned no kind of vapor has an advantage over another—a result which was to be expected from the earlier general investigations, for it has already been emphasized that in the Carnot cycle it does not matter what the nature of the mediating body is.

Therefore no advantage is to be expected from an engine working with a combination of two different vapors.

Du Tremblay[1] proposed more than fifty years ago to

[1] Annales des mines, 1853, Vol. IV, p. 203; Dingler's polytechn. Journal, Vol. 134, p. 161; Polytechn. Centralblatt, 1854, p. 385.

unite a water-vapor engine with an ether-vapor engine in such a way that the vapor of water is condensed by the heat withdrawn from it by liquid ether; in so doing the ether evaporates and after it has performed work is condensed in the usual fashion. But here one cannot go higher than the upper temperature limit T_1 of the ordinary steam engine, nor lower than the external atmospheric temperature, which is the lower temperature limit T_2; it is therefore evident why Du Tremblay's engine system, which he applied particularly to marine engines, did not come into vogue. Nevertheless in recent years engineers have come back to such combined engines, with the difference that sulphurous acid is used in place of ether, but ammonia vapor might also have been employed. At the conclusion of these articles we shall go into the question somewhat more fully.

Equation (I) gives the maximum of work corresponding to a particular heat quantity Q_1 and given temperature limits. Every deviation in the arrangement of a constructed steam engine from the perfect form presented in Fig. 36, and every departure from the assumptions underlying the above developments, leads to a smaller performance of work for the same heat quantity and the same temperature limits. Every kind of deviation, therefore, involves a corresponding loss of work, and it is one of the tasks of the theory of machines to establish analytical expressions for these losses in order to recognize from them the means which must be used to reduce these losses to the minimum in actually constructed steam engines; furthermore the different losses of work corresponding to the various deviations from perfection are compared with each other numerically, and then, when striving to improve the steam engine, attention will first be directed to the greatest losses of work.

The following investigations will specially treat the questions indicated, but the fundamental equation (I) enables one to make certain remarks in this general direction. This equation shows that for a particular quantity of heat Q_1 supplied to the boiler the work L_m produced depends upon the temperature limits, and that this work is greater the higher we can take the temperature T_1 and the lower we can take the temperature T_2.

Since the pressure only grows with the temperature in saturated vapors, we can say that the heat quantity Q_1 will be more advantageously utilized the higher the temperature T_1 in the boiler, the higher therefore the boiler and admission pressure p_1, the same temperature T_2 being assumed.

A part of the astonishing progress in the steam-engine constructions of to-day is to be ascribed to observing this proposition; in the last decades the boiler pressure has in fact been continually raised.

It would be incorrect, from the theoretical standpoint, to support the occasional efforts to increase the boiler pressure, and thus the boiler temperature, far beyond the extent to which it is done at the present time. In the foregoing the question has been assumed to be the utilization of the heat quantity Q_1 which is really introduced into the boiler; on the other hand, if we consider that quantity of heat which is released during the combustion of the fuel, and, therefore, in addition to the action of the steam engine proper, consider the process of boiler heating also, then the circumstances change. The question has already been quite fully discussed in Vol. I, p. 448, with reference to the steam engine too, and it has been proved that there is a maximum value for the temperature T_1 which should not be transcended.

Moreover it has already been emphasized that the lower temperature limit T_2 should be taken as low as possible in the cycle of the perfect engine. This limit is perfectly well fixed in all heat engines; its extreme value is the mean atmospheric temperature and is usually assumed as varying from 15° to 20° C. [59° to 68° F.] and is ascribed to the cooling body, the cooling water in the condenser. Within the condenser itself, the temperature prevailing, which is exactly what is meant by the temperature T_2, always possesses a higher value. If, as is customary in actually constructed steam engines, we assume the condenser pressure (for steam) to be one-tenth of an atmosphere, then this would correspond to a temperature of $t_2 = 46.21°$ [115.18°]; the lower temperature limit would therefore be $T_2 = 319.21°$ [574.58°].

Equation (I) can also be written in the form

$$\frac{L_m}{Q_1} = \frac{T_1 - T_2}{AT_1}, \quad \ldots\ldots\ldots \quad (Ia)$$

and here the value $L_m : Q_1$ represents the work, measured in meter-kilograms [foot-pounds], which corresponds to the unit of heat; here, too, we must think of the heat at the temperature T_1 at which it is supplied to the engine by the boiler; we may call this ratio the engine's mechanical equivalent of the unit of heat, that is, if we could completely convert the heat quantity Q_1 into work, (which is impossible, for in the Carnot cycle we must then have $T_2 = 0$), equation (Ia) would give for $L_m : Q_1$ the value 424 in kg. [772.83 ft-lb.], the mechanical equivalent of the unit of heat. From the point of view just given, the value $L_m : Q_1$ of equation (Ia) can furnish a scale of comparison for judging the excellence of an engine.

If we substitute in equation (Ia) the heat quantity Q_1 which enters the boiler in each hour, and designate it by Q_h, then L_m represents the work per hour in meter-kilograms [foot-pounds]; if N_m represents the work of the engine in horse-powers, there will obtain the relation

$$L_m = 75 \times 3600 N_m$$
$$[L_m = 550 \times 3600\, N_m].$$

The substitution of Q_h and L_m in equation (I) then gives

$$\frac{Q_h}{N_m} = 75 \times 3600 \frac{AT_1}{T_1 - T_2} \quad \ldots\ldots \quad (19)$$
$$\left[\frac{Q_h}{N_m} = 550 \times 3600 \frac{AT_1}{T_1 - T_2}\right],$$

from which we calculate, for this engine, the necessary heat quantity per hour per horse-power. On the other hand if D_h is the steam quantity in kilograms [pounds]

which is generated per hour in the steam boiler, because $Q_h = r_1 D_h$, there follows, from equation (19),

$$\frac{D_h}{N_m} = \frac{75 \times 3600}{r_1} \frac{AT_1}{T_1 - T_2} \quad \ldots\ldots \quad (20)$$

$$\left[\frac{D_h}{N_m} = \frac{550 \times 3600}{r_1} \frac{AT_1}{T_1 - T_2}\right],$$

or

$$\frac{D_h}{N_m} = \frac{1}{r_1} \frac{Q_h}{N_m}, \quad \ldots\ldots\ldots \quad (20a)$$

according to which there is found the steam quantity, per hour per horse-power, which must be generated.

Finally, if D represents the steam quantity in kilograms [pounds] per engine stroke, then we must substitute in equation (I) $Q_1 = r_1 D$, and there follows

$$\frac{L_m}{D} = \frac{r_1}{AT_1}(T_1 - T_2), \quad \ldots\ldots \quad (21)$$

or, utilizing equation (20),

$$\frac{L_m}{D} = \frac{75 \times 3600}{\left(\frac{D_h}{N_m}\right)} \quad \ldots\ldots\ldots \quad (21a)$$

$$\left[\frac{L_m}{D} = \frac{550 \times 3600}{\left(\frac{D_h}{N_m}\right)}\right].$$

From this can be found what corresponds to the work of a kilogram [pound] of steam in meter-kilograms [foot-pounds].

Example. In a perfect steam engine let the boiler pressure $p_1 = 6$ atmospheres, then, according to Table 11 of the Appendix, $t_1 = 159.22$ [318.60], hence $T_1 = 432.22°$ [778.00°], and $r_1 = \rho_1 + Ap_1u_1 = 494.124$ [889.423].

Let the pressure in the condenser amount to $p_2 = 0.1$ atmospheres, and

therefore $t_2 = 46.21°$ [115.178°], and $T_2 = 319.21°$ [574.572°]; the foregoing equations then respectively give

$$\frac{L_m}{Q_1} = 110.86, \quad \frac{Q_h}{N_m} = 2435.5 \text{ Cal.}, \quad \frac{D_h}{N_m} = 4.929 \text{ kg., and}$$

$$\frac{L_m}{D} = 54779 \text{ mkg}$$

$$\left[\frac{L_m}{Q_1} = 202.065, \quad \frac{Q_h}{N_m} = 9798.8 \text{ B.t.u.}, \quad \frac{D_h}{N_m} = 11.016 \text{ lb., and} \right.$$

$$\left. \frac{L_m}{D} = 179733 \text{ ft-lb.} \right].$$

If Q_h' is the quantity of heat which is to be withdrawn per hour by the condenser, then, according to equation (II), the heat quantity per hour per horse-power is

$$\frac{Q_h'}{N_m} = \frac{T_2}{T_1} \frac{Q_h}{N_m} = 1798.7 \text{ [7236.8].}$$

With the foregoing as a basis, the following survey has been prepared with the help of the data furnished by Table 12.

In this survey the pressure is expressed in kilograms to the square centimeter in new atmospheres, and it is expressed in absolute measure, not in gauge pressure. [In the English table following the pressure is pounds per square inch, and likewise in absolute measure.] Now we clearly see the advantage of using high boiler pressure; the values given in Cols. 3 to 5 are suitable for judging of the excellence of actually constructed condensing engines, and for fixing the goal to which all further betterments should be directed. In making comparisons we use the values occurring in the middle of the vertical columns. In the best engines of the present day (compound multiple-cylinder engines) we have obtained at 6 to 8 kg. [85.34 to 113.78 lb. per sq. in.] of boiler pressure an indicated consumption per hour per horse power of 7 kg. [15.6 lb.] of steam, and even less, and consequently we have approached pretty closely to the limit values 4.933 and 4.673 [10.73 and 10.16] respectively.

1.	2.	3.	4.	5.
Pressure in Boiler in Kilograms per sq. centimeter (absolute). p_1	$\frac{Q_h}{N_m}$	$\frac{D_h}{N_m}$	$\frac{L_m}{D}$	$\frac{L_m}{Q_1}$
	In the Condenser: Pressure $p_2=0.1$ kg. and Temperature $t_2=45.58$.			
	cal.	kg.	mkg.	mkg.
2	3378.6	6.465	41763	79.91
4	2723.0	5.382	50168	99.15
6	2442.3	4.933	54728	110.55
8	2274.4	4.673	57776	118.71
10	2158.6	4.499	60017	125.08
12	2071.9	4.372	61760	130.31
14	2003.6	4.275	63163	134.77

1.	2.	3.	4.	5.
Pressure in Boiler in Pounds per square inch (absolute). p_1	$\frac{Q_h}{N_m}$	$\frac{D_h}{N_m}$	$\frac{L_m}{D}$	$\frac{L_m}{Q_1}$
	In the Condenser: Pressure $p_2=1.5$ lb. per sq. in. and Temperature $t_2=115.85°$ F.			
	B.t.u.	lb.	ft-lb.	ft-lb.
30	13528	14.415	137361	146.358
60	10904	12.006	164922	181.582
90	9780	11.009	179857	202.455
120	9108	10.431	189816	217.391
150	8644	10.044	197127	229.050
180	8297	9.763	202806	238.630
210	8024	9.548	207382	246.766

Of special importance are the numerical values in the fourth column, which give the work value of o n e kilogram [pound] of steam in the perfect engine at different boiler pressures. If we are considering the above-quoted heat condensing engine, with an indicated steam consumption per hour per horse power of 7 kg. [15.6 lb.], then this corresponds, according to equation (21*a*), to a really achieved work value of

$$\frac{75\times 3600}{7} = 38571 \text{ mkg.}$$

$$\left[\frac{550\times 3600}{15.647} = 126544 \text{ ft-lb.}\right].$$

The theoretical work value at a boiler pressure of 6 kg. [85.34 lb.] is, according to the preceding table, 54728 mkg. [176142 ft-lb.]; the ratio of these two values is 0.705, and already a very favorable one.

The preceding table and the remarks added refer to condensing steam engines.

On the other hand, if, in an actually constructed steam engine, condensation cannot be employed, either because the necessary quantity of cooling water is not available or because it cannot be carried along, as in locomotive engines, then we must institute a comparison with a perfect engine in which the pressure in the condenser is taken as one atmosphere, and the temperature prevailing there is assumed to be $t_2 = 100°$ [212°]. For such a perfect steam engine we obtain the results tabulated on page 389.

The comparison of this table with the preceding one shows very clearly the value of employing condensation, and shows the influence which accompanies a reduction of the lower temperature limit. We see, what was long ago discovered in practice, that non-condensing engines should work with as high a boiler pressure as possible and that then only do they approach in steam utilization the condensing steam engines when the latter are worked with a low boiler pressure.

Now, as regards the dimensions which are to be chosen for the perfect steam engine, we must here consider the dimensions of both cylinders. The volume V_1 of the steam cylinder C (Fig. 36, p. 376) is determined according to equation (14), p. 379, and the volume V_2 of the feed cylinder D is found from equation (14*a*). As we may neglect the value σ in both formulas, there follows

$$V_1 = G x_2 u_2 \quad \text{and} \quad V_2 = G x_3 u_2, \quad . \quad . \quad . \quad . \quad (22)$$

where G represents the weight of steam and liquid per stroke, and where it is assumed that the piston in the feed cylinder has as many periods as the one in the steam cylinder. The ratio of the two values is found to be

$$\frac{V_2}{V_1} = \frac{x_3}{x_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

1. Pressure in Boiler in Kilograms per sq. centimeter (absolute). p_1	2. $\frac{Q_h}{N_m}$	3. $\frac{D_h}{N_m}$	4. $\frac{L_m}{D}$	5. $\frac{L_m}{Q_1}$
	In the Condenser: Pressure $p_2 = 10333$ kg. and Temperature $t_2 = 100°$.			
	cal.	kg.	mkg.	mkg.
2	12784.0	24.443	11046	21.12
4	6183.8	12.222	22091	43.66
6	4735.9	9.567	28223	57.01
8	405 .4	8.335	32395	66.56
10	3647.8	7.602	35515	74.02
12	3369.0	7.109	37982	80.14
14	3163.6	6.743	40004	85.35

1. Pressure in Pounds per square inch (absolute). p_1	2. $\frac{Q_h}{N_m}$	3 $\frac{D_h}{N_m}$	4. $\frac{L_m}{D}$	5. $\frac{L_m}{Q_1}$
	In the Condenser: Pressure $p_2 = 14.6967$ lb. per sq. in. and Temperature $t_2 = 212°$.			
	B.t.u.	lb.	ft-lb.	ft-lb.
30	47543	50.657	39086	41.646
60	23925	26.342	75165	82.758
90	18484	20.806	95163	107.119
120	15897	18.206	108754	124.552
150	14331	16.652	118904	138.161
180	13258	15.600	126922	149.341
210	12466	14.833	133487	158.838

and, utilizing equation (11) and (12), there is found

$$\frac{V_2}{V_1} = \frac{T_1(\tau_1 - \tau_2)}{x_1 r_1 + T_1(\tau_1 - \tau_2)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23a)$$

If we determine the value of G from the equation of work (18) and substitute it in the first of equations (22), we get for the volume of the steam cylinder

$$V_1 = \frac{A T_1 x_2 u_2}{x_1 r_1 (T_1 - T_2)} L_m.$$

If the engine is double-acting and makes n revolutions per minute, then, if the performance in horse powers is designated by N_m, we get the relation

$$L_m = \frac{30 \times 75}{n} N_m \quad \ldots \ldots \ldots \quad (24)$$

$$\left[L_m = \frac{30 \times 550}{n} N_m \right],$$

and, designating the cross-section and stroke of the steam cylinder by F_1 and s_1, we find from the last equation but one

$$\frac{F_1 s_1 n}{30} = 75 \times \frac{A T_1 x_2 u_2}{x_1 r_1 (T_1 - T_2)} N_m \quad \ldots \ldots \quad (25)$$

$$\left[\frac{F_1 s_1 n}{30} = 550 \times \frac{A T_1 x_2 u_2}{x_1 r_1 (T_1 - T_2)} N_m \right].$$

If the steam quality x_1 at the end of admission is known, we must here determine the steam quality x_2 from equation (11) and x_3 according to equation (12). If F_2 is the area of the piston and s_2 the stroke of the piston in the feed cylinder, then we have, by equation (23),

$$\frac{F_2 s_2}{F_1 s_1} = \frac{x_3}{x_2}. \quad \ldots \ldots \ldots \quad (26)$$

The expression in the left member of equation (25) evidently represents the average space swept through per second by the piston of the cylinder.

E x a m p l e. A perfect engine works with a boiler pressure of $p_1 = 7$ kg. [99.562 lb.], and a condensation pressure of $p_2 = 0.1$ kg. [1.422 lb.]; here, according to Table 12 of the Appendix,

$$t_1 = 164.03, \quad T_1 = 437.03, \quad \tau_1 = 0.47531, \quad \frac{r_1}{T_1} = 1.1227,$$

$$t_2 = 45.58, \quad T_2 = 318.58, \quad \tau_2 = 0.15463, \quad \frac{r_2}{T_2} = 1.8041,$$

$$u_1 = 0.2725 \quad \text{and} \quad u_2 = 15.0121.$$

$$\left[\begin{array}{l} t_1 = 327.254, \quad T_1 = 786.654, \quad \tau_1 = 0.47531, \quad \frac{r_1}{T_1} = 1.1227, \\ t_2 = 114.044, \quad T_2 = 573.464, \quad \tau_2 = 0.15463, \quad \frac{r_2}{T_2} = 1.8041, \\ \qquad u_1 = 4.3651 \quad \text{and} \quad u_2 = 240.478. \end{array}\right]$$

There follows, from equations (11) and (12),

$$\frac{x_2 r_2}{T_2} = \tau_1 - \tau_2 + \frac{x_1 r_1}{T_1} \quad \text{or} \quad x_2 = 0.1777 + 0.6223\, x_1,$$

$$\frac{x_3 r_2}{T_2} = \tau_1 - \tau_2 \qquad \text{or} \quad x_3 = 0.1777.$$

If the steam quality at the end of admission is $x_1 = 0.90$, i.e., if 10% water is mixed with the admission steam, we shall have $x_2 = 0.7378$, and, from equation (25),

$$\frac{F_1 s_1 n}{30} = 0.016369\, N_m \; [= 0.58616\, N_m],$$

and furthermore, from equation (26),

$$\frac{F_2 s_2}{F_1 s_1} = 0.241.$$

If the engine is to develop a performance of $N_m = 250$ horse powers and is to make $n = 65$ revolutions per minute, then, according to the preceding, the cubic capacity of the two cylinders is

$$F_1 s_1 = 1.889 \text{ cbm.} \quad \text{and} \quad F_2 s_2 = 0.455 \text{ cbm.}$$
$$[F_1 s_1 = 67.634 \text{ cu. ft.} \quad \text{and} \quad F_2 s_2 = 16.300 \text{ cu. ft.}].$$

From the first of equations (22) we can compute the weight G of the mixture of steam and water, and find it to be

$$G = \frac{F_1 s_1}{x_2 u_2} = 0.1705 \text{ kg. } [0.3759 \text{ lb.}].$$

The feed cylinder has therefore to draw in from the condenser, during each stroke, the steam weight $G x_3 = 0.0303$ kg.[0.0668lb.] and the water weight $G(1 - x_3) = 0.1402$ kg. [0.3091 lb.]. The volume of the steam cylinder, which in a compound engine would correspond to the volume of the low-pressure cylinder, here seems to be very great, because very great expansion is

assumed; the admission volume V is determined from equation (13), and is $V = G\ (x_1 u_1 + \sigma)$, or, utilizing the given values, it is

$$V = 0.0420 \text{ cbm. } [1.4833 \text{ cu. ft.}];$$

we therefore have an expansion of

$$\frac{F_1 s_1}{V} = 44.97.$$

Such an expansion will, of course, not be sought nor attained in practice; if it could be reached, the hourly steam consumption per horse power would be, according to equation (20),

$$\frac{D_h}{N_m} = 4.789 \text{ kg. } [10.705 \text{ lb.}],$$

while at the present time the best compound engines give, for the assumed pressure conditions, a minimum of, say, 5.5 kg. [12.3 lb.] of steam per indicated horse power.

The formulas given in the preceding, for the calculation of the dimensions of a perfect engine, are of subordinate value for practice, because the actually constructed engine differs in structure from the perfect one inasmuch as it does not contain the feed or compression cylinder; but it is otherwise with the formulas for work or performance; these are well-suited, on account of their simplicity, for judging of new engine arrangements.

The combined engine of Du Tremblay, mentioned at the beginning of this article (p. 381), and which consists of a combination of a steam with an ether engine, is an example of this. Quite recently the fundamental idea underlying this engine has been taken up again by Behrend and Zimmermann, only that they employ the vapor of sulphurous acid in place of the vapor of ether; the advantage of the former is that the power-cylinder needs no lubrication, otherwise the vapors of ammonia and other fluids might be employed. After the trials had been wrecked by practical difficulties Josse took up the experimental investigations. The following numerical data are obtained from his preliminary report;[1] a theoretical treatment

[1] Josse, "Versuche zur Erhöhung des thermischen Wirkungsgrades der Dampfmaschine." Mitteilungen aus dem Maschinen-Laboratorium der K Technischen Hochschule zu Berlin. Book II, Munich and Leipsic, 1899.

of the question is not given by J o s s e , but his experiments do show very favorable results.

c Suppose a steam engine to describe a C a r n o t cycle between the temperatures T_1 and T_2, receiving the heat quantity Q_1 and delivering the work L_1; then according to equation (I), p. 380,

$$L_1 = \frac{Q_1}{AT_1}(T_1 - T_2), \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

and the heat quantity Q_2 which is withdrawn from the condenser bears to Q_1, according to equation (II), p. 381, the relation

$$\frac{Q_2}{T_2} = \frac{Q_1}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The heat quantity Q_2 released during condensation now serves to evaporate a second liquid, for example sulphurous acid, and the vapor developed describes a C a r n o t cycle in a s e c o n d engine between the temperature limits T_2 and T_3, and this produces the additional work L_2; we therefore have

$$L_2 = \frac{Q_2}{AT_2}(T_2 - T_3),$$

or, with the help of equation (28),

$$L_2 = \frac{Q_1}{AT_1}(T_2 - T_3). \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

If to this we add equation (27), we get the work L of the two engines (the steam engine may be designated as the f i r s t engine):

$$L = L_1 + L_2 = \frac{Q_1}{AT_1}(T_1 - T_3). \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Division of the two equations (29) and (27) furnishes the ratio

$$\frac{L_2}{L_1} = \frac{T_2 - T_3}{T_1 - T_2} = \frac{t_2 - t_3}{t_1 - t_2}. \quad . \quad . \quad . \quad . \quad . \quad (31)$$

From these equations we get, in the first place, the following:

If the same lower temperature limit t_3 in the condenser could be reached in the first engine as in the second engine, so that $t_2=t_3$, we should have $L=L_1$ and $L_2=0$; there would then be no sense in adding the second engine.[1]

On the other hand if we can go lower with the condenser temperature t_3 in the second engine than in the first, so that $t_3<t_2$, we may, to be sure, under certain circumstances, expect an advantage; but this is of less consequence, as equation (31) shows, the smaller the temperature difference (t_2-t_3), and, what is noteworthy, the higher the temperature t_1, i.e., the higher the pressure in the steam boiler.

If the first engine works alone equation (27) holds; if both engines work together equation (30) is valid; if in the latter case the same work is to be produced so that $L=L_1$, then we must, in the latter case, supply to the (water) steam boiler the heat quantity Q_1' instead of Q_1, so that the relation

$$L_1=\frac{Q_1'}{AT_1}(T_1-T_3)$$

obtains. From combination with equation (27) then follows

$$\frac{Q_1'}{Q_1}=\frac{T_1-T_2}{T_1-T_3}=\frac{t_1-t_2}{t_1-t_3}. \quad \ldots\ldots \quad (32)$$

The two quantities of steam which must be generated in the steam boiler in these two cases also bear the same ratio to each other. If D_h is the hourly steam consumption per horse power

---

[1] The second engine is called by Josse the "cold-vapor engine." The designation was suggested by me (Zivilingenieur, 1881, Vol. 27, p. 449), but I meant by the term a cold-producing machine; if we follow Josse's practice in this matter we shall not know in the future whether a prime mover or an operating machine is referred to. Such a utilization of terms which already exist for arrangements of an entirely different sort unfortunately occurs too frequently in technical circles.

when the first engine works alone, and D_h' the steam quantity when both engines work together, then we also have

$$\frac{D_h'}{D_h} = \frac{t_1 - t_2}{t_1 - t_3}. \quad \ldots\ldots\ldots \quad (33)$$

The two weights of water vapor differ less from each other the less t_3 differs from t_2 and the higher the steam pressure in the boiler.

Example. Let the boiler temperature amount to $t_1 = 179°$ [354.2°], which corresponds to a pressure of 10 kg. per sq. cm. [142.23 lb. per sq. in.]; let the temperature in the first condenser be $t_2 = 70°$ [158°] and in the second $t_3 = 15°$ [59°] (see Josse elsewhere). Here there follows from equation (31)

$$L_2 = 0.505\, L_1,$$

and from equation (33)

$$D_h' = 0.665 D_h.$$

These values, which agree closely with Josse's experimental values, seem to be very favorable; the second engine gives approximately 50% of the work of the first engine, and the steam consumption seems to be about $^2/_3$ that of the water-vapor engine. Of course the condenser temperature t_2 is much higher than is usual in good condensing engines, and probably the low temperature $t_3 = 15°$ [59°] will rarely be available.

If we make $t_1 = 179°$ [354.2], and make $t_2 = 50°$ [122°] and $t_3 = 25°$ [27°], which corresponds better with average conditions, then calculation gives

$$L_2 = 0.194 L_1 \quad \text{and} \quad D_h' = 0.838 D_h,$$

values which, comparatively speaking, are still favorable.

From the preceding it follows that the type of engine discussed may, under certain circumstances, be introduced in spite of the increased complication of the engine, provided large quantities of sufficiently cold water are available.

A general introduction of the system is, however, not to be expected; and in high-pressure engines with multiple cylinders and with superheated steam the same economy of heat and steam has been effected.

The calculated results are found here under the assumption of the Carnot cycle, but there is no difficulty in taking the ordi-

nary cycle as a basis for the same purpose. The tables given in the Appendix for sulphurous acid and ammonia can be easily extended if necessary, and can be computed for higher temperatures than are there assumed.

The formulas just given are applicable to every sort of vapor for the second engine; the table would have to be extended if we desired to employ the formulas for the calculation of the dimensions of the engine, but such an extension could easily be effected.

§ 51. CYCLE OF THE ACTUAL STEAM ENGINE AND LOSS OF WORK DUE TO IMPERFECTION OF THE CYCLE.

The cycle which the steam engine of the construction of the present day actually describes differs from the Carnot cycle; therefore for equal heat consumption the work produced will be smaller than with the perfect engine.

This smaller amount of work involves a loss of work; the determination of this loss, due to the imperfection of the cycle of the existing steam engines, is an important problem, for if this loss proves to be considerable it behooves us to change the cycle of the actual steam engine and bring it nearer to that of the perfect one. Moreover it may be expected in advance that the discussion of the analytical expression, derived by close investigation, will indicate the path that must be pursued in order to effect the desired betterment. In the following discussion we shall, in the first place, ignore all other losses occurring in steam engines. We shall therefore assume:

(*a*) That no clearance space is present in the cylinder; (*b*) that the steam pressure in the cylinder during steam pressure is equal to the boiler pressure; (*c*) that the steam pressure on the exhaust side is identical with the condenser pressure, or, in the case of non-condensing engines, identical with the external atmospheric pressure; (*d*) that no heat exchange occurs between the steam and the cylinder walls while the cycle is being described within the cylinder; (*e*) that the expansion takes place adiabatically and, moreover, completely, i.e., that the pressure of the

steam during expansion sinks from the boiler pressure down to the condenser or atmospheric pressure, as the case may be; and finally (*f*) that the valve gear is so constituted that during steam admission the entrance port is completely open and during the exhaust stroke the exhaust space is connected with the condenser or open air by a completely open discharge conduit.

Every departure from the assumptions made here involves a new and special loss of work; a few of the most important of these losses will be discussed in what follows.

For the purpose of discussing the question just proposed we reproduce Fig. 37 from p. 376; it gives the schematic representation

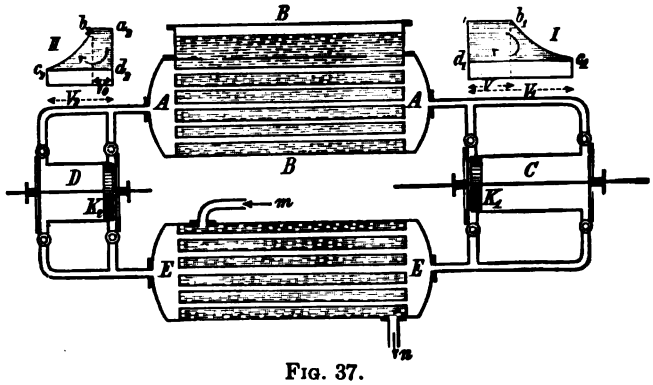

FIG. 37.

of a perfect engine; but it is also suited to schematically represent an actually constructed steam engine, provided the engine under consideration works with condensation. *AA* represents a steam boiler, *EE* the condenser, *C* is the steam cylinder of the engine, and *D* is the feed cylinder; the sole difference is that the steam coming from cylinder *C* to the condenser is completely condensed, and the feed cylinder, which may here be called the feed pump, in accordance with practice, does not, as in the perfect engine, move a prescribed mixture ratio of water and steam, but sucks in water only and forces it into the boiler.

The two indicator diagrams of Fig. 37 will therefore possess the form given in Fig. 38; diagram I for steam cylinder *C* corresponds exactly to the similar diagram in Fig. 37.

The work produced in the cylinder per stroke is determined here, as in the perfect engine, according to equation (16) or (16*a*), p. 379, and is therefore found from

$$AL_1 = G\left[\frac{x_1 r_1}{T_1}(T_1 - T_2) + q_1 - q_2 - T_2(\tau_1 - \tau_2) + A\sigma(p_1 - p_2)\right], \quad . \quad (34)$$

when the same notation is used here as there.

It is different with the feed cylinder, for which the indicator diagram II in Fig. 38 is valid.

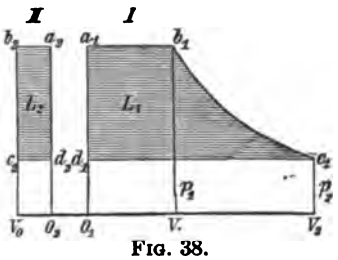

FIG. 38.

During the suction stroke of the pump piston the water volume $G\sigma$ is drawn in under the constant pressure p_2, and the work $G\sigma p_2$ is thus produced; during the return stroke this volume of water is forced into the boiler against the constant pressure p_1, which corresponds to a work consumption of $G\sigma p_1$. The work L_2 which the pump requires is, when expressed in units of heat, given by

$$AL_2 = AG\sigma(p_1 - p_2), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

and this equation takes the place, in the actual engine, of equations (17) and (17*a*), p. 380, which were derived for the perfect engine.

The difference of the two equations (34) and (35) now gives the work produced per stroke in the actual engine; if we designate this work by L there follows

$$AL = G\left[\frac{x_1 r_1}{T_1}(T_1 - T_2) + q_1 - q_2 - T_2(\tau_1 - \tau_2)\right], \quad . \quad . \quad (36)$$

while for the perfect engine, according to equation (18), p. 380, the corresponding work L_m is calculated from the formula

$$AL_m = G\frac{x_1 r_1}{T_1}(T_1 - T_2).$$

A comparison of the last two formulas and a comparison of the indicator diagrams of Figs. 37 and 38 shows that for the same temperature limits and for the same weight G of the working mass of steam and water the work L of the actual engine is greater than in the perfect engine. But this kind of comparison of the two engine varieties is not permissible, for there should be considered besides the heat quantity Q_1 supplied to the boiler per stroke, and this heat quantity is different in the two engines.

For the perfect engine equation (9), p. 378, gives $Q_1 = Gx_1r_1$, but in the actual engine Q_1 is found in the following manner.

If the feed pump takes the water from the condenser, then this water has the temperature t_2; in the boiler it must be heated to the boiler temperature t_1 and only then does evaporation begin. The necessary heat quantity is therefore

$$Q_1 = G(x_1r_1 + q_1 - q_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

If we are dealing with a non-condensing engine and the feed water which is led to the engine has a temperature t_0, then the heat quantity which must be supplied to the boiler per stroke is given by

$$Q_1 = G(x_1r_1 + q_1 - q_0),$$

and this equation is the more general one; if we are considering a condensing engine we must simply take $t_0 = t_2$, and therefore substitute $q_0 = q_2$.

The work L_m, produced by the Carnot cycle of the perfect engine, is determined by the general equation

$$AL_m = \frac{Q_1}{T_1}(T_1 - T_2).$$

If we therefore assume that the heat quantity given by equation (37) is employed in the perfect engine, then the work L_m delivered by the engine is determined from the following formula:

$$AL_m = G\left[\frac{x_1r_1}{T_1}(T_1 - T_2) + \frac{(q_1 - q_0)(T_1 - T_2)}{T_1}\right] \quad . \quad . \quad (38)$$

It deserves to be inserted here that in the actual, as in the perfect, engine we can transform the indicator diagram (as was done before in Fig. 35, p. 373), this diagram being represented in Fig. 39*a* as the difference of the diagrams of Fig. 38. Let us assume the working mass to be a unit of weight of water and steam; a unit of weight of water is first raised in the boiler from t_0 to t_1, and then is generated the steam quality x_1 at constant pressure p_1 or constant temperature t_1.

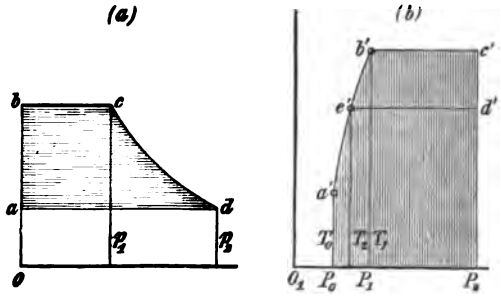

FIG. 39.

Now the entropy P of a unit of weight of water at the temperature T (see equation (4), p. 374) is

$$P=\frac{\tau}{A}.$$

Therefore if we lay off P as abscissa and T as ordinate, we get in the curve $a'e'b'$ (Fig. 39*b*) the transformation of the pressure curve ab (Fig. 39*a*). For the initial and final temperatures T_0 and T_1 we have the corresponding abscissas

$$\overline{O_1P_0}=\frac{\tau_0}{A} \quad \text{and} \quad \overline{O_1P_1}=\frac{\tau_1}{A},$$

and the hatched area lying under curve $a'b'$ therefore gives the heat quantity q_1-q_0, measured in units of work.

If we make, in the figure,

$$\overline{O_1P_2}=\frac{1}{A}\left(\tau_1+\frac{x_1r_1}{T_1}\right),$$

then, according to propositions developed earlier, it follows that the hatched rectangle $P_1b'c'P_2$, lying under the horizontal line $b'c'$, measures the heat quantity x_1r_1, measured in units of work. Accordingly the whole hatched area of Fig. 39*b* represents the value $Q_1:A$ or 424 Q_1 [772.83 Q_1], where Q_1 represents the heat quantity which must be supplied from the boiler to the actual engine per piston stroke.

In like manner it is easy to see that the hatched area lying under the portion $a'e'd'$ represents, in units of work, the heat quantity Q_2 which is to be withdrawn during the cycle provided that the water produced during condensation is subsequently cooled from the condenser temperature t_2 to the temperature t_0.

If the feed pump takes the water directly from the condenser, as is the case in condensing engines, then in Fig. 38*b* the point a' will coincide with the point e'.

In both cases the area $e'b'c'd'$ represents the indicated work; the contents of the area are identical with those of the hatched area in Fig. 39*a*.

Let us return to the consideration of equations (36) and (38). The former gives the work produced per stroke in the actual engine, while the other represents the work which will be produced when an equal quantity of heat Q_1 is employed; the difference $L_m - L$ accordingly represents the loss of work which corresponds to the imperfection of the cycle of the actual engine; if we designate this loss of work by L_1 and its ratio to the disposable work L_m by ζ_1, then the latter term may be called the loss of effect due to the imperfection of the cycle; we then get, with the help of equations (36) and (38), for the loss of work

$$L_1 = \frac{G}{AT_1}[(q_1-q_0)(T_1-T_2)-T(q_1-q_2)+T_1T_2(\tau_1-\tau_2)], \quad (39)$$

and for the loss of effect

$$\zeta_1 = \frac{(q_1-q_0)(T_1-T_2)-T_1(q_1-q_2)+T_1T_2(\tau_1-\tau_2)}{(x_1r_1+q_1-q_0)(T_1-T_2)} \quad . \quad . \quad . \quad (40)$$

If we assume for the specific heat c of the water a constant mean value, as is always permissible in steam-engine calculations with moderate differences of temperature, we can write

$$q_1-q_0=c(T_1-T_0) \quad \text{and} \quad q_1-q_2=c(T_1-T_2);$$

consequently there follows, from equation (39), for the loss of work L_1 the approximate value

$$L_1=\frac{G}{AT_1}[T_1T_2(\tau_1-\tau_2)-T_0(q_1-q_2)], \quad \ldots \quad (39a)$$

and in like manner we get from equation (40) for the loss of effect

$$\zeta_1=\frac{T_1T_2(\tau_1-\tau_2)-T_0(q_1-q_2)}{(x_1r_1+q_1-q_0)(T_1-T_2)}. \quad \ldots \quad (40a)$$

If the difference between the temperature limits is not considerable we can, by making further use of approximate calculation, obtain from equation (54a), p. 59,

$$\tau_1-\tau_2=c\log_e\frac{T_1}{T_2}=c\log_e\left(1+\frac{T_1-T_2}{T_2}\right),$$

or with sufficient accuracy

$$\tau_1-\tau_2=\frac{c(T_1-T_2)}{T_2}=\frac{q_1-q_2}{T_2},$$

and accordingly we shall have in place of equations (39) and (40) the following formulas:

For the loss of work

$$L_1=\frac{G}{AT_1}(q_1-q_2)(T_1-T_0), \quad \ldots \quad (39b)$$

and for the loss of effect

$$\zeta_1=\frac{q_1-q_0}{x_1r_1+q_1-q_0}. \quad \ldots \quad (40b)$$

These approximate formulas at least show what quantities have the most influence on the loss of effect.

The preceding formulas are derived under the supposition that the feeding of the boiler is accomplished by an ordinary feed pump,

which supplies the water to the boiler at the temperature t_0; and that before the beginning of the evaporation the water is first raised to the temperature t_1 of the boiler. Every kilogram [pound] of feed water therefore needs, in the boiler, a heat quantity $(q_1 - q_0)$ which cannot be utilized for the work development of the engine.

We obtain exactly the same result under the hypothesis that the feeding of the boiler is effected by a Giffard injector; its use requires, according to the investigation made in § 20, p. 149, and with equation (52), exactly the same heat quantity per unit of weight of feed water;—here, of course, as with the ordinary feed pump, the terms $A[\sigma(p_1 - p_0) + h]$ adduced on pp. 149 and 151 are neglected because they are so small.

In the perfect engine, however, the feed water, by means of the feed cylinder, enters the boiler at the temperature t_1, and therefore does not need to be first brought, by extra heating, to the boiler temperature. The preheating is accomplished here by adiabatic compression of a particular, prescribed, mixture of water and steam, taken from the condenser.

If, in the actual engine, the water drawn in by the feed pump is heated beforehand to the boiler temperature by a supply of heat otherwise unavailable for the heating of the boiler, then we must substitute in the preceding equations $t_1 = t_0$ and therefore also $q_1 = q_0$, and then from equation (40*b*) we obtain the loss of effect $\zeta_1 = 0$, or by the exact formula (40) a quantity so small that the actual engine is nearly identical with the perfect one. Such an hypothesis cannot be realized, and still less can the injector here accomplish the purpose, for in the main it can only be run with feed water of a comparatively low temperature.

From all this it follows that an improvement of the cycle of the actual engine can only be attained by an effort to bring its cycle nearer to that of the perfect engine.

Whether there is a real necessity or not can be decided by ascertaining whether at the ordinarily occurring temperature values in engines the loss of effect ζ_1, determined by equation (40) or (40*a*), is really so considerable that we ought to take

thought of its diminution. The question is best settled by special examples.

Example 1. Let a steam engine work without condensation at an admission pressure of $p_1 = 7$ kg. [99.56 lb.]; then, according to Table 12 of the Appendix, $t_1 = 164.03$ [327.254], $q_1 = 165.89$ [298.6], $r_1 = 490.64$ [883.15], and $\tau_1 = 0.47531$.

The lower temperature limit is $t_2 = 100°$ [212°], also $p_2 = 1.0333$ kg. [14.697 lb. per sq. in.], and to these values correspond $q_2 = 100.50$ [180.9] and $\tau_2 = 0.31356$.

Let us now assume that the engine works with dry steam, so that $x_1 = 1$, and that the temperature of the feed water $t_0 = 15°$ [59°], then we have for the actual engine, according to equation (36),

$$AL = 76.939\ G \text{ Cal. } [138.49\ G \text{ B.t.u.}],$$

and for the work L_m of a perfect engine under like conditions we have, according to equation (38),

$$AL_m = 93.984\ G \text{ Cal. } [169.17\ G \text{ B.t.u.}];$$

hence the loss of work, in consequence of the imperfection of the cycle, is

$$AL_1 = A(L_m - L) = 17.045\ G\ [30.68\ G \text{ B.t.u.}],$$

and the corresponding loss of effect is

$$\zeta_1 = \frac{L_1}{L_m} = 0.181.$$

On the other hand if we assume that the engine works with wet steam and that the steam quality in two cases is $x_1 = 0.90$ and $x_1 = 0.80$, then in the same way we find

$$\zeta_1 = 0.196 \quad \text{and} \quad \zeta_1 = 0.214.$$

We see that the loss of effect increases with the wetness of the driving steam, and this, moreover, is revealed by a glance at equations (40), (40*a*), and (40*b*); we furthermore see that the loss of effect is very considerable and that the cycle of the actual engine must be characterized as a very imperfect one.

If we closely examine the equations quoted, we see that the temperature t_0 of the feed water has a marked influence on the loss of effect.

If we assume in the preceding example that the waste products of combustion preheat the feed water to $t_0 = 80°$ [176°], then the loss of effect, according to equation (40*a*), is for practical purposes given with sufficient accuracy by this tabulation:

for $x_1 = 1$	0.90	0.80
$\zeta_1 = 0.089$	0.097	0.107.

Comparison with the results found above therefore shows that a decided advantage is secured by simply preheating the feed water, for the loss of effect is reduced one-half by the preheating.

Example 2. Let an engine work with condensation and let it have an admission pressure of $p_1 = 7$ kg. [99.56 lb.], and let the pressure of condensation amount to $p_2 = 0.1$ kg. [1.422 lb.]; furthermore let us assume dry live steam, that is, presuppose $x_1 = 1$, then, with the help of the corresponding values from Table 12 of the Appendix, we get, because $q_0 = q_2$, from equations (36) and (38),

$$AL = 151.06\,G \quad \text{and} \quad AL_m = 165.57\,G$$
$$[AL = 271.908\,G \quad \text{and} \quad AL_m = 298.026\,G].$$

There follows for the loss of work

$$AL_1 = 14.51\,G\,[AL_1 = 26.118\,G],$$

and for the loss of effect

$$\zeta_1 = \frac{L_1}{L_m} = 0.087.$$

Under the supposition that in two cases the steam quality of the live steam is $x_1 = 0.90$ and $x_1 = 0.80$, the calculation gives

$$\zeta_1 = 0.095 \quad \text{and} \quad \zeta_1 = 0.104.$$

The examination of the preceding calculated results and the closer consideration of the underlying formulas show that the loss of effect that is due to the imperfection of the actual cycle of our steam engines is not an unimportant one.

In non-condensing engines this is considerably diminished by the preheating of the feed water, and theoretical investigation shows that this preheating (by the waste fire gases) is more useful than has hitherto been generally assumed, although it has long been known that it is advantageous. As it is, the loss of effect ζ_1 is too large not only in the first of the preceding examples but also in the second example, the one on the condensing engine; the loss of work in both cases amounts to nearly 10% of the disposable work, and a glance at equations (40), (40*a*), and (40*b*) shows that it becomes still more considerable at higher admission pressures.

As the admission pressure, in recent steam engineering, is very properly being raised more and more, the question here considered acquires a greater significance; accordingly it is to be expected that,

in time, an effort will be made to change the cycle of our present-day engines so that it will approach the Carnot cycle described by the perfect engine, and that this will be done by utilizing the feed or compression cylinder D represented by Fig. 37, p. 397.[1]

In §§ 49 and 50 (pp. 371 and 381) the question has been so completely treated that we may now refer the reader to those presentations.

§ 52. LOSS OF WORK DUE TO INCOMPLETE EXPANSION.

In the preceding investigations it has been assumed that the expansion of the steam in the steam cylinder continues down to the lower temperature limit, and therefore down to the pressure p_2 corresponding to this limit. But in reality this does not happen; in general the terminal pressure of the expansion is greater and hence the whole work is not utilized. If the expansion only takes place from the volume V_1 to the volume V instead of from V_1 to V_2, then an amount of work remains unutilized which is represented by the vertically hatched area of the curved triangle xx_2a; the corresponding loss of work L_2 can easily be calculated. Let p be

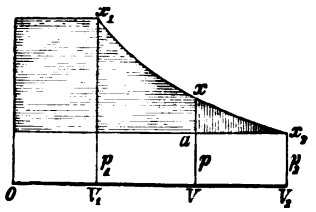

FIG. 40.

[1] The possibility of improving in principle the action of the steam engine was already pointed out in the second edition of this book (1866). Since that time one attempt has been made by the Englishman R. M. Marchant to change the customary method of feeding the boiler, and in the discussion of this engine in the English periodicals my proposition is incidentally mentioned. In the second half of the yearly series 1886, Vol. 62 of "The Engineer," the Marchant engine is repeatedly discussed in a discouraging way, and, in so doing, my proposition, which differs essentially from that of Marchant's, is presentcd as being theoretically untenable but these discussions are based on a misunderstanding of my presentation, and on a lack of knowledge of the behavior of wet vapors during compression. Suitable rejoinders, however, have already been made in the English periodical "Industries," etc., Vol. I, 1886, Nos. 12 and 19.

the actual terminal pressure of the expansion, and x the steam quality at its end, then, because the expansion curve can be regarded as an adiabatic, we have for the course x_1x the relation

$$\tau+\frac{xr}{T}=\tau_1+\frac{x_1r_1}{T_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

and for the whole course x_1x_2,

$$\tau_2+\frac{x_2r_2}{T_2}=\tau_1+\frac{x_1r_1}{T_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

from which the two values x and x_2 of the steam quality can be computed, provided we know the steam quality x_1 at the beginning of the expansion, and can regard the pressures p_1, p, and p_2 as given. For the portion xx_2 (Fig. 40) we get, from equation (15a), p. 379, for the work of expansion which is produced, when measured in units of heat, the expression

$$G(q-q_2+x\rho-x_2\rho_2),$$

where G is understood to be the weight of the mixture of steam and liquid confined in the cylinder; if we subtract from this preceding work that which corresponds to the back pressure p_2 along the path V_2-V, also expressed in units of heat, we get

$$A(V_2-V)p_2=AG(x_2u_2-xu)p_2,$$

and for the loss of work L_2, due to the incomplete expansion, the expression

$$AL_2=G[q-q_2+x\rho-x_2\rho_2-Ap_2(x_2u_2-xu)],$$

or

$$AL_2=G[q-q_2+xr-x_2r_2-Axu(p-p_2)], \quad . \quad . \quad . \quad (43)$$

where x and x_2 are determined by equations (41) and (42).

The work L_m of the perfect engine is determined from equation

(38), p. 399; if we divide equation (43) by AL_m, we get for the loss of effect ζ_2, which is due to incomplete expansion, the equation

$$\zeta_2 = \frac{T_1 \ [q - q_2 + xr - x_2 r_2 - Axu(p - p_2)]}{(x_1 r_1 + q_1 - q_0)(T_1 - T'_2)} \quad . \ . \ . \ . \quad (44)$$

In condensing engines we must here substitute $q_0 = q_2$.

To the preceding we may now add the determination of two others, a loss of work and a loss of effect.

The above calculations are valid under the assumption that the admission pressure is identical with the boiler pressure p_1, and, also, that the back pressure during exhaust is equal to the external or condenser pressure p_2. But in actual construction this assumption is not realized; let p_1' represent the real admission pressure, and p_2' the back pressure, then the areas distinguished by vertical hatchings in Fig. 41 represent the two corresponding losses of work. The loss of work L_3, which is due to the difference between boiler and admission pressure, is determined with sufficient accuracy from

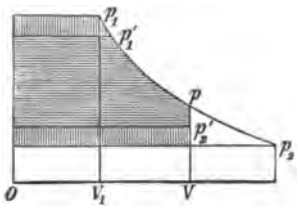

FIG. 41.

$$AL_3 = AV_1(p_1 - p_1') = AGx_1u_1(p_1 - p_1'), \quad . \ . \ . \quad (45)$$

and the loss of work L_4, which is due to the difference between external pressure and back pressure, is given by the equation

$$AL_4 = AV(p_2' - p_2) = AGxu(p_2' - p_2). \quad . \ . \ . \quad (46)$$

If we divide the two equations by expression (38), p. 399, representing the work of the perfect engine, we get the corresponding losses of effect ζ_3 and ζ_4, i.e., get

$$\zeta_3 = \frac{Ax_1u_1(p_1 - p_1')T_1}{(x_1r_1 + q_1 - q_0)(T_1 - T_2)} \quad . \ . \ . \ . \quad (45a)$$

and

$$\zeta_4 = \frac{Axu(p_2' - p_2)T_1}{(x_1 r_1 + q_1 - q_0)(T_1 - T_2)}. \quad . \quad . \quad . \quad . \quad (46a)$$

To be sure both formulas are only approximate expressions, but they can be regarded as sufficiently exact for the present investigations.

In non-condensing engines we ordinarily assume $p_2' = 1.1\ p_2$, and in condensing engines $p_2' = 2\ p_2$, while the relation between admission and boiler pressure is taken as varying from $p_1' = 0.80\ p_1$ to 0.90 p_1 when throttling is excluded.

Example. Let a condensing engine work with a pressure $p_1 = 7$ kg. [99.56 lb.], let the terminal pressure of the expansion be $p = 0.6$ kg. [8.534 lb.], and let the condenser pressure amount to $p_2 = 0.1$ kg. [1.422 lb.].

If we take the steam quality at the beginning of the expansion as equal to $x_1 = 1$, $x_1 = 0.9$, and $x_1 = 0.8$, and if we calculate the work L_m according to equation (38), p. 399, and substitute $q_0 = q_2$, there will follow, with the help of corresponding values of Table 12 of the Appendix, respectively,

$x_1 =$	1,	0.90,	0.80,
$AL_m =$	165.57 G,	152.27 G,	138.97 G
[$AL_m =$	298.026 G,	274.086 G,	250.146 G].

Equations (41) and (42) respectively give

$x =$	0.8684,	0.7948,	0.7212,
$x_2 =$	0.7997,	0.7375,	0.6752.

Hence, according to equation (43), we have for the loss of work L_2 due to incomplete expansion:

$AL_2 =$	27.138 G,	25.037 G,	22.893 G
[$AL_2 =$	48.848 G,	45.067 G,	41.207 G];

and for the loss of effect ζ_2 resulting from the incomplete expansion:

$$\zeta_2 = 0.164, \quad 0.164, \quad 0.165.$$

Let us assume that, with boiler pressure $p_1 = 7$ kg. [99.56 lb]., the admission pressure is $p_1' = 0.9$, $p_1 = 6.3$ kg. [89.606 lb.]; then we get for the loss of work L_3 and for the loss of effect ζ_3, which is due to the difference between boiler and admission pressure, from equations (45) and (45a) respectively:

$AL_3 =$	4.499 G,	4.049 G,	3.599 G,
$\zeta_4 =$	0.027,	0.027,	0.026
[$AL_3 =$	8.098 G,	7.288 G,	6.478 G].

Finally, if the back pressure on the piston on the exhaust side is $p_2' = 2\ p_2 = 0.2$ kg. [2.845 lb.], then we find for the loss of work L_4 and for the loss of effect ζ_4 due to the difference between back pressure and condenser pressure, according to equations (46) and (46a) respectively:

$$\begin{aligned} AL_4 &= 5.194\,G, \quad 4.790\,G, \quad 4.694\,G, \\ \zeta_4 &= 0.031, \quad 0.032, \quad 0.034 \\ [AL_4 &= 9.349\,G, \quad 8.622\,G, \quad 8.449\,G]. \end{aligned}$$

For the same engine, example 2, p. 405, gave for the loss of effect ζ_1 due to the imperfections of the cycle process the respective values

$$\zeta_1 = 0.087, \quad 0.095, \quad 0.104.$$

Of these four values, the one due to incomplete expansion is the greatest; recent practice has therefore fully justified reducing more and more the terminal pressure of the expansion.

In the preceding example the conditions in a compound engine were kept in view. In a single-cylinder engine the terminal pressure p would have to be taken greater, and hence the loss of effect would become considerably greater than in the preceding case.

§ 53. LOSS OF WORK DUE TO CLEARANCE. ACTION OF COMPRESSION.

When investigating the influence of the clearance on the utilization of the steam, we encounter very complicated formulas even under the simplest hypotheses; the laws of thermodynamics to be sure permit of an exact following of the problem, but the corresponding formulas contain a quantity concerning which, in numerical calculations, we are compelled to make an arbitary assumption; it is the steam quality existing in the clearance just as the live steam enters it from the boiler. Even in the calorimetric investigation of the steam engine given below the same uncertainty arises in general, and probably will be removed only by careful experiments on steam engines.

On account of this uncertainty the following consideration of the action of the clearance will be carried only so far as is necessary to supplement the above investigations.

Let us assume that the engine describes completely the cycle of the actual engine, and that therefore the admission pressure is

identical with the boiler pressure p_1, and that the exhaust pressure p_2 is identical with the external pressure, and that, accordingly, complete expansion obtains; let us, furthermore, assume that no compression takes place and that the influence of the cylinder walls on the steam can be disregarded.

Fig. 42 represents the corresponding indicator diagram.

Let V_0 be the volume of the hurtful space, V_1 the volume traversed during admission, so that V_0+V_1 is the admission volume, i.e., the space occupied by the mixture of steam and liquid at the beginning of expansion; let x_1 be the steam quality of the mixture at this instant, let x_k represent the quality of the steam coming from the boiler and entering the cylinder, and let G kg. [lb.] of mixture enter per stroke.

Let the clearance space just before admission contain a mixture of G_0 kg. [lb.] possessing the steam quality x_0 and subject to the pressure p_2, because compression is excluded. At the entrance of the steam to the cylinder there occurs, in the first place, a mixture with the unexpelled steam, and not till then does admission action begin, and then it continues to the point x_1 (Fig. 42). The heat quantity Q' which, in so doing, must be imparted to the boiler amounts to

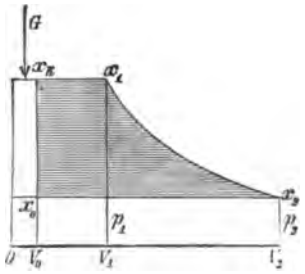

FIG. 42.

$$Q' = Gx_k r_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

and the work L' produced during admission, measured in units of heat, is

$$AL' = A[(G+G_0)(x_1u_1+\sigma) - G_0(x_0u_2+\sigma)]p_1. \quad . \quad . \quad (48)$$

The process discussed here is a non-reversible one. In order to determine the relation between the several magnitudes of the preceding formulas let us now suppose the whole mass to be brought back in reversible fashion to the initial state, and let us make use of the propositions in § 15, Vol. I, p. 74, and, in particular, make use of equation (54), Vol. I, p. 78. Let us suppose,

for this purpose, the mass to be compressed from the point x_1 under constant pressure p_1, and converted into water, but suppose G kg. [lb.] of it to be forced back into the boiler; the quantity of heat Q'' which in so doing must be withdrawn amounts to

$$Q'' = -(G+G_0)x_1 r_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

and the work which must be expended is

$$AL'' = -A(G+G_0)x_1 p_1 u_1 - AG\sigma p_1. \quad . \quad . \quad . \quad (50)$$

In the cylinder G_0 kg. [lb.] of water of the temperature t_1 remains behind; this must, in the first place, be cooled to the temperature t_2 of the clearance space, and then $G_0 x_0$ kg. [lb.] of it must be converted into steam under constant pressure p_2; the necessary heat quantity Q''' amounts to

$$Q''' = -G_0(q_1 - q_2) + G_0 x_0 r_2, \quad . \quad . \quad . \quad . \quad . \quad (51)$$

and the work L''' produced amounts to

$$AL''' = AG_0 x_0 p_2 u_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

If we collect the heat quantities and work quantities adduced in the foregoing, according to the equation $\Sigma(Q) = \Sigma(AL)$, there follows

$$Q' + Q'' + Q''' = AL' + AL'' + AL''',$$

and utilizing the preceding formulas, we get after a simple reduction

$$Gr_1(x_k - x_1) = G_0[x_1 r_1 - x_0 r_2 + q_1 - q_2 - Ax_0 u_2(p_1 - p_2)] \quad . \quad (53)$$

as a first result of the preceding considerations.

When determining the specific volume of the unit of weight of mixture from the formula $v = xu + \sigma$, we can unhesitatingly neglect the quantity σ as very small; hence we obtain $v = xu$, and, in the case before us,

$$V_1 + V_0 = (G+G_0)x_1 u_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

and

$$V_0 = G_0 x_0 u_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

If we utilize these two formulas in equation (53) we get after some reductions the equation

$$Gx_k r_1 = V_1 \frac{r_1}{u_1} + V_0 \left[\frac{\rho_1}{u_1} - \frac{\rho_2}{u_2} + \frac{q_1 - q_2}{x_0 u_2} \right]. \quad . \quad . \quad . \quad (53a)$$

It is evident from equation (53) that on account of the mixing during the admission a change of steam quality takes place from x_k to x_1; if there is no clearance space, so that $G_0 = 0$, then such a change does not occur, because then $x_1 = x_k$.

Furthermore it is evident from equation (53*a*) that, in engines with clearance space, the steam consumption Gx_k is always greater than in engines without such a space, because here the second term of the right member of the equation is always positive. If the steam quality x_0 in the clearance space is known and if the quality x_k of the boiler steam is also known, then the weight G can be computed from equation (53*a*), and likewise the weight G_0 from equation (55), whereupon equation (54) determines the steam quality x_1 at the end of the admission.[1]

To the problem treated in the foregoing there is now added a second one bearing on the indicated work of the engine.

With Fig. 42 as a basis, we find for the work L' during admission, measured in units of heat, the expression already given above, namely,

$$AL' = A[(G + G_0)(x_1 u_1 + \sigma) - G_0(x_0 u_2 + \sigma)]p_1.$$

The work of expansion L'', according to the propositions given earlier, is

$$AL'' = (G + G_0)[q_1 - q_2 + x_1 \rho_1 - x_2 \rho_2].$$

[1] The above problem I have discussed in a more general way in an article "Über die Wirkung des Drosselns und den Einfluss des schädlichen Raumes auf die bei Dampfmaschinen verbrauchte Dampfmenge" in Zivilingenieur, Vol. 21, p. 1, 1875; I there treated the problem so as to include the influence of an artificial difference between boiler and admission pressure, i.e., so as to include throttling; the problem was also extended to the case of superheated vapors. The results to be sure are only of theoretical interest, because the influence of the cylinder walls is left out of account, as it is also, for the present, in the text above.

The work L''' done in overcoming the back pressure is

$$AL''' = -[A(G+G_0)(x_2u_2+\sigma)p_2 - AG_0(x_0u_2+\sigma)p_2].$$

Finally the work L^{IV} which is necessary to feed the boiler follows from

$$AL^{IV} = -AG\sigma(p_1-p_2).$$

After some reductions the sum of the preceding quantities of work gives the indicated work L of the engine under the assumptions used here, and is

$$AL = (G+G_0)(q_1-q_2+x_1r_1-x_2r_2) - AG_0x_0u_2(p_1-p_2), \quad (56)$$

or, utilizing equation (53), it is

$$AL = G[q_1-q_2+x_kr_1-x_2r_2]+G_0r_2(x_0-x_2), \quad (56a)$$

where the product x_2r_2 can be determined from the known equation

$$\tau_2+\frac{x_2r_2}{T_2} = \tau_1+\frac{x_1r_1}{T_1}.$$

If there is no hurtful space on hand, so that $G_0=0$, then equation (56) passes over into equation (36), p. 398; now if the boiler is fed with water of a temperature t_0, the heat quantity which must be supplied to the boiler is

$$Q_1 = G(x_kr_1+q_1-q_0),$$

and therefore the work L_m of the perfect engine is

$$AL_m = G\frac{(x_kr_1+q_1-q_0)(T_1-T_2)}{T_1}. \quad (57)$$

Then, utilizing equations (56), (57), and (53), and assuming the steam qualities x_k and x_0 as known, there follows for the loss of work

$$AL_m - AL,$$

and for the loss of effect

$$\zeta = \frac{L_m-L}{L_m},$$

which quantities correspond to the imperfection of the cycle of the actual engine when the influence of the clearance space

is simultaneously considered. The loss of effect ζ in a special case can be divided into two parts, so that we can distinguish between the part which is due to the imperfection of the cycle and that which is due to the presence of the clearance space. Investigation can then be extended to the determination of the loss of effect which results from incomplete expansion, and, on the whole, can be extended in the manner followed in § 51.

If the expansion is assumed to be a complete one, as presupposed in the foregoing and in accordance with Fig. 42, p. 411, we may interchange x_0 with x_2, because during the exhaust the mass is quietly pushed out of the cylinders; but with the incomplete expansion the discharge is a stormy one, and no means exists for determining analytically the steam quality x_0 in the clearance space. If we suppose the steam in the clearance space to be dry saturated, so that we may take $x_0 = 1$, the above-developed formulas lend themselves easily to the solution of particular numerical examples.

But it is of great practical importance to investigate the influence of compression, for this was left out of account in the preceding, and we will now take it up under the special assumption that the compression is a complete one and therefore that the steam pressure in the clearance space at the end of compression is identical with the pressure p_1 of the entering steam. Fig. 43 shows the indicator diagram for the case in hand.

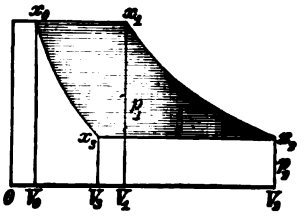

FIG. 43.

Let x_0 be the steam quality in the clearance space at the end of compression; let x_1 be the quality at the end of admission, and x_k that of the boiler steam. Then we get directly

$$(G+G_0)x_1 = Gx_k + G_0x_0, \quad \ldots\ldots \quad (58)$$

because during admission we have simply a mixing under constant pressure, and this is an equation which might also have been derived by the method given on p. 411.

The determination of the indicated work measured in units of heat is now easily obtained as follows; and in so doing we will utilize the notation hitherto employed and also that given in Fig. 43.

The work of admission L' is

$$AL' = A[(G+G_0)(x_1u_1+\sigma) - G_0(x_0u_1+\sigma)]p_1.$$

The work of expansion L'' for complete expansion is

$$AL'' = (G+G_0)[q_1 - q_2 + x_1\rho_1 - x_2\rho_2].$$

The work L''' against the back pressure along the path x_2x_3 follows from

$$AL''' = A[(G+G_0)(x_2u_2+\sigma) - G_0(x_3u_2+\sigma)]p_2,$$

and the work of compression L^{IV} from

$$AL^{IV} = G_0[q_1 - q_2 + x_0\rho_1 - x_3\rho_2].$$

Hence we get the indicated work from

$$L = L' + L'' - L''' - L^{IV},$$

and utilizing the preceding formulas we obtain, after some reduction,

$$AL = G(q_1 - q_2 + x_1r_1 - x_2r_2) + G_0[(x_1 - x_0)r_1 - (x_2 - x_3)r_2], \quad (59)$$

after we have first deducted the work of feeding,

$$AG\sigma(p_1 - p_2).$$

As the two curves x_1x_2 and x_0x_3 can be regarded as adiabatics, we have the known relations

$$\tau_2 + \frac{x_2r_2}{T_2} = \tau_1 + \frac{x_1r_1}{T_1},$$

$$\tau_2 + \frac{x_3r_2}{T_2} = \tau_1 + \frac{x_0r_1}{T_1},$$

and from these, by subtraction, we get

$$\frac{(x_2-x_3)r_2}{T_2}=\frac{(x_1-x_0)r_1}{T_1}, \quad . \quad . \quad . \quad . \quad . \quad (60)$$

so that equation (59) passes over into the following one:

$$AL=G\left[\frac{x_1r_1}{T_1}(T_1-T_2)+q_1-q_2-T_2(\tau_1-\tau_2)\right]. \quad . \quad . \quad (59a)$$

This equation furnishes an interesting result; the quantities which relate to the clearance space have disappeared from the formula, and the equation is identical with equation (36), which gave the indicated work of an engine without clearance space. In this comparison it is, to be sure, assumed that in both cases the steam quality x_1 at the end of admission is the same. But, as we may substitute $x_3=x_2$ after complete expansion, we also find from equation (60) that $x_0=x_1$ and according to equation (58) that $x_k=x_1$; the identity of the two quoted equations is therefore complete.

The compression of the steam in the steam-engine cylinder up to the pressure of admission is therefore to be recommended, because the loss of work occasioned by the hurtful space is thus avoided; but improvement of the cycle of our actual engines, i.e., of close approach to the cycle of the perfect engine, is, however, n o t thereby achieved, although externally the indicator diagram in Fig. 43 shows the same form as the diagram of a perfect engine (compare Fig. 35*a*, p. 373). The process on the compression side of the piston is essentially different in the two cases; but the investigation has shown that the two diagrams Fig. 43 and Fig. 42, pp. 415 and 411, the latter being valid for an engine without hurtful space, have the same indicated work.

As a conclusion of the investigations we will derive two approximate formulas for the indicated work L in the case represented by Fig. 43, because we shall make use of them farther on.

If the two adiabatics x_1x_2 and x_0x_3 are subject to the law $pv^\kappa=$constant, then if the notation in Fig. 43 is employed we get

from the propositions on p. 140, Vol. I, for the work of expansion

$$\frac{(V_1+V_0)p_1}{\kappa-1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}}\right],$$

and for the work of compression

$$\frac{V_0p_1}{\kappa-1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}}\right].$$

The work of admission is V_1p_1, and the work of overcoming the back pressure along the path x_2x_3 is

$$(V_2-V_3)p_2.$$

If we combine these work quantities in proper fashion, and at the same time make use of the relations

$$(V_1+V_0)^\kappa p_1=(V_2+V_0)^\kappa p_2 \quad \text{and} \quad V_0{}^\kappa p_1=(V_0+V_3)^\kappa p_2,$$

we find after some reductions the indicated work:

$$L=\frac{\kappa}{\kappa-1}V_1p_1\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}}\right]. \quad . \quad . \quad . \quad . \quad . \quad (61)$$

On the other hand, if, with Poncelet (see p. 114), we had assumed that the two curves x_1x_2 and x_0x_3 were equilateral hyperbolas, and therefore subject to the law $pv=$constant, we should have obtained for the work of expansion

$$(V_1+V_0)p_1\log_e\frac{p_1}{p_2}.$$

The work of compression would have been

$$V_0p_1\log_e\frac{p_1}{p_2},$$

and we should get furthermore for the admission work V_1p_1 and for the work against the back pressure $(V_2-V_3)p_2$. If we com-

bine the four values and consider the relations

$$(V_1+V_0)p_1=(V_2+V_0)p_2 \quad \text{and} \quad V_0p_1=(V_3+V_0)p_2,$$

we find, after some reductions, for the indicated work L, represented by the hatched area in Fig. 43, the expression

$$L=V_1p_1\log_e\frac{p_1}{p_2}, \quad \ldots\ldots\ldots \quad (62)$$

which equation might have been derived from equation (61) by the assumption of $\kappa=1$.

The two formulas (61) and (62) both hold for the assumption that there is no clearance space and no compression, and that therefore the exhaust takes place during the whole piston stroke (compare Fig. 39*a*, p. 400).

§ 54. LOSS OF WORK DUE TO THE ACTION OF THE CYLINDER WALLS. CALORIMETRIC INVESTIGATION OF STEAM ENGINES.

In all the preceding investigations it has been assumed that the walls of the steam cylinder are completely indifferent to the changes of temperature to which the steam in the cylinder is subjected during its changes of state. Of course this assumption is really wide of the mark; we should rather assume that a peculiar exchange of heat takes place between the steam and the cylinder walls and is of such a nature that the walls withdraw heat from the steam during a certain portion of its change of state, namely, so long as its temperature is less than that of the steam, and conversely that the walls supply heat to the steam during those portions of their pressure changes in which the walls have the higher temperature. If the engine is in normal running condition, then during a double stroke of the piston an equalization will occur of such a charact r that the walls will give back to the steam, in one part of the process, the heat quantity which they withdrew from the steam during another part of the process,

ignoring in so doing the heat quantity which has been lost in the meanwhile by radiation to the outside.

The afore-mentioned action of the cylinder walls is manifested in the indicator diagram, to a certain extent, by the changed course of some of its curves; its influence is manifested particularly during the expansion and compression periods in which the steam runs through its changes of state while confined in the cylinder; but this is accompanied by a change of the indicated work of the engine, and it is to be expected that certain losses of work and losses of effect will occur whose determination will require a more thorough study of the problems involved.

The distinction of having first pointed out emphatically the necessity for taking account of the influence of the walls on the working steam, when specially investigating the steam engine, belongs to Hirn, and he also conducted the first experiments on a large engine, and he inspired a series of other experiments which were conducted in Alsace by Hallauer, Leloutre, and others, under his guidance and cooperation. In the "Bulletin de la Société industrielle de Mulhouse" we find, from the year 1855 on, a series of articles and reports on such experiments, all of which pursued the same aim; in working up the results my formulas for the heat contents of mixtures of vapor and liquid, and also my steam tables, were employed with advantage.

It is not possible beforehand to pursue analytically the occurrences under consideration, and therefore it is necessary to reverse the problem and to present clearly, by analytical expressions, the connections between the experimental results for the purpose of discussion; for this sort of investigation Gustav Schmidt introduced the suitable designation "calorimetric investigation of steam engines."

Ledoux[1] first touched the question with the help of the corresponding laws of thermodynamics without attaching himself directly to Hirn's presentations and experimental results; this was first done by Dwelshauvers-Dery[2] in a more

[1] Ledoux, "De la condensation de la vapeur à l'intérieur des cylindres des machines." Annales des mines, 1877.

[2] Dwelshauvers-Dery, "Exposé succinct de la théorie pratique des moteurs à vapeurs." Revue universelle des mines, 1880.

complete fashion and by setting up certain analytical expressions. The equations in question, however, on account of their general form, are not suited for utilizing in a numerical way existing experimental results for the purpose of elucidating the problems arising here; in general they only furnish a basis for the following of the complicated occurrences. For the purpose of direct practical utilization I have cast the fundamental equations into a particular form and have thus more fully elucidated the course of Hirn's[1] thought; at the same time I pointed out that account ought to be taken of other questions which were not considered in working up the Alsatian experiments.

The fundamental equations set up by me have since then come into general use, but concerning the extensions mentioned, of which we will speak later, there are still different opinions, and unanimity will probably only be attained after further experiments on actual engines.

For the following investigations we will assume a single-cylinder engine, and before discussing the occurrences in the cylinder itself we will determine the heat quantity Q which must be imparted to the steam boiler per stroke. Let G be the weight of the mixture of steam and water which enters the cylinder during every stroke; let x be the quality of the steam which is formed in the boiler under the pressure p and the corresponding temperature t. For the sake of simplicity and because the following investigations are not limited thereby, let us assume that the boiler is fed with water of the temperature of 0° [32°]; then the heat quantity Q supplied to the boiler per stroke is

$$Q = G(q + xr). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63a)$$

If, in the assumed experiments, G' kg. [lb.] of steam condenses in the steam pipe on account of the radiation of the heat, then, in order to avoid specially introducing it into the calculation, it must be discharged to the outside and must not be led back for the purpose of feeding the boiler as is usually done. In

[1] "Kalorimetrische Untersuchung der Dampfmaschinen." Zivilingenieur, Vol. 27, 1881, p. 385, and Vol. 28, 1882, p. 353.

equation (63*a*) Q refers expressly to the steam mixture G which actually enters the cylinder.

On the other hand, if the steam engine works with superheated vapor, and its temperature at the entrance to the cylinder is designated by t_x, then the heat quantity necessary for the generation of the steam is

$$Q = G[q + r + c_p(t_x - t)], \quad . \quad . \quad . \quad . \quad . \quad . \quad (63b)$$

where $c_p = 0.4805$ represents the specific heat at constant pressure for the vapor of water.

Hereafter, in the one case as in the other, the heat quantity supplied to the boiler will be represented simply by

$$Q = \lambda G, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

where the significance of λ follows from comparison with equations (63*a*) and (63*b*).

After this preparation we can proceed to a closer consideration of the occurrences within the steam cylinder of a condensing engine.

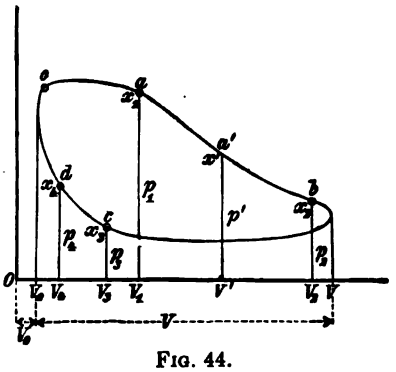

FIG. 44.

Fig. 44 is a sketch of an indicator diagram taken with the indicator.

The end of the admission is at the point a, and it is here that the expansion begins; at the point b we have the end of expansion and therefore the beginning of the exhaust of the steam. At this instant suppose the piston to have traversed in the cylinder the volume V_2, while the volume swept through during the admission is designated by V_1. The volume traversed by the piston during the whole stroke is designated by V, and as the figure gives $V_2 < V$ we have here preexhaust.

At the point c the exhaust port is closed and compression begins. Along the path bc the interior of the steam cylinder is constantly in communication with the condenser, while compression takes place along the path cd, preadmission occurring as early as the point d. Let the space swept through by the steam piston from the shut-off at c to the end of the stroke be designated by V_3, and let V_4 be the piston travel from the beginning of the preadmission to the end of the piston stroke; let the clearance be characterized by V_0 and let G_0 represent the weight of mixture of steam and water confined in the cylinder while compression takes place along the path cd.

In the further investigations it will be assumed that the diagram shown in Fig. 44 is given by particular indicator experiments, and that it is now a question of obtaining, with its help and appropriate measurements, a clear conception of the heat exchange between steam and cylinder walls.

At the four points a, b, c, and d we find respectively the pressure p_1, p_2, p_3, and p_4, and for the volume of steam at each of the four points V_1+V_0, V_2+V_0, V_3+V_0, and V_4+V_0.

In the first place let us assume that the steam is wet in all parts of the whole cycle, and let the steam quality at the four cardinal points be, respectively, x_1, x_2, x_3, and x_4; then we immediately obtain the following relations:

$$\left.\begin{aligned} V_1+V_0&=(G+G_0)(x_1u_1+\sigma),\\ V_2+V_0&=(G+G_0)(x_2u_2+\sigma),\\ V_3+V_0&=G_0(x_3u_3+\sigma),\\ V_4+V_0&=G_0(x_4u_4+\sigma). \end{aligned}\right\} \quad \ldots\ldots \quad (64)$$

If we designate by J_1, J_2, J_3, and J_4 the heat contents of the mixture of steam and fluid at the four points, then by a much-used proposition we have

$$\left.\begin{aligned} J_1&=(G+G_0)(q_1+x_1\rho_1),\\ J_2&=(G+G_0)(q_2+x_2\rho_2),\\ J_3&=G_0(q_3+x_3\rho_3),\\ J_4&=G_0(q_4+x_4\rho_4). \end{aligned}\right\} \quad \ldots\ldots \quad (65)$$

By combining the two corresponding equations of (64) and (65) and considering the transformation discussed on p. 77, we obtain

$$\left.\begin{aligned} J_1 &= (G+G_0)\omega t_1 + (V_1+V_0)\frac{\rho_1}{u_1}, \\ J_2 &= (G+G_0)\omega t_2 + (V_2+V_0)\frac{\rho_2}{u_2}, \\ J_3 &= G_0\omega t_3 + (V_3+V_0)\frac{\rho_3}{u_3}, \\ J_4 &= G_0\omega t_4 + (V_4+V_0)\frac{\rho_4}{u_4}, \end{aligned}\right\} \quad \ldots \quad (65a)$$

where we may substitute for the vapor of water $\omega = 1$.

Finally equation (64) can be written in simpler fashion when we neglect σ as very small and introduce the specific weight γ of dry saturated vapor:

$$\left.\begin{aligned} (G+G_0)x_1 &= (V_1+V_0)\gamma_1, \\ (G+G_0)x_2 &= (V_2+V_0)\gamma_2, \\ G_0x_3 &= (V_3+V_0)\gamma_3, \\ G_0x_4 &= (V_4+V_0)\gamma_4. \end{aligned}\right\} \quad \ldots \quad (64a)$$

On the other hand if the steam in the cylinder is in a superheated condition, which happens when in one or all of the cardinal points the preceding equations give the impossible value $x > 1$, then the temperature T takes the place of x.

According to p. 237 the equation of condition for a unit of weight of steam is

$$pv = BT - C\sqrt[4]{p},$$

where, when p is given in kg. per sq. m. [lb. per sq. in.], we must substitute

$$B = 50.933 \quad \text{and} \quad C = 192.50$$
$$[B = 0.64459 \quad \text{and} \quad C = 22.5819].$$

The heat content of a unit of weight of superheated steam, on the other hand, is calculated from equation (56), p. 249, and is

$$J = J_0 + \frac{Apv}{\kappa - 1}.$$

If for this we write $J=\alpha+\beta pv$, where, in accordance with the values of J_0 and κ given earlier on p. 249, we must substitute $\alpha=476.11$ [857.00] and $\beta=0.007075$ [0.003982], then in place of equations (64) we have

$$\left.\begin{aligned}(V_1+V_0)p_1&=(G+G_0)[BT_1-C\sqrt[4]{p_1}],\\(V_2+V_0)p_2&=(G+G_0)[BT_2-C\sqrt[4]{p_2}],\\(V_3+V_0)p_3&=G_0[BT_3-C\sqrt[4]{p_3}],\\(V_4+V_0)p_4&=G_0[BT_4-C\sqrt[4]{p_4}],\end{aligned}\right\} \quad . \ . \ . \quad (66)$$

and for the heat contents we get in place of equations (65) or (65*a*) the following equations:

$$\left.\begin{aligned}J_1&=\alpha(G+G_0)+\beta(V_1+V_0)p_1,\\J_2&=\alpha(G+G_0)+\beta(V_2+V_0)p_2,\\J_3&=\alpha G_0+\beta(V_3+V_0)p_3,\\J_4&=\alpha G_0+\beta(V_4+V_0)p_4.\end{aligned}\right\} \quad . \ . \ . \ . \quad (67)$$

In Fig. 45 the indicator diagram of Fig. 44 is repeated, but here there is also indicated the heat exchange that occurs. During admission along the path *ao* a work L_a is produced which is determined by the total area lying under *ao* and can easily be found from the diagram by the planimeter and be expressed in meter-kilograms [foot-pounds], or if we divide this value by 424 [772.83] we get the expression in units of heat. In the following, in order to simplify the formulas, we will assume that the quantities of work occurring in the diagram are always expressed in units of heat.

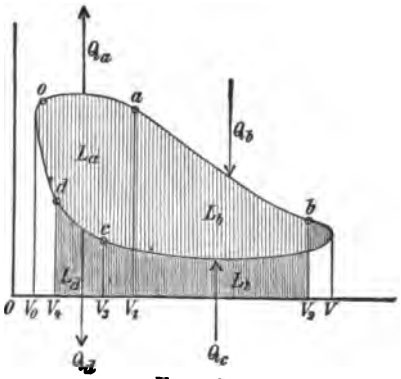

FIG. 45.

Hirn now concludes that during the discharge from the cylinder, i.e., when the interior of the cylinder is in communication with the condenser, the steam along the path *bc*, by

virtue of its lower temperature, cools the walls of the cylinder and consequently these walls in turn cool the live steam coming from the boiler during the admission; let Q_a designate the heat quantity which is thus transferred to the cylinder walls, the direction of the arrow attached to Q_a in Fig. 45 indicating that the heat passes from the steam to the cylinder walls.

During the expansion along the path *ab* the work L_b is produced; it is designated in the figure by the total hatched area lying underneath the portion *ba* of the curve; in so doing the heat quantity Q_b is, in general, imparted to the expanding steam by the walls, which flow is indicated by the reversed direction of the arrow.

During the exhaust of the steam the quantity of heat Q_c is imparted to the steam, and the work consumed amounts to L_c and is measured by the closely hatched area lying under *bc*.

Finally during compression along the path *cd* there is consumed the work L_d which is measured by the closely hatched area lying under *cd*, and in so doing a certain quantity of heat Q_d is imparted by the steam to the cylinder walls.

The problem here is to determine the four heat quantities Q_a, Q_b, Q_c, and Q_d; it is Q_c in particular whose determination is of practical importance, as Hirn very properly points out; this heat is completely lost for the running of the engine, because it is given to the steam by the cylinder walls just as it is leaving for the condenser.

The determination of the four heat quantities requires four fundamental equations, the setting up of which now occasions no difficulty. During admission the boiler receives the heat quantity Q (compare equation (63)), the heat quantity Q_a enters the walls, and the heat quantity L_a is converted into work; therefore $Q-Q_a-L_a$ must have been expended to increase the heat contents of the steam in the cylinder. Now J_1 is the heat content at the end of admission (at the point *a*), and J_4 the content at the beginning of the preadmission of the steam (at the point *d*), and therefore J_1-J_4 is the increase of the heat contents; accordingly we get for the first fundamental equation

$$L_a+Q_a=\lambda G+J_4-J_1, \quad \ldots\ldots \quad \text{(I)}$$

in which the terms of the right member can be replaced by equations (63*a*) or (63*b*) as the case may be, and by the corresponding equations (65) or (65*a*) or (67).

During expansion the heat quantity L_b is produced as work and the heat quantity Q_b in general is supplied to the expanding steam; the difference therefore represents the diminution $(J_1 - J_2)$ of the heat contents; we therefore get

$$L_b - Q_b = J_1 - J_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(II)}$$

as the second fundamental equation.

For the period of steam exhaust we must take into consideration the occurrences in the condenser. Let G_i be the weight of the cooling water in kg. (lb.) per piston stroke which enters with the temperature t_5 and, when a surface condenser is considered, flows away with the temperature t_6; let t_0 be the temperature in the condenser itself, and let us suppose that finally the G mixture of steam and water is completely converted into water.

The heat content at the beginning of compression, i.e., at the end of the exhaust, amounts to Gq_0 in the water of condensation, to $G_i q_6$ in the cooling water, and to J_3 in the mass remaining behind in the cylinder; on the other hand the heat content at the beginning of the exhaust is $G_i q_5$ in the cooling water and is J_2 in the mass contained in the cylinder. Accordingly the increment J' in the heat content is

$$J' = Gq_0 + G_i(q_6 - q_5) + J_3 - J_2.$$

During the exhaust, however, the heat quantity Q_c has been imparted to the mass, and the work L_c has been converted into heat; if, for the sake of completeness, we also assume that in the pipe connecting with the condenser, in the condenser itself, and in the air pump the heat quantity Q_i is lost by radiation, there will follow

$$J' = L_c + Q_c - Q_i,$$

and consequently the equating of these two values will furnish the third fundamental equation:

$$L_c + Q_c = Gq_0 + G_i(q_6 - q_5) + J_3 - J_2 + Q_i \quad . \quad . \quad . \quad \text{(III)}$$

If a jet condenser is employed instead of a surface condenser, we must substitute $q_6 = q_0$ in this equation.

Finally we find for the period of compression, as is at once evident,

$$L_d - Q_d = J_4 - J_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (IV)$$

for the fourth fundamental equation.

To the fundamental equations just derived two others will now be added.

In the first place we get for the indicated work L_i, measured in units of heat,

$$L_i = L_a + L_b - L_c - L_d, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (V)$$

and there exists, besides, a simple relation between the introduced heat quantities. Let Q_v represent the heat quantity lost by radiation from the steam cylinder during one piston stroke, let Q_r be the heat quantity which is simultaneously generated by the piston friction, and, presupposing the presence of a steam jacket, let Q_m represent the heat quantity which enters the cylinder from the jacket; then for the normal running condition we shall have the relation

$$Q_a - Q_b - Q_c + Q_d + Q_r + Q_m - Q_v = 0. \quad . \quad . \quad . \quad (VI)$$

If we unite the first four fundamental equations in the fashion (I+II−III−IV), then we get, with the help of (V) and (VI),

$$L_i = G(\lambda - q_0) - G_i(q_6 - q_5) + (Q_m + Q_r - Q_v - Q_i), \quad . \quad (Va)$$

an equation which might have been written directly.

§ 55. DISCUSSION OF THE EQUATIONS FOR THE HEAT EXCHANGE BETWEEN STEAM AND CYLINDER WALLS.

The formulas developed in the preceding may, under certain circumstances, be employed to determine, from experiments on a steam engine, the heat exchange between steam and cylinder walls.

From the dimensions of the engine and the indicator diagrams taken, let us suppose the following quantities (see Figs. 44 and 45) to be determined:

$$V, V_0, V_1, V_2, V_3, V_4.$$

Furthermore the boiler pressure p is known, and the indicator diagram gives, for the four principal points, the pressures

$$p_1,\ p_2,\ p_3,\ p_4.$$

We determine, moreover, from the indicator diagram with the planimeter the magnitudes

$$L_a,\ L_b,\ L_c,\ L_d, \text{ and } L_i.$$

There is observed the weight G_i of the cooling water needed for the condenser and the weight of feed water needed for the steam boiler, both measured in kg. [lb.] and both reduced to one piston stroke. Now measure the steam weight G' which condenses in the steam pipe, by discharging and collecting it periodically; similarly, if the cylinder is provided with a jacket, measure the steam weight G_m in the jacket and reduce both to the piston stroke; let G_k represent the weight of the feed water; then the value designated by G in the above formulas is determined from the relation

$$G = G_k - G' - G_m. \quad \ldots\ldots\ldots \quad (68)$$

But here the formulas established are valid only under the supposition that the said waters of condensation are not led back to the boiler during the experiments.

Let q be the heat of the liquid corresponding to the temperature of the boiler; then we can calculate with sufficient accuracy the heat quantity $G'q$ lost by radiation from the steam pipe, and can also compute the heat quantity Q_m which enters the cylinder from the jacket, the latter being found from the relation

$$Q_m = G_m q, \quad \ldots\ldots\ldots \quad (69)$$

whether the steam flows through the jacket to the cylinder or whether it stagnates in the jacket.

If we temporarily assume that there is known the weight G_0 of the mixture of steam and water contained in the cylinder at the closure of the exhaust, and hence at the beginning of the com-

pression (at point *d*, Fig. 45), we can determine from equations (64) and (64*a*) the values

$$x_1,\ x_2,\ x_3,\ x_4,$$

provided the steam is wet in all its stages during the whole process within the cylinder; then from the steam tables we find for the pressures p_1, p_2, p_3, p_4 the corresponding values of u_1, u_2, u_3, u_4, those of ρ_1, ρ_2, ρ_3, ρ_4, and also of γ_1, γ_2, γ_3, γ_4.

On the contrary if the steam is superheated at the four principal points or in one or more of them, the corresponding temperature T_1, T_2, T_3, or T_4 of the superheated steam can be calculated from equations (66), and therefore in the first case the values of the four heat contents J_1, J_2, J_3, and J_4 can be found according to equations (65) and (65*a*), and in the second case according to equation (67). We can now proceed to utilize the four fundamental equations (I) to (IV) and with their help determine the heat quantities Q_a, Q_b, Q_c, and Q_d, and this indeed constitutes the main purpose of the whole investigation.

However, in doing this we encounter, in an engine working with wet vapor, a difficulty in determining the value λ in equation (I), which determination is to be accomplished by equation (63*a*); the equation contains the quantity x for whose observation no certain means is at present known; we can only assert this, that in engines running quietly and not too rapidly the steam quality x differs but little from unity.

Another difficulty is encountered when using equation (III) for the determination of the heat quantity Q_i; but probably we can determine the latter accurately enough from equation (Va) if we regard the remaining heat quantities Q_m, Q_r, and Q_v, occurring in this equation, as already determined. Q_m can be found from equation (69), Q_r from the piston friction, and Q_v, according to Grashof,[1] in steam-jacket engines, may be determined by

[1] Grashof, "Kalorimetrische Untersuchung der Dampfmaschinen." Zeitschrift des Vereins deutscher Ingenieure, Vol. 27, 1883, p. 161.

In this article Grashof discusses my paper of the same title and in the main agrees with my views and also bases his considerations on my four funda-

filling the jacket space with boiler steam when the engine is at rest and after the cylinder walls are thoroughly heated determining the amount of steam condensation in the jacket.

Of the two difficulties specified here, the former disappears entirely when we are dealing with superheated steam, and both can be overcome; but matters are different with respect to another question.

Almost all the formulas adduced contain the quantity G_0, which in the preceding was temporarily assumed as already known and which states the weight of the steam in the cylinder at the end of the exhaust and therefore the weight at the beginning of compression.

The question arises as to the constitution of this steam, i.e., as to the amount of water mixed with it—in other words, as to the steam quality x_4 at the point d of the indicator diagram, Figs. 44 and 45, pp. 422 and 425.

If the value x_4 is known, then according to the fourth of equations (64) or (64a) we can determine the important quantity G_0 which influences all the other quantities to be calculated, as the above equations show.

This question led to the interchange of views between Hirn and myself [1] which is pretty well known to mechanical engineers.

Since we have not yet succeeded in determining the quantity G_0 in an experimental way, directly or indirectly, recourse has been had to hypothesis.

---

mental equations, with my notation, but he adds to the right member of equation (I) a term K which arises in equation (II) with a negative sign which is intended to measure in heat the kinetic energy of the mass contained in the cylinder at the end of admission. I have less objection to this suggestion of Grashof's, because I myself first called attention to it, and because it takes account of the influence of the stormy motion of the mass. I have not introduced the term in the text above, because it is probably better to base the investigations on the sum of the two equations (I) and (II) (this is pointed out later in the text), and then the term K will disappear from the analytical expressions.

Grashof moreover emphasizes that the term Q_1 should be added in the fundamental equation (III), and this has been done in the text above, although in most cases the said value may be neglected as insignificant.

[1] Revue universelle des mines, 1882, 2d Series, Vol. XI, p. 15, and 1883, 2d Series, Vol. XIII, p. 1.

Hirn and his coworkers, as well as other engineers, said that the assumption could be made with sufficient certainty that the steam is dry saturated at the beginning of compression, so that we may assume $x_3 = 1$. They conclude that the considerable quantity of water ordinarily present at the end of the expansion is completely swept into the condenser considering the stormy exit during the exhaust and the simultaneous heat supply Q_c from the cylinder walls after some preliminary evaporation.

It cannot be denied that this case is possible and conceivable, particularly when the terminal pressure of the expansion is considerably greater than the steam pressure in the cylinder during exhaust, as was the case in the experiments on Hirn's engine; these experiments will receive further mention later on.

On the other hand the assumption must not be contradicted (we are always thinking of a single-cylinder engine) that, at the beginning of compression, the confined steam may be mixed with water, so that we may have $x_3 < 1$.

Under the assumption of this sort of wet steam the third of equations (64) or (64*a*) gives a greater value for G_0, and a different value for each of the heat quantities Q_a, Q_b, Q_c, and Q_d is therefore found from the fundamental equations (I) to (IV); indeed it is possible that sometimes, in certain engines, the steam quality x_3 may be considerably smaller than unity, so that the wetness of the steam may be considerable.

As was said before, these questions can be decided only by further and careful investigations of steam engines; in general (see remark on p. 87) we can infer from the course of the compression curve something as to the degree of wetness of the steam; with a large amount of moisture the compression curve rises less rapidly than with a less admixture of water (see example on p. 86).

Finally we will direct attention to a question which has already been touched on p. 107. Because of the stormy motion of the steam in the cylinder during admission and during the first part of the expansion, the pressure curve drawn by the indicator cannot be regarded as an equilibrium-pressure curve; consequently there is an uncertainty in measuring the pressure p_1 at the end of admis-

sion, at the point a (Fig. 44, p. 422), because the equilibrium-pressure curve to which the values u_1, γ_1, t_1, etc., belong is, in reality, higher. Therefore if we use the measurement of p_1 the first of equations (64a) will give a value for x_1 that is too small; the same thing follows for J_1, according to the first of equations (65), and so the heat quantities Q_a and Q_b found from equations (I) and (II) will appear somewhat greater than they really are; their difference will not be affected by the uncertain measurement of p_1 because the pressure p_2 at the end of expansion can be assumed as the equilibrium-pressure; likewise nothing is affected that can be derived from (III), (IV), (V), and (VI). It would better suit our purpose, on the whole, not to separate equations (I) and (II),.but use their sum in the calculation because this eliminates the uncertain quantity J_1; we therefore get

$$L_a + L_b + Q_a - Q_b = \lambda G + J_4 - J_2,$$

from which we can obtain the heat quantity $Q_a - Q_b$ given off to the cylinder walls during the course of the admission and expansion.

As regards the now known experiments which permit of a utilization of the preceding developments, those by Hirn on his Logelbach-Colmar engine are in advance of all others because of their completeness. The engine is a beam engine without steam jacket and with slide valve gear; it possesses four slide valves, two for the admission and two for the exhaust; the boiler steam can be directly led to the cylinder or indirectly through a superheater. The diameter of the cylinder is given as 0.605 m. [23.82 in.] and the piston stroke as 1.702 m. [67.01 in.]; when running normally the engine makes 30 revolutions per minute.

Altogether eight experiments were made on the engine; two of them were made on November 18th and 28th, 1873, and the others in 1875, August 26th and 27th, September 7th, 8th, and 29th, and October 28th.

Seven of these eight experiments were accompanied by condensation and different ratios of expansion; five of the seven were

experiments with superheated vapor, and only two of them were conducted with a saturated (wet) vapor; the eighth was carried on with superheated steam, but without condensation.

Hallauer[1] first reported on these experiments in 1877, and later a more detailed report with a statement of essential data was made by Dwelshauvers-Dery.[2] The reports of later experiments by others (some of them made in England) are not sufficiently complete to be followed by calculation; the only exceptions are experiments by Delafond[3] on a Corliss engine. These experiments were conducted with great technical knowledge and were subjected to discussion; they were made on an engine especially arranged for such experiments, and were conducted at different boiler pressures with different ratios of expansion, and also with and without a steam jacket; unfortunately Delafond was not in a position to observe the quantity of cooling water supplied to the condensers, so that his experiments could not be utilized in the direction of the above investigations.

For the elucidation of the preceding investigations let us consider more fully Hirn's experiment of August 27th, 1875; I have already analyzed the experiment (see note, p. 421) and Grashof (see remark, p. 430) then worked it out as an example; more recently this experiment was treated by Dwelshauvers-Dery,[2] who subjected all eight experiments to investigation.

Example. Hirn's engine without a steam jacket; the pressure in the boiler is $p = 48{,}075$ kg. [68.38 lb. per sq. in.], the temperature $t = 150°$ [302°]. The working steam is superheated and has the temperature $t_x = 223°$ [433.4°].

There is a jet condenser, steam weight per stroke is $G = 0.2822$ kg. [0.622 lb.]; the weight of injected water is $G_1 = 8.5983$ kg. [18.956 lb.] and its initial temperature is $t_5 = 16.5°$ [61.7°]; the final temperature, which is here identical with condenser temperature, is $t_6 = t_0 = 35.26°$ [95.47°]; accordingly $q_5 = 16.51$ Cal. [29.718 B.t.u.] and $q_6 = q_0 = 45.30$ Cal. [63.54 B.t.u.].

---

[1] Hallauer, "Expériences sur les moteurs à vapeur." Bulletin de la société industrielle de Mulhouse, 1877, Vol. XLVII.

[2] Dwelshauvers-Dery, "Méthode nouvelle pour représenter l'échange de chaleur entre le métal et la vapeur." Bulletin de Mulhouse, 1888.

[3] Delafond, "Essais effectués sur une machine Corliss aux usines du Creuzot." Annales des mines, 1884.

According to equation (63*b*) we then get $\lambda = 687.29$ Cal. [1237.122 B.t.u.]; and therefore the heat necessary for the generation of the steam is

$$Q = 687.29 \times G = 193.95 \text{ Cal. } [769.66 \text{ B.t.u.}].$$

The heat quantity taken up by the injected water is

$$G_i(q_0 - q_5) = 161.56 \text{ [641.12 B.t.u.]}.$$

Now, according to Hallauer and Dwelshauvers-Dery,

$$\begin{array}{llll} V_0 + V_1 = 0.2224 \text{ cbm.}, & V_0 + V_2 = V_0 + V = 0.4900 \text{ cbm.}, \\ V_0 + V_3 = 0.0400 \text{ cbm.}, & V_0 + V_4 = V_0 = 0.0050 \text{ cbm.} \\ \left[\begin{array}{l} V_0 + V_1 = 7.911 \text{ cu. ft.}, \\ V_0 + V_3 = 1.413 \text{ cu. ft.}, \end{array}\right. & \left.\begin{array}{l} V_0 + V_2 = V_0 + V = 17.295 \text{ cu. ft.}, \\ V_0 + V_4 = V_0 = 0.1766 \text{ cu. ft.} \end{array}\right], \end{array}$$

and hence in this engine there is approximately neither pre-admission nor pre-exhaust of the steam.

The steam pressures at the four principal points (Fig. 44) are

$$\begin{array}{llll} p_1 = 23070 \text{ kg.}, & p_2 = 8417 \text{ kg.}, & p_3 = 1447 \text{ kg.}, & p_4 = 5787 \text{ kg.} \\ [p_1 = 32.813, & p_2 = 11.972, & p_3 = 2.058, & p_4 = 8.231 \text{ lb. per sq. in.}]. \end{array}$$

These values, supposing wet steam to exist at all four points, give, according to steam tables 11 and 12 of the Appendix, the quantities

$$\begin{array}{llll} q_1 = 125.00, & q_2 = 94.79, & q_3 = 52.96, & q_4 = 84.80 \text{ cal.} \\ \rho_1 = 477.22, & \rho_2 = 500.76, & \rho_3 = 533.59, & \rho_4 = 505.58 \text{ cal.} \\ \gamma_1 = 1.2900, & \gamma_2 = 0.4996, & \gamma_3 = 0.0945, & \gamma_4 = 0.3507 \text{ kg.} \\ \left[\begin{array}{l} q_1 = 225.00, \\ \rho_1 = 859.00, \\ \gamma_1 = 0.0806, \end{array}\right. & \begin{array}{l} q_2 = 170.62, \\ \rho_2 = 901.37, \\ \gamma_2 = 0.0312, \end{array} & \begin{array}{l} q_3 = 95.33, \\ \rho_3 = 960.46, \\ \gamma_3 = 0.0059, \end{array} & \left.\begin{array}{l} q_4 = 152.64 \text{ B.t.u.} \\ \rho_4 = 910.04 \text{ B.t.u.} \\ \gamma_4 = 0.0219 \text{ lb.} \end{array}\right]. \end{array}$$

Finally these other work quantities were taken from the indicator diagram and were

$$\begin{array}{llll} L_a = 14.79 \text{ Cal.}, & L_b = 9.24 \text{ Cal.}, & L_c = 1.97 \text{ Cal.}, & L_d = 0.20 \text{ Cal.} \\ [L_a = 58.691, & L_b = 36.667, & L_c = 7.818, & L_d = 0.794 \text{ B.t.u.}], \end{array}$$

from which follows, according to equation (V),

$$L_i = 21.86 \text{ Cal. } [86.747 \text{ B.t.u.}],$$

and from equation (*Va*)

$$Q_i + Q_v - Q_r - Q_m = 0.57 \text{ Cal. } [2.262 \text{ B.t.u.}].$$

As there is no steam jacket, $Q_m = 0$; if we further estimate, as Grashof did, $Q_r = 0.22$ Cal. [0.873 B.t.u.] and $Q_v = 0.75$ Cal. [2.976 B.t.u.], we get

$$Q_i = 0.04 \text{ Cal. } [0.159 \text{ B.t.u.}],$$

a very small quantity in the present case.

There furthermore follows from equation (64*a*)

$$\left.\begin{array}{ll}(G+G_0)x_1=0.2869\text{ kg.}, & (G+G_0)x_2=0.2448\text{ kg.}\\ G_0x_3=0.0038\text{ kg.}, & G_0x_4=0.0018\text{ kg.}\end{array}\right\} \quad \ldots \quad (a)$$

$$\left[\begin{array}{ll}(G+G_0)x_1=0.633\text{ lb.}, & (G+G_0)x_2=0.538\text{ lb.},\\ G_0x_3=0.0084\text{ lb.}, & G_0x_4=0.0040\text{ lb.}\end{array}\right].$$

Now these equations furnish a remarkable result. If we really make the assumption that, at the end of the exhaust, the steam present is exactly dry saturated, then $x_3=1$, and, according to the third of the preceding equations, $G_0=0.0038$ kg. [0.0084 lb.], hence $G+G_0=0.2860$ kg. [0.6305 lb.], and therefore, according to the first of equations (*a*), there follows $x_1>1$, and the steam is still superheated at the end of the admission, although, as it happens, to a very small degree in the present case. A still greater superheating would obtain if it were assumed that the steam at the end of compression is dry saturated, so that $x_4=1$; then, in order to further follow the problem, it would be necessary to have recourse to some of the equations given under (66) and (67); but if we hold fast to the assumption that the steam is not superheated at any of the four main points of the indicator diagram, that is the limiting case for which $x=1$ and $G+G_0=0.2869$ kg. [0.633 lb.], and in such a case we must assume $G_0=0.0047$ kg. [0.0104 lb.]. We then get from equations (*a*)

$$x_1=1, \quad x_2=0.853, \quad x_3=0.808, \quad x_4=0.383;$$

therefore we calculate from equations (65) and from the fundamental equations (I) to (IV):

$$\begin{array}{llll}J_1=172.78, & J_2=149.78, & J_3=2.28, & J_4=1.31,\\ Q_a=7.69, & Q_b=-13.76, & Q_c=22.09, & Q_d=1.17\\ J_1=685.65, & J_2=594.38, & J_3=9.05, & J_4=5.20,\\ Q_a=30.587, & Q_b=-54.604, & Q_c=87.660, & Q_d=4.642\end{array}$$

On the other hand, if we make an assumption which is well within the region of possibility and take $x_3=0.1$, we then have $G_0=0.0380$ kg. [0.0838 lb.] and $G+G_0=0.3202$ [0.7059], and then we find, according to equations (*a*),

$$x_1=0.896, \quad x_2=0.765, \quad x_3=0.100, \quad x_4=0.047,$$

and furthermore,

$$\begin{array}{llll}J_1=176.94, & J_2=152.94, & J_3=4.04, & J_4=4.14\text{ Cal.},\\ Q_a=6.38, & Q_b=-14.75, & Q_c=20.70 & Q_d=0.10\\ J_1=702.16, & J_2=606.91, & J_3=16.03, & J_4=16.43\text{ B.t.u.},\\ Q_a=25.318, & Q_b=-58.533, & Q_c=82.144, & Q_d=0.397\end{array}$$

The circumstance that in both cases Q_b appears negative shows that heat is given off by the steam to the cylinder walls, not only during the admission, but also, taken as a whole, during the expansion. The heat

quantity Q_c which escapes to the condenser appears to be very considerable; according to the above calculations the total heat quantity which is expended per stroke in steam formation is $Q = 193.95$ cal. [769.66 B.t.u.], and then there follows, according to the preceding two assumptions,

$$\frac{Q_c}{Q} = 0.114 \quad \text{and} \quad 0.107 \text{ respectively,}$$

and this loss will nearly correspond to the loss of effect which is called forth when, during exhaust, the interior of the cylinder is set directly into communication with the condenser space.

From the numerical result of the preceding example it may well be inferred that, with the indeterminate value G_0, there still exists a lamentable uncertainty in the whole question discussed here, and that, according to the assumption made with respect to the value x_3, we obtain different values for those heat quantities which are to afford an insight into the heat exchange between steam and walls. It is surely a dubious proceeding to adopt the hypothesis $x_3 = 1$, i.e., that the steam in the cylinder at the beginning of compression is always exactly dry saturated, and to say that it is generally valid for all s i n g l e - c y l i n d e r condensing engines.

Recently, with this hypothesis as a basis, these computed and exchanged heat quantities have been represented graphically in the indicator diagram by representing the heat quantities expressed in units of work by rectangles having bases corresponding to the piston travel.

Let h_a, h_b, h_c, and h_d be the heights of these rectangles, then they can be computed from the formulas

$$V_1 h_a = 424\, Q_a, \quad (V_2 - V_1) h_b = 424\, Q_b \text{ [772.83]},$$
$$(V_1 - V_3) h_c = 424\, Q_c \text{ and } (V_3 - V_4) h_d = 424\, Q_d \text{ [772.83]},$$

and these heights are to be introduced into the diagram with reference to the algebraic sign, a procedure into which we will not enter here.

D w e l s h a u v e r s - D e r y has gone more thoroughly into this question (see remark, p. 434) and tried to show at the same time, by specially drawn curves, t h e c o u r s e of the heat

transferred, and thus to construct, along with the indicator diagram (diagramme des pressions), heat-exchange diagrams (diagrammes des changes).

But the representation in question is a very cumbersome one, and it is much better to replace it by the method of representation often employed above in the transformation of the pressure curves. Here, therefore, we will simply refer the reader to Dwelshauvers-Dery's article in which the procedure is also applied to Hirn's eighth experiment. On the other hand, we will briefly state how the method of transformation can be employed for the same purpose.

In the first place let us specially consider the expansion curve *ab* of the indicator diagram in Fig. 46, then the transformation of this curve will show graphically the heat exchange between steam and cylinder walls during the course of the expansion.

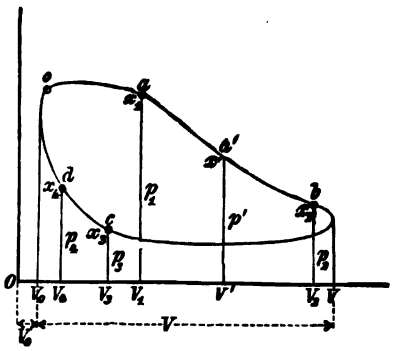

FIG. 46.

If for any point a' of this curve there is taken from the indicator diagram the corresponding piston travel V' and the steam pressure p', then the steam tables will give for the latter value the quantities t', u', ρ', and γ', also τ' and r'.

For the considered point a', therefore, the steam quality x' is determined from the equation

$$(G+G_0)x' = (V'+V_0)\gamma', \quad \ldots\ldots \quad (70)$$

and, according to the propositions discussed on p. 64, the corresponding heat weight P' is found from the equation

$$AP' = (G+G_0)\left(\tau' + \frac{x'r'}{T'}\right). \quad \ldots\ldots \quad (71)$$

If we now determine for a series of points of the expansion curve the value P', and lay off this value as abscissa and the cor-

responding absolute temperature T' as ordinate, then the curve passing through the points thus obtained is the transformation of the expansion curve, and the areas of the strips lying under this curve give directly (as was explained on p. 64) the law of the variation of the heat exchange; moreover, the direction of the curve in its various parts shows whether the heat flows from the walls to the steam or in the opposite direction.

In the same way the compression curve can be transformed; but in effecting the transformation of the admission- and exhaust-pressure curves we encounter the difficulty that G appears to be variable in equation (70); in the former curve G appears to increase and in the other to decrease; accordingly we must either assume a particular hypothesis with respect to the change, or sketch in the transformation curve for the admission and emission according to what seems probable and possible, and this is what Dwelshauvers-Dery really did in the method proposed by him. But to all this there is opposed the circumstance that the admission and the expansion curves in certain portions of their course already mentioned cannot be regarded as reversible pressure curves. We will therefore omit a further consideration of the question.

On the other hand the consideration of the third fundamental equation under (III), p. 427, gives rise to another remark. Let us substitute in it for J_2 and J_3 the formulas given under (65), then there follows

$$L_c+Q_c=Gq_0+G_i(q_6-q_5)-(G+G_0)(q_2+x_2\rho_2)+G_0(q_3+x_3\rho_3)+Q_i.$$

If we assume that the steam exhaust takes place under the constant mean pressure p_3, we can write

$$L_c=(G+G_0)Ap_3x_2u_3-G_0Ap_3x_3u_3,$$

and therefore we have, from the preceding equation,

$$G_i(q_6-q_5)=(G+G_0)[q_2+x_2(\rho_2+Ap_3u_2)] \\ -Gq_0-G_0(q_3+x_3r_3)+Q_c-Q_i,$$

from which can be found the weight G_i of the cooling water for the

condenser; but for this purpose we can substitute $G_0=0$ and $Q_i=0$; we therefore obtain

$$G_i(q_6-q_5)=G[q_2-q_0+x_2(\rho_2+Ap_3u_2)]+Q_c. \quad . \quad . \quad . \quad (72)$$

If we compare this formula with equation (33), which was given on p. 137 for the surface condenser, we find that in the preceding case the heat quantity Q_c has been added to the right member of the equation, but that in other respects there is an identity, provided we consider that in the preceding developments other subscripts have been chosen for the same quantities.

That one must take account of the value Q_c in calculating the weight of cooling water has been emphasized by Dwelshauvers-Dery;[1] of course this assumes that Q_c is known. For this purpose Dwelshauvers-Dery supposes that Ap_3u_2 can be neglected, that during exhaust the heat quantity Q_c is, under all circumstances, just sufficient to completely evaporate the quantity of water $G(1-x_2)$ present at the end of expansion, and that we may assume $Q_c=G(1-x_2)\rho_2$; from equation (72) then follows

$$G_i(q_6-q_5)=G(q_2+\rho_2-q_0),$$

an equation which has been proposed for practical use and in structure agrees with the formulas proposed for the same purpose on pp. 137 and 140.

The presentations of the whole preceding section, which deals with the calorimetric investigation of single-cylinder engines, have shown that the final discussion of certain doubtful points can be reached only by further experiments. The method will also be of importance in multiple-cylinder engines, and will be specially helpful in clarifying the subject of the supplementary heating of the cylinders and receivers.

The fundamental equations developed above can easily be extended to engines with several cylinders; but the further pursuit of this question would lead us too far from the purpose of the book.

[1] Dwelshauvers-Dery, "Note sur la théorie des condenseurs." Revue universelle des mines, 1889, Vol. V, 3d Series, p. 225.

The great progress in the construction of two- and three-cylinder engines is due to the use of high steam pressure, far-going expansion, generous steam-jacketing, and to the fact that the interior of the high-pressure cylinder is no longer directly in communication with the interior of the condenser during the exhaust of the steam.

b) Engines for Superheated Steam.

§ 56. THE INDICATED WORK AND THE THERMAL EFFICIENCY OF THE STEAM ENGINE. THE STANDARD CYCLE AND THE CORRESPONDING EFFICIENCY.

In the foregoing articles the investigations referred expressly to engines working with saturated steam; in this connection a perfect steam engine was understood to be one describing the Carnot cycle, and this cycle was used as the "standard cycle" in judging of the cycle of the actual engine (see § 51, p. 396).

As long as we ignore the heating plant and only consider the heat quantity entering the boiler and which is at our disposal under the upper temperature limit, there can be no objection from the purely theoretical standpoint to this use of the Carnot cycle.

As time passed, however, engines working with superheated steam were more and more extensively used and steam temperatures up to 350° C. [662° F.] were employed, so that with such engines it has become necessary to drop the Carnot cycle as a standard for comparison.

This cycle demands that the steam generation take place at constant, highest temperature, and this temperature must therefore exist in the steam boiler itself, in which the water is first converted into saturated steam, and then it must be brought, in the superheater, at constant temperature with diminishing pressure, into an available condition. The steam temperature of 350° C. [662° F.] is, however, already close to the critical temperature $t_k = 365°$ [689°], which corresponds to the steam pressure $p_k = 200.5$ atmospheres (p. 207); enormous and entirely inadmissible boiler pressures would therefore obtain.

If in judging the superheated steam engine we wish to proceed in the same manner as in the engine working with saturated steam, we must first settle what cycle is necessary for perfection in the former engine; it is certainly not the Carnot cycle; but then, supposing the problem capable of solution, we should not obtain the same measure for the two types of engines; only engines of the same kind could be compared with respect to their excellence, we could not compare an engine of one type with that of another.

The question raised here has, in the course of time, called forth different investigations and propositions, and quite recently, particularly in England, has led to lively discussions among mechanical engineers. The question is evidently one of great technical importance; the difficulty involved is, however, due to the fact that in comparing two engines we have accustomed ourselves too greatly to taking the steam consumption as a basis, instead of the heat quantity required for the production of the steam in the steam boiler and superheater combined.

Let L_i represent the indicated work of any kind of steam engine, measured in mkg. [ft-lb.] and reduced to one kilogram [one pound] of steam consumed, and let Q be the heat quantity necessary to produce the steam, then the ratio

$$L_i : Q$$

at once furnishes an unobjectionable measure for the excellence of the engine considered;[1] the value has already been designated (p. 384) as the engine-power equivalent of the heat; it was especially calculated for a perfect engine working with saturated steam and tabulated in Col. 5, p. 387.

If we express the indicated work in units of heat, write AL_i in place of L_i, and designate the ratio by η_t, we get

$$\eta_t = \frac{AL_i}{Q} \quad \ldots\ldots\ldots \quad (72)$$

[1] See article by the author, "Zur Theorie und Beurtheilung der Dampfmaschinen." Zivilingenieur, 1896, Vol. 42, p. 665.

for the "thermal efficiency η_t" of the engine. The indicated work L_i is found from the indicator diagram, while the heat quantity Q can be easily calculated in the present case. If t_1 is the temperature of the saturated steam corresponding to the normal boiler pressure p_1 and if t_0 is the temperature of the feed water, then, from what has preceded, we get for an engine working with dry saturated steam

$$Q = q_1 - q_0 + r_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73a)$$

On the other hand, if we are dealing with superheated steam of the temperature t, which is always generated under the constant pressure p_1, we must write

$$Q = q_1 - q_0 + r_1 + c_p(t - t_1), \quad . \quad . \quad . \quad . \quad . \quad (73b)$$

where $c_p = 0.4805$.

Example 1. In an experiment on a triple-expansion engine[1] working with saturated steam, the indicated work in horse-powers was found to be $N_i = 1007.23$ [993.431], and the steam weight generated per hour in the boiler amounted to $D_h = 5944.1$ kg. [13104.5 lb.], or per second $D_s = 1.6511$ kg. [3.64 lb.], and hence the indicated work per kg. [lb.] of steam was

$$L_i = \frac{75 \times N_i}{D_s} = 45752 \text{ mkg.}$$

$$\left[L_i = \frac{550 \times N_i}{D_s} = 150101 \text{ ft-lb.}\right].$$

For the boiler pressure p_1 we have the temperature $t_1 = 165.2°$ [329.36°], and the total heat $q_1 + r_1 = 656.9$ Cal. [1182.42 B.t.u.]; the heat of the liquid for the feed water is $q_0 = 32.5$ [56.7], accordingly $Q = 624.4$ Cal. [1125.72 B.t.u.], and from equation (72) there follows the "thermal efficiency"

$$\eta_t = 0.1728,$$

and then we also have, from equation (72),

$$L_i : Q = 424\ \eta_t = 73.273 \text{ mkg.: Cal.}$$
$$[L_i : Q = 772.837\ \eta_t = 133.346 \text{ ft-lb.: B.t.u.}]$$

as the indicated work per unit of heat.

[1] Schröter, "Vergleichende Versuche mit gesättigtem und überhitztem Dampfe an einer 1500-pferdigen Dreifach-Expansionsmaschine." Zeitschrift des Vereins deutscher Ingenieure, Vol. 40, 1896, p. 249.

Example 2. This engine worked at the same boiler pressure $p_1 = 7.2$ kg. with superheated steam of a temperature $t = 231°$ [447.8° F.]. The indicated work in horse-powers was found to be $N_i = 1042.4$ [1028.12], and the steam weight consumed per hour is $D_h = 5606.4$ kg. [12360.0 lb.].

Here, as before, the indicated work per kg. [lb.] of steam is

$$L_i = 50201 \text{ mkg. [164320 ft-lb.]}.$$

The temperature of the feed water during this experiment was $t_0 = 25.5°$ [77.9°], and accordingly we calculate, from equation (73*b*),

$$Q = 663.0 \text{ Cal. [1193.4 B.t.u.]},$$

and then, from equation (72), the thermal efficiency

$$\eta_t = 0.1786,$$

and the indicated work, per unit of heat,

$$\frac{L_i}{Q} = 75.718 \text{ [} = 138.03\text{]}.$$

Comparison with the results of Example 1 shows the advantage of employing superheated steam.

The method of judging of the excellence of our steam engines developed in the foregoing has hitherto rarely been employed when working up experiments; it is still the general custom to state the hourly steam weight (D_h) in kg. [lb.] which is consumed with the indicated work N_i; and it is usually stated as a ratio $D_h : N_i$, as the indicated steam consumption " per hour per horse-power."

It has often been pointed out that this statement must be accepted with caution where it is a question of comparing an engine for saturated steam with one for superheated steam; we will therefore touch upon the question briefly.

Let L_i be the indicated work of the engine reduced to one kg. [lb.] of steam and expressed in mkg. [ft-lb.], and let D_h be the hourly steam consumption, then we get at once

$$N_i = \frac{L_i D_h}{3600 \times 75} \quad \left[N_i = \frac{L_i D_h}{3600 \times 550} \right],$$

or

$$\frac{D_h}{N_i} = \frac{3600 \times 75}{L_i} \quad \left[\frac{D_h}{N_i} = \frac{3600 \times 550}{L_i} \right]. \quad . \quad . \quad . \quad (74)$$

But this can also be written

$$\frac{D_h}{N_i}=\frac{3600\times 75A}{Q}\,\frac{Q}{AL_i}\qquad\left[\frac{D_h}{N_i}=\frac{3600\times 550A}{Q}\,\frac{Q}{AL_i}\right],$$

and hence, with equation (72),

$$\frac{D_h}{N_i}=\frac{3600\times 75A}{Q\eta_t}\qquad\left[\frac{D_h}{N_i}=\frac{3600\times 550A}{Q\eta_t}\right],$$

or

$$\eta_t=\frac{636.79}{Q\left(\frac{D_h}{N_i}\right)}\quad\left[\eta_t=\frac{2562.0}{Q\left(\frac{D_h}{N_i}\right)}\right]. \qquad (75)$$

We see from this equation that the ratio $D_h:N_i$ does not take the place of an efficiency factor, but that we must also take into account the heat quantity Q which is necessary to generate one kg. [lb.] of steam. For the same type of engine the value $D_h:N_i$ can serve as a standard of comparison, but if we wish to compare an engine working with superheated vapor with one using saturated steam, then, as has been repeatedly proposed, we can proceed as follows:

For equal thermal efficiency η_t we find, from equation (75),

$$Q'\left(\frac{D_h}{N_i}\right)'=Q\left(\frac{D_h}{N_i}\right),$$

where the symbols relating to superheated steam are primed to distinguish them from the ones belonging to the saturated steam, and then we get, with equations (73*a*) and (73*b*),

$$\left(\frac{D_h}{N_i}\right)'=\frac{Q}{Q'}\left(\frac{D_h}{N_i}\right)=\frac{q_1-q_0+r_1}{q_1-q_0+r_1+c_p(t-t_1)}\left(\frac{D_h}{N_i}\right). \qquad (76)$$

The value $\left(\frac{D_h}{N_i}\right)'$ has been designated as the reduced steam weight.

Example. In the data furnished by Schröter for the examples on p. 443 there was given for an engine with saturated steam (example 1)

$$D_h = 5944.1, \quad N_i = 1007.23, \quad Q = 624.4 \text{ cal.}$$
$$[D_h = 13104.5, \quad N_i = 993.431, \quad Q = 1123.92 \text{ B.t.u.}];$$

therefore

$$\frac{D_h}{N_i} = 5.901 \; [= 13.191].$$

For the engine with superheated steam (example 2)

$$D_h = 5606.4, \quad N_i = 1042.4, \quad Q' = 663.0$$
$$[D_h = 12360.0, \quad N_i = 1028.12, \quad Q' = 1193.4];$$

therefore

$$\frac{D_h}{N_i} = 5.378 \; [= 12.05].$$

Accordingly we find in a correct comparison for the second engine, in accordance with equation (76), the reduced steam weight

$$\left(\frac{D_h}{N_i}\right)' = \frac{624.4}{663.0} \times 5.901 = 5.557 \; [= 12.423],$$

which is to be compared with the value 5.901 [13.191] of the first engine.

The thermal efficiency η_t is always very small and allows the utilization of the heat to appear more unfavorable than it really is; moreover, it does not enable one to recognize the parts of the cycle of an actual engine which are capable of improvement. Thought has therefore been given to devising for the steam engine a working cycle that will serve as a standard of comparison, the work of the cycle being compared with the actually observed indicated work L_i. Different suggestions have been made as to this standard cycle; doubtless the most suitable cycle is the one already discussed in § 51, p. 396, as the "cycle of the actual steam engine," but in that case the losses there mentioned under (*a*) to (*f*) should be ignored; it is therefore assumed that in the cycle (so free from loss) the admission takes place under constant boiler pressure, the expansion is adiabatic and extends to the con-

denser pressure, the exhaust from the cylinder occurs against the constant back pressure of the condenser, the condensation of the steam is complete, and the feeding of the boiler is effected with water at the lower temperature limit t_2. The process therefore corresponds to an indicator diagram of the form of Fig. 38, p. 398.

For present purposes I have designated the cycle as a "normal cycle"[1]; this cycle has also been taken as a basis of Mollier's discussion,[2] and he calculated the corresponding efficiency for a whole series of very different but good experiments on steam engines which, in the course of time, have been conducted at different places.

For a steam engine working with saturated steam, under the assumptions indicated, equation (36), p. 398, is at once valid. If we assume dry steam, so that $x_1 = 1$, and designate by L_n the work of the normal cycle for the unit of weight of steam, then (for $G = 1$) the quoted equation gives

$$AL_n = q_1 - q_2 - T_2(\tau_1 - \tau_2) + \frac{r_1}{T_1}(T_1 - T_2). \quad . \quad . \quad (76a)$$

On the contrary, if we are dealing with an engine using superheated steam and if T is its temperature, then by preceding propositions we have

$$AL_n = q_1 - q_2 - T_2(\tau_1 - \tau_2) + \frac{r_1}{T_1}(T_1 - T_2) + c_p(T - T_1) - c_p T_2 \log_e \frac{T}{T_1}, \quad (76b)$$

where of course it is assumed that the steam at the end of the expansion is saturated and wet, a case that will always exist in practice.

If, in this manner, there has been calculated the value L_n for one case or the other, then, with the help of the indicated work L_i observed in the engine, we get the efficiency η_i, which may be designated as the indicated efficiency

$$\eta_i = \frac{L_i}{L_n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

[1] Zivilingenieur, 1896, Vol. 42, p. 665.

[2] Mollier, "Über die Beurtheilung der Dampfmaschinen." Zeitschrift des Vereins deutscher Ingenieure, 1898, Vol. 42, p. 685.

It would be easy to calculate the several losses of work for this assumed normal cycle in the manner similar to that pursued in §§ 52 and 53; but in determining the losses of effect, which are characterized by ζ, we should have to use, in place of the quantity L_m, the preceding value of L_n.

The two formulas (76*a*) and (76*b*), for the calculations of the work L_n in the normal cycle, are not simple, for they presuppose the use of steam tables; but they can easily be replaced by other simpler formulas.

If the engine works with dry saturated steam, then the admission work is p_1v_1, the back-pressure work p_2v_2, and the work of expansion, according to equation (77), p. 84, is

$$\frac{1}{\mu-1}(p_1v_1-p_2v_2).$$

It therefore follows that the engine work is

$$L_n=\frac{\mu}{\mu-1}(p_1v_1-p_2v_2),$$

or, because of the relation

$$p_2v_2^{\mu}=p_1v_1^{\mu},$$

$$L_n=\frac{\mu}{\mu-1}p_1v_1\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{\mu-1}{\mu}}\right] \quad . \quad . \quad . \quad . \quad (78a)$$

(with $\mu=1.135$), in place of equation (76*a*).

On the other hand, if the engine works with superheated steam, the work of expansion must be determined by equation (14), p. 279, and this value, combined with the admission work p_1v_1 and the back-pressure work p_2v_2, then gives, instead of equation (76*b*),

$$L_n=\frac{\kappa}{\kappa-1}(p_1v_1-p_0v_0)+\frac{\mu}{\mu-1}(p_0v_0-p_2v_2). \quad . \quad . \quad (78b)$$

Here we must take $\kappa=1.333$ and $\mu=1.135$, the symbols p_0 and v_0 referring to the point of intersection T_0 of the expansion curve with the limit curve (see Fig. 27, p. 279).

The values p_0, v_0, and p_0v_0 are easily determined from equations (11) and (12*a*) to (12*c*), p. 278, and moreover we find

$$p_2v_2=p_0v_0\left(\frac{p_2}{p_0}\right)^{\frac{\mu-1}{\mu}},$$

with which we can determine v_2 and consequently can find the expansion ratio $v_1 : v_2$, finally obtaining from $v_2 = x_2 u_2 + \sigma$ the state of the steam x_2 at the end of the expansion.

In order to make use of all the preceding expressions the following data would have to be established by observations on an existing steam engine: the steam consumption D_h per hour, the indicated work L_i in horse-powers, the normal boiler pressure p_1, the corresponding temperature t_1, the pressure p_2 and the corresponding temperature t_2 in the condenser, the temperature of the feed water t_0, and the temperature t of the superheated steam.

§ 57. ADDENDUM. THEORY OF FIRELESS STEAM ENGINES.

Let a vessel, for example a steam boiler without heating plant, be placed on a wagon and be filled with water and steam under high pressure and at corresponding temperature, and let the steam flow from an orifice in the steam space and towards a place in which a lower pressure prevails, then the boiler will be gradually and partially emptied, till finally the same pressure will exist in the boiler and in the receiving chamber. The water evaporates under the diminishing pressure caused by the efflux, and the heat necessary for steam formation is furnished by the superheated water, whose temperature diminishes with the sinking steam pressure.

If the boiler is large enough and is filled with a sufficiently large quantity of water at the start, the generated steam can be used for quite a period to run a steam engine situated on the same wagon, and we thus get a fireless locomotive,—or, if the boiler is placed upon a ship, we obtain a fireless marine engine.

When the pressure and temperature in the boiler have reached the lower limit, a certain weight of water and steam will remain behind, and the boiler must be newly filled with highly heated water at the initial temperature in order to make it again capable of driving.

This filling takes place at particular supply stations pro-

vided with stationary steam boilers in which there is produced, by ordinary heating, the high-pressure steam and the highly heated water. Now in filling, either the hot water is allowed to flow through a connecting pipe, or, more advantageously, so as to avoid losses of heat, the steam is led from the stationary boiler to the locomotive boiler, where it is condensed by the low-temperature water remaining there, until finally the pressure is almost the same in both boilers, and the water consumed in the preceding trip is again replaced and a new trip can begin.

To utilize the steam from a fireless boiler in the manner indicated was first thought of by Perkins (1823), but the idea was taken up later by Dr. Lamm and carried out practically for the running of street locomotives (1872). In Europe the first perfect machines of the sort were built by the French engineer Léon Franc in Paris (1875), while in Germany G. Lentz of Düsseldorf promoted, with great technical knowledge, the building of such engines.

Fig. 47 gives a sketch of a fireless locomotive boiler. The steam flows out of the boiler A through the steam efflux pipe a from the dome B and from there through a reducing valve at b and through the steam pipe cc to the steam cylinder. The steam pipe cc, greatly enlarged, is led through the steam and water space in order to avoid cooling and perhaps, it is thought, to evaporate some entrained water. A throttling takes place at the reducing valve b which is necessary, particularly at the beginning of the run, when the steam pressure in the boiler is very considerable, generally amounting to from 16 to 17 kg. per sq. cm. [227 to 242 lb. per sq. in.], and this throttling reduces the steam to about 6 to 7 kg. [85 to 100 lb.]. As the pressure in the boiler diminishes the throttling is diminished, and the admission part of the piston stroke in the steam cylinders is gradually increased so as to keep the steam work of the engine nearly con-

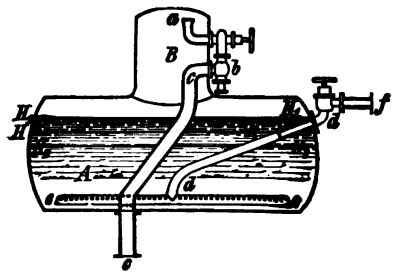

FIG. 47.

stant. The pipe *dd* (Fig. 47) is for filling purposes; it is therefore connected at *f* with the steam pipe of the stationary boiler; in the interior of the latter the pipe ends in a horizontal, perforated piece of pipe *ee*; through the openings the steam enters into the boiler water.

The fireless engines certainly possess advantages for certain driving purposes on street railways, in switching, etc.: they are easy to run and need no special stoker; there is no soot and smoke formation, and the danger of fire is completely absent.

The theoretical questions which arise here relate to the law of steam formation and to the determination of the mechanical work which corresponds to a freshly filled boiler, and finally to the mode of filling.

Let H_1H_1 (Fig. 47) be the level of the water in the boiler at the beginning, and let H_2H_2 be that at the end of the performance of the work; at any instant let there be G kg. [lb.] of water and steam of the pressure p and temperature t in the boiler and let the water level stand at HH. Let the volume of the boiler be V cbm. (cu. ft.).

Now let Gx' designate the weight of the steam present and $G(1-x')$ represent that of the water present, then the heat content J of the whole mass is

$$J = G(q + x'\rho),$$

and, moreover, the total volume is

$$V = G(x'u + \sigma).$$

Eliminating x' from both equations there follows

$$J = G\left[q + \frac{\rho}{u}\left(\frac{V}{G} - \sigma\right)\right]$$

The second term in the bracket is so small that under the conditions here prevailing it may always be neglected, and we therefore get for the heat content

$$J = Gq,$$

and from this, when the weight diminishes by dG, on account of the efflux, we have for the corresponding increment of the heat content

$$dJ = -d(Gq). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

Now if x is the steam quality of the steam flowing toward the steam pipe, we have $(q+x\rho)$ for the heat content of a unit of weight of this steam. Within an infinitesimal interval of time we can regard the steam formation as taking place under the constant pressure, and therefore the unit of weight of steam takes up the additional heat quantity $Apux$; consequently the heat quantity carried off by the discharging mass dG is

$$(q+x\rho+Apux)dG,$$

or

$$(q+xr)dG.$$

This value is identical with the diminution $-dJ$ of the heat contents in the boiler; from equation (79) we therefore get

$$d(Gq) = (q+xr)dG,$$

or

$$\frac{dG}{G} = \frac{dq}{xr}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

If we now proceed from the assumption which is here always permissible, that the steam quality x of the steam flowing toward the steam pipe is constant, then this equation can be integrated, because q and r are known as functions of t.

For facilitating the calculations I use the empirical formula

$$q = \alpha - \beta r, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

and make $\alpha = 862.58$ [1552.64] and $\beta = 1.420$ [1.420].

It will be shown immediately how well this formula agrees with the more exact formulas within the limits existing here.

From equation (81) there follows $dq = -\beta dr$, and then, from equation (80), by integration,

$$\log \frac{G}{G_1} = \frac{\beta}{x} \log \frac{r_1}{r}, \quad \ldots\ldots\ldots \quad (82)$$

where G_1 represents the initial weight of water and steam in the boiler. If G_2 is the weight at the end of the run, then we have

$$\log \frac{G_2}{G_1} = \frac{\beta}{x} \log \frac{r_1}{r_2}. \quad \ldots\ldots\ldots \quad (83)$$

But G_2 and G always differ slightly from G_1 and, likewise, r_2 and r differ little from r_1; we can therefore represent the logarithms by the initial values of their series, and get

$$\frac{G_1 - G}{G_1} = \frac{\beta}{x} \frac{(r - r_1)}{r}, \quad \ldots\ldots\ldots \quad (82a)$$

and at the end we also have

$$\frac{G_1 - G_2}{G_1} = \frac{\beta}{x} \frac{(r_2 - r_1)}{r_2}. \quad \ldots\ldots\ldots \quad (83a)$$

Between the pressure and temperature limits occurring here we can substitute with sufficient accuracy

$$r = 609.58 - 0.726\, t = 0.726\, [839.64 - t]. \quad \ldots\ldots \quad (84)$$
$$[r = 1097.24 - 0.726\, (t - 32) = 0.726\, (1543.25 - t)].$$

Let us, for the sake of simplicity, represent this constant 839.64 [1543.25] by b, and remember that $(G_1 - G)x$ represents only the steam weight which is generated while the weight of steam and water diminishes from G_1 to G. Finally, let D represent the weight of the steam, then according to equation (82a), we have

$$\frac{D}{G_1} = \frac{\beta(t_1 - t)}{b - t}, \quad \ldots\ldots\ldots \quad (82b)$$

and here we must substitute $\beta = 1.420$ according to equation (81); for the end of the run we may substitute $t = t_2$, corresponding to the lowest pressure, in order to calculate the total steam weight D which corresponds to a single filling of the boiler.

That the empirical formulas (81) and (84) are sufficiently exact, the following tabulation shows:

Boiler pressure.....	$p = 2$ kg.	5 kg.	10 kg.	15 kg. (absolute).
According to Table 12 of the Appendix.....	$t = 119.570$	150.991	178.886	197.244
	$q = 120.369$	152.480	181.243	200.324
	$r = 522.600$	500.072	479.817	466.335
According to equation (81)....	$q = 120.49$	152.48	181.24	200.38
" " " (84)...	$r = 522.77$	499.96	479.71	466.38

In these fireless boilers the terminal pressures will rarely sink below 2 kg. per sq. cm. [28.45 lb. per sq. in.].

Example. In a fireless boiler, during a single run, the steam pressure falls from $p_1 = 16$ kg. [227.57 lb.] to $p_2 = 2$ kg. [28.45 lb.]. Here, according to table 12 of the Appendix, $t_1 = 200.32$ [392.58], $t_2 = 119.57$ [247.23], and because $r = \rho + Apu$, $r_1 = 464.06$ [835.31], $r_2 = 522.60$ [940.68].

Hence, there follows, from equation (83),

$$\log \frac{G_2}{G_1} = -\frac{0.0732653}{x},$$

and from this follows for

$$x = \quad 1 \qquad 0.9 \qquad 0.8$$
$$G_2 = 0.8447\, G_1, \quad 0.8291\, G_1, \quad 0.8099\, G_1,$$

and we get, for the total steam weight generated,

$$D = (G_1 - G_2)x = 0.1553\, G_1, \quad 0.1538\, G_1, \quad 0.1521\, G_1,$$

and, from the approximate formula (82*b*), we get

$$D = 0.1592\, G_1,$$

and therefore, respectively,

$$G_2 = 0.8408\, G_1, \quad 0.8231\, G_1, \quad 0.8018\, G_1.$$

For practical purposes the approximate formula (82*b*) furnishes sufficiently exact values. We see, moreover, that the weight G_1 of the steam and water at the beginning is not very different from the weight G_2 at the end. The boiler at the end of the run is still largely filled with water. The boiler therefore would have to be almost entirely emptied of water if the filling were to take place with hot water; but filling with steam from a stationary boiler can take place directly.

Equation (82*b*) shows that for the same initial fill G_1, and for the same drop in pressure, the steam weight generated is mainly dependent on the initial pressure; with diminishing steam pressure the steam quantity increases, as the following survey shows:

Drop of pressure.	$\frac{D}{G_1}$.
15 kg. to 14 kg.	0.00713 kg.
12 " " 11 "	0.00839 "
9 " " 8 "	0.01024 "
6 " " 5 "	0.01433 "
3 " " 2 "	0.02609 "

Now, as regards the other question, of the amount of work available in a filled fireless boiler, it may be answered for practical purposes and with sufficient accuracy in the following manner:

In ordinary high-pressure non-condensing steam engines we estimate

$$\frac{D_h}{N_e} = 25 \text{ to } 16 \text{ kg. [55.88 to 35.76 lb.]},$$

where D_h represents the hourly steam consumption and N_e the effective work in horse-powers; the preceding ratio is the hourly steam consumption per horse-power, the smaller value corresponding to the higher pressure of admission and to the greater expansion.

Let D be the weight of steam and L the work which can be produced by it, expressed in mkg. [ft-lb.], then, from the preceding values, we determine

$$\frac{L}{D} = 10800 \text{ to } 16875 \text{ mkg. [35434 to 55237 ft-lb.]}. \quad . \quad (85)$$

If we multiply both members of equation (82*b*) by L we get

$$L = \frac{\beta(t_1 - t)}{(b - t)} \frac{L}{D} G_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

from which we can calculate for a fireless boiler with a given fill and known pressure limits the work which is available.

For example we get, for a drop of pressure from 16 kg. [228 lb.] to 2 kg. [28.5 lb.], with the help of the values in equation (85):

$$L = 1719\,G_1 \text{ to } 2686\,G_1, \text{ on the average } L = 2200\,G_1 \text{ mkg.}$$
$$[L = 5640\,G_1 \text{ to } 8812\,G_1, \text{ on the average } L = 7218\,G_1 \text{ ft-lb.}].$$

On the other hand, for a drop of pressure from 16 kg. [228 lb.] to 4 kg. [57 lb.]:

$$L = 1266\,G_1 \text{ to } 1978\,G_1, \text{ on the average } L = 1600\,G_1 \text{ mkg.}$$
$$[L = 4154\,G_1 \text{ to } 6490\,G_1, \text{ on the average } L = 5249\,G_1 \text{ ft-lb.}].$$

Birk[1] estimates $L = 2000\,G_1$ [$6562\,G_1$] and Lentz, for the sake of safety and very properly, takes $L = 1500\,G_1$ [$4921\,G_1$] because it would be very embarrassing to have the steam give out during the trip.

Suppose that the weight of steam and water in a fireless boiler at the beginning of a run is $G_1 = 2000$ kg. [4409 lb.], then according to Lentz there will be available a work of 3,000,000 mkg. [21,700,000 ft-lb.]; from this can easily be computed, for a locomotive with a prescribed load and the tractive force calculated from it, the whole distance which can be traversed by the engine with a single boiler charge.

In conclusion we may briefly take up the question of newly charging the boiler.

If at the end of the run the weight of the steam in the boiler is $G_2(1-x'')$ then the corresponding heat content J_2 is equal to

$$J_2 = G_2(q_2 + x''\rho_2).$$

Suppose that there is transferred from the stationary boiler (the feed boiler) in which the pressure p_0 prevails, the steam weight G_0 possessing the steam quality x_0, the transfer continuing till the temperature in the fireless boiler has again risen to the initial value. The heat quantity J_0 contained in this steam is

[1] Birk, "Die feuerlose Locomotiv in inrer Theorie und Anwendung." Wien, 1883. The theoretical investigations in this treatise, which in other respects is to be recommended, are inexact, because Birk throughout confuses the total heat λ with the steam heat J of saturated steam; but this still frequently happens in other technical treatises.

therefore $G_0(q_0+x_0\rho_0)$, and since the steam takes up the work $G_0 p_0 u_0 x_0$ during the transfer, the heat contents of the total weight at the end amount to

$$J_1+J_0+AG_0p_0x_0u_0=G_2(q_2+x''\rho_2)+G_0(q_0+x_0r_0).$$

On the other hand, these heat contents also amount to

$$(G_2+G_0)(q_1+x'\rho_1).$$

Therefore by equating we get

$$G_2(q_1-q_2+x'\rho_1-x''\rho_2)=G_0(q_0-q_1+x_0r_0-x'\rho_1).$$

But the steam weight in the fireless boiler can be neglected in comparison with the weight of water present, so that we can make $x'=0$ and $x''=0$ and may assume the steam coming from the feed boiler as dry saturated, so that $x_0=1$ may be assumed. The last equation therefore gives, because q_0+r_0 represents the total heat λ_0 of the transferred steam, the relation

$$G_2(q_1-q_2)=G_0(\lambda_0-q_1),$$

from which follows

$$G_2+G_0=G_2\left(\frac{\lambda_0-q_2}{\lambda_0-q_1}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

The value G_2+G_0 represents the boiler charge when the initial temperature has again been reached, and this ought to be identical with G_1 when the initial condition has again been completely restored.

We find, however, from the above given relation $D=(G_1-G_2)x$:

$$\frac{G_2}{G_1}=1-\frac{D}{xG_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

From the last two equations the ratio G_2+G_0 to G_1 can be derived. If the former value is the greater then after a few

charges a part of the water of the new supply must be discharged from the boiler, otherwise some water must be added.

Example. Let the drop of pressure during the run be from 16 kg. [227.57 lb.] to 2 kg. [28.45 lb.]; let the pressure in the feed boiler be 18 kg. [256.01 lb. per sq. in.].

Here we get from equation (87) with the help of Table 12

$$G_2 + G_0 = 1.178\, G_2.$$

On the other hand, when the quality of the working steam is

$$x = 1, \quad 0.90, \quad 0.80,$$

we get from equation (88) (see example, p. 454)

$$\frac{G_2}{G_1} = 0.841, \quad 0.823, \quad 0.802$$

and

$$\frac{G_2 + G_0}{G_1} = 0.991, \quad 0.970, \quad 0.945.$$

If the final pressure is greater than 2 kg. [28.45 lb. per sq. in.], then somewhat greater values must take the place of the last given quantities; anyway we can conclude from the results of the calculation, although they are approximate, that with a fireless boiler a whole series of fillings may take place before the original water level must again be restored by blowing off or pumping in. A special uncertainty affects the calculation, inasmuch as the value of x' can really be very different from 0, because it may well be that at the end of the charging the water contains a greater number of steam bubbles.

In the fireless engine discussed here the steam flows from the cylinder into the open air. Now Honigmann proposed a highly interesting completion of the process and also realized it practically.

Honigmann surrounds the steam boiler with a second boiler filled with soda lye of a certain degree of concentration into which the steam is discharged from the steam cylinder. Here the steam is condensed, accompanied by a development of heat, and the heat released is utilized to produce new steam in the driving boiler. The process continues till the soda lye has been diluted to such a degree that a further absorption of the steam is no longer possible; the boiler is then filled with a new charge of concentrated lye and the diluted lye is brought back into an available condition by evaporation in special vessels.

The chemical process occurring here has thus far not been adequately explained to enable us to submit a theoretical investigation sufficient for practical needs; we therefore only refer to the two excellent articles by Riedler and Gutermuth. Riedler[1] gives a clear view of the results of excellently made experiments on a Honigmann engine. Gutermuth[2] continued the experiments in the same manner and at the same time discussed all the physical and chemical investigations bearing on this subject which were known at the time.

B. Cold-vapor Engines.

§ 58. REFRIGERATING MACHINES IN GENERAL.

We have already called attention to the difference between the ordinary steam engine, or hot-vapor engine, and the cold-vapor engine and have emphasized the point that the simple reversal of the cycle will permit passing easily from one kind to the other. In saying this we are only considering the cold-vapor engines, which are compression engines, and in these, in the course of the cycle, there is only an exchange of evaporation and condensation with corresponding heat supply and heat withdrawal, and also an exchange of expansion and compression.

In all cold-vapor engines liquids are employed which, even at low temperature, evaporate under great pressure. The liquid confined in a tubular boiler (the evaporator) withdraws heat from its environment, for example when it evaporates, taking the heat from another liquid surrounding the tubes of the evaporator and cooling it down nearly to the evaporation temperature. To produce a continuous process the vapor developed must be removed and be restored in the liquid condition to the evaporator.

---

[1] Riedler, "Die Honigmann'schen Dampfmaschinen mit feuerlosem Natronkessel." Zeitschrift des Vereins deutscher ingenieure, Vol. 27, p. 729, 1883.

[2] Gutermuth, "Untersuchungen an Honigmann'schen feuerlosem Natronkessel." Zeitschrift des Vereins deutscher Ingenieure, Vol. 28, pp. 69 and 533, 1844. "Das Honigmann'sche Natronverfahren." *Ibid.*, Vol. 29, p. 101, 1885

There are two means of doing this: either steam is drawn off by a pump and then compressed and condensed by cooling to the liquid condition, whereupon it is returned to the evaporator,— or the steam is led to a second space, where it is absorbed by a liquid (water); this liquid is transferred by a pump to a vessel (boiler) which is supplied with heat for the purpose of driving off the steam taken in by the absorbing liquid, and this steam is then led to a condenser from which it is removed to the evaporator in a liquid condition.

We therefore distinguish between the compression system and the absorption system. The machines of the former we shall treat more fully in the following under the name "cold-vapor engines"; they are the ones which are most widely used and accessible to a theoretical investigation.

The circumstances are different, however, with the absorption machines which are mostly known under the name of the Carré ice-machines; of course in these machines only such vapors can be used as are easily and largely absorbed by water or some other liquid and can then be driven off again at higher temperature.

Ammonia vapor, with water as an absorbing liquid, is the best suited to this purpose, and it is ammonia vapor which is exclusively used in the Carré machine.

There is no difficulty in following the occurrences in the absorption machine and setting up general equations according to thermodynamic laws; but the equations cannot be practically utilized because they contain quantities which have not been determined accurately enough by experiment. Thus we do not know the heat quantity which is released and which must be withdrawn from the mass when a liquid absorbs a vapor under different conditions of pressure and temperature, for example when water absorbs the vapor of ammonia; just as little do we know the heat quantity necessary to drive off the absorbed vapor from the liquid at a higher temperature and at a higher pressure. The latter case is rendered still more complicated because the exit of the absorbed steam is accompanied by a partial evaporation of the absorption liquid itself; any way, this circumstance must be considered in Carré's

machine, because in it the steam is driven off under high temperature and high pressure.

Under such circumstances and because the equations to be set up cannot be utilized for numerical calculations, we will here omit a presentation of a theory of the absorption system, and all the more because the other type of refrigerating machine, the compressor type, has almost completely displaced it at this time.

The compressor type, which is the real cold-vapor engine and which we will now examine more closely, employs a vapor which, in the saturated state, exhibits a high pressure even at low temperatures. The vapors which have thus far been practically employed for this purpose are the vapors of ether, sulphurous acid, ammonia, and carbonic acid; there have been suggested, besides, the vapors of methylic ether, chloride of methyl, bisulphide of carbon, and others.

The first machines which were constructed worked with ether vapor; in 1835 Perkins[1] patented a cold-vapor engine, but it seems to have been little known, because an American, Harrison, is usually credited with being the inventor, although he did not get a patent on his machine until 1856. The two machines differ but little in construction. There are decided improvements in construction shown in Carré's machine, which became known in 1860, but which must not be confused with his absorption machine. Shortly afterward (1862) Siebe patented an ether machine, and it is often spoken of at the present time under his name, although in principle it is in no way distinguishable from the machine of his predecessors

That the machines mentioned were very slow in being introduced is mainly due to the fact that they used the costly vapor of ether, which has a very slight pressure at low temperatures. If,

[1] Dingler's "Polytechnic Journal," from 1837 (Vol. 64) on, probably contains the description and illustration of all the cold-air engines and cold-vapor engines proposed or constructed; therefore we will refer the reader to this journal for fuller particulars, and especially to two excellent articles:

Prof. Dr. Meidinger: "Die Fortschritte in der künstlichen Entwickelung von Kälte und Eis." Vols. 217 and 218.

F. Fischer: "Über die Herstellung von Eis." Vol. 224.

as is actually the case, the temperature of the vapor within the machine varies between $-20°$ and $+20°$ [$-4°$ and $+68°$], then the pressure variations of the ether will be between 0.9 and 0.57 atmospheres; a penetration of the external atmospheric air into the machine is therefore unavoidable. The access of air, however, is in the highest degree injurious with all cold-vapor engines; it reduces their effect and is probably the main cause of the great heating of certain parts of the compression cylinder with which we must contend.

Marked progress was made by Raoul Pictet when he employed sulphurous acid for these machines; moreover, later, in his known experiment on the condensation of gases, he utilized carbonic acid; in a very clever way he united two engines, one working with sulphurous acid and the other with carbonic acid. A glance at Table 10 of the Appendix shows what a great lowering of temperature can be effected by carbonic acid when we reduce the pressure of the vapor drawn in from the evaporator down to only the boiler pressure that is customary in hot-vapor engines.

Great credit is due to Prof. Linde in perfecting and extending the cold-vapor engine. Originally Linde (like Tellier) suggested methylic ether, but then went over to the use of ammonia vapor and constructively, also, brought this machine to a high degree of perfection.

Machines working with ammonia are at this time the ones most widely used; recently, in accordance with Windhausen's suggestion, carbonic acid has again been employed.

The occurrences in the machines working with the above-mentioned simple vapors, can easily be discussed [1] by the laws of

[1] See the article by the author: "Zur Theorie der Kaltdampfmaschinen." With an Appendix: "Über das Verhalten der Ammoniakdämpfe." Zivilingenieur, Vol. 27, 1881, p. 449.

This work was preceded only by very differently treated articles presented by Carl Linde, "Über die Wärmeentziehung bei niedrigen Temperaturen durch mechanische Mittel." Bayerisches Industrie- und Gewerbeblatt, 2d year, 1878, pp. 205, 321, 363, and "Theorie der Kälteerzeugungsmaschinen," Verhandlungen des Vereins zur Beförderung des Gewerbefleisses, 54th year, 1875, p. 357, and 55th year, 1876, p. 185; also Ledoux, "Théorie des machines à froid." Extrait des Annales des mines. Dunod, editor, Paris, 1878.

Thermodynamics, as the following will more fully show; the investigations presented above on the behavior of sulphurous acid, ether, ammonia, and carbonic acid vapors had for their main purpose the establishing of the necessary basis for calculating and judging the cold-vapor engines.

§ 59. CYCLE OF THE THEORETICALLY PERFECT COLD-VAPOR ENGINE.

As a basis for the following let us take the schematic representation (Fig. 48) which has already been employed in treating the closed, hot-air engine (Vol. I, p. 377) and has also been used in this volume when considering the theoretically perfect hot-vapor engine. Let the figure here represent a cold-vapor engine.

The tubular boiler AA filled with liquid and steam, i.e., with the mediating body and is called the evaporator; it is surrounded by a casing B, which is filled with another liquid.

For example, let us suppose the evaporator is filled with ammonia in the liquid and vapor form, of a pressure of 2.3 atmospheres, so that a temperature of $-15°$ [$+5°$] will obtain there, and

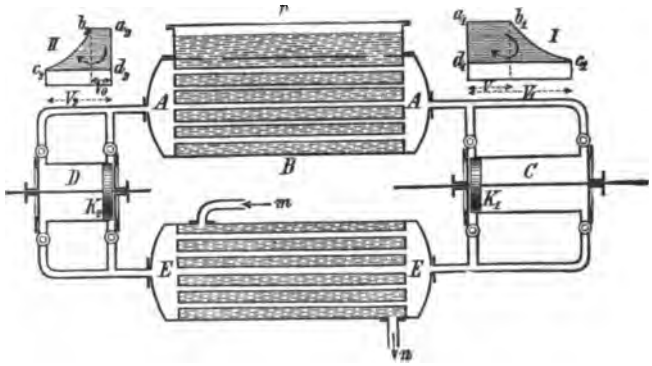

FIG. 48.

that the liquid in the envelope B is cooled, because it gives off heat to the evaporator; in order that this latter liquid may not

freeze it is in the form of a solution of table salt or chloride of calcium. In order to produce ice there are immersed in this solution, usually cooled to $-10°$ [$+14°$], tin cells containing fresh water which is frozen. In other cases the cold brine is led in pipes through spaces in which air or liquids are to be cooled, and the warm brine is then led back to the space B, where it is again cooled off. The process is a continuous one, and in order to facilitate the cooling of the heating fluid, by which name we may briefly designate the brine, it is artificially set in motion in space B. The performance of the machine is usually estimated by the heat units which are given off per hour by the heating fluid to the evaporator, in other words, withdrawn by the latter from the fluid.

The procedure in the interior of the engine must likewise be a continuous one; during the forward stroke the piston in the power- or compressor-cylinder C draws in vapor, which is mixed with more or less liquid from the evaporator A, and during the return stroke the mass is compressed in the first portion of the piston travel and is then forced during the remaining portion into a second tubular boiler EE, the cooling apparatus or condenser, which is also surrounded by a casing into which cooling water flows at m, discharging through the pipe n after it has been warmed. In the condenser the vapor is completely reduced to a liquid; if its temperature there is $+20°$ [$+68°$], and if we are dealing with ammonia, then, according to Table 8 of the Appendix, the pressure in the condenser is 8.5 atmospheres.

It is now a question of bringing the liquid back to the evaporator, where it is again vaporized and the cycle begins anew. As the pressure in the condenser is greater, and in general considerably greater, than in the evaporator, no special device is needed for leading the liquid back; in actual cold-vapor engines the condenser E is simply connected with the evaporator A by a pipe provided with a regulating valve, and then as much liquid is allowed to flow back as there is vapor taken from the evaporator in the same time.

This procedure, however, is a theoretically imperfect one, as we will show; rather the leading back of the liquid

should be effected as follows by the insertion of a special cylinder D, which I call the feed-cylinder. The piston in cylinder D first sucks in that liquid under constant pressure, which was just formed per stroke in the condenser, then it is confined in the cylinder and allowed to expand till the evaporator pressure is reached at the end of the piston stroke. Steam is formed during expansion, and during the return stroke of the piston the whole mass is forced back into the evaporator against the constant steam pressure in the latter. In the figure the theoretical indicator diagrams are added. Let p_1 represent the pressure in the evaporator and p_2 that in the condenser; the horizontally hatched area $a_1b_1c_1d_1$ of diagram I represents the work consumed per stroke in the power-cylinder, and the area $a_2b_2c_2d_2$ of diagram II is the work produced in the feed-cylinder; the difference of the two is the work needed to run the machine. The curve b_1c_1 is the compression curve, b_2c_2 the expansion curve; both are adiabatic curves in the perfect cycle, while the horizontal boundaries of the indicator diagrams are isothermal lines, because we are here dealing with saturated vapors for which the temperature is constant when the pressure is constant.

The arrows in the two indicator diagrams show the direction in which the cycle is described in the hot-vapor engine; for the present case the directions of the two arrows must be regarded as reversed.

The following investigations will show whether or not the feed-cylinder should be added to the actual engine in order to perfect the cycle.

The theory of the perfect cold-vapor engine is developed, with Fig. 48 as a basis, in the following manner:

Let G represent the weight of liquid and vapor which is sucked in per stroke by the power-cylinder from the evaporator and which is brought back to the condenser by the feed-cylinder, and let x_1 be the steam quality at the end of the piston stroke in the power-cylinder C and therefore the quality at the point c_1 of the diagram (Fig. 48); let x_2 be the steam quality at the end of compression at the point b_1, and x_3 at the end of the stroke in the feed-cylinder, i.e., at the point c_2.

Therefore Gx_3 is the steam weight forced per stroke into the evaporator and Gx_1 is the weight which the power-cylinder, or so-called compressor, takes from the evaporator. Accordingly there is formed in the latter per stroke the steam quantity $G(x_1-x_3)$, and it is formed under the constant pressure p_1 and the corresponding temperature t_1. Using the above notation and letting r_1 be the latent heat we get, in the first place, the heat quantity Q_1, which the evaporator withdraws per stroke from the brine or heating fluid

$$Q_1=Gr_1(x_1-x_3). \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The steam quantity pushed by the power-piston into the condenser is Gx_2, and as this steam is completely condensed at the constant pressure p_2 and the temperature t_2, we have for the heat quantity Q_2 absorbed by the cooling water in the condenser

$$Q_2=Gr_2x_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In the following discussion we will first assume that the cylinder walls neither receive nor give off heat, that there is no clearance space, and that the steam is wet during its cycle in the power-cylinder, and therefore that the steam is nowhere superheated in the cycle; moreover, it is presupposed that the temperature t_1 exists everywhere in the evaporator and also that in every part of the condenser the temperature t_2 exists. We therefore have for the heat quantity dQ, required by the unit of weight of mixture of steam and liquid for an infinitesimal change of state,

$$dQ=Td\left(\tau+\frac{xr}{T}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

or, it may be written in the form

$$dQ=dq+d(xp)+AdL, \quad . \quad . \quad . \quad . \quad . \quad (3b)$$

where dL is the outer work corresponding to the infinitesimal expansion.

In our perfect machine the compression takes place adiabatically along the path b_1c_1 (Fig. 48), and therefore $dQ=0$, and accordingly there follows from equation (3*a*) when we integrate between the given limits

$$\tau_1+\frac{x_1r_1}{T_1}=\tau_2+\frac{x_2r_2}{T_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In like manner there is adiabatic expansion in the feed-cylinder along the path b_2c_2, and there follows from the same equation, because at the beginning $x=0$,

$$\tau_1+\frac{x_3r_1}{T_1}=\tau_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

With the help of the last two formulas x_2 and x_3 can be calculated from the value x_1 and the limit temperatures. Subtracting the equations from each other we get

$$\frac{(x_1-x_3)r_1}{T_1}=\frac{x_2r_2}{T_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Uniting the preceding formula with that obtained by combining equations (1) and (2) we obtain the well-known relation for the perfect cycle

$$\frac{Q_1}{Q_2}=\frac{T_1}{T_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The technical investigation is now extended to the determination of the work necessary to run the machine and of the principal dimensions for a prescribed performance.

The work L_1, required by the power-cylinder C per stroke is easily determined as follows:

The volume v_1 of the cylinder C is

$$V_1=G(x_1u_1+\sigma), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and the volume V_2 of the mass at the end of compression is

$$V_2=G(x_2u_2+\sigma). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Accordingly the work produced during suction is $Gp_1(x_1u_1+\sigma)$ and the work consumed, in order to push the mass under constant pressure p_2 into the condenser, is $Gp_2(x_2u_2+\sigma)$; the compression work, on the other hand, is found from equation (3*b*), when we make $dQ=0$ and integrate,

$$\frac{G}{A}(q_2-q_1+x_2\rho_2-x_1\rho_1)$$

There results from the combination of the last three expressions, when we consider the relation $r=\rho+Apu$, the whole work of the power-cylinder per stroke, measured in units of heat, namely,

$$AL_1=G[q_2-q_1+x_2r_2-x_1r_1+A\sigma(p_2-p_1)]. \quad . \quad . \quad (10)$$

In the feed-cylinder, on the other hand, the work L_2 is produced, first the work $G\sigma p_2$ during the suction of the liquid from the condenser, and then, in consequence of the expansion,

$$\frac{G}{A}(q_2-q_1-x_3\rho_1).$$

However, since the transfer of the mass to the evaporator requires the work $Gp_1(x_3u_1+\sigma)$ we get, by the combination of the preceding three values, the work expressed in units of heat, namely,

$$AL_2=G[q_2-q_1-x_3r_1+A\sigma(p_2-p_1)]. \quad . \quad . \quad . \quad (11)$$

Designating the work necessary per stroke to run a perfect machine by L_m, and because $L_m=L_1-L_2$, we get, from the utilization of equations (10) and (11),

$$AL_m=G[x_2r_2-(x_1-x_3)r_1], \quad . \quad . \quad . \quad . \quad . \quad (12)$$

or, with the help of equations (1) and (2),

$$AL_m=Q_2-Q_1,$$

and therefore, according to equation (7),

$$AL_m = \frac{Q_1}{T_1}(T_2 - T_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

This equation is the one which has been repeatedly derived in the present treatise for the Carnot cycle. In our case the formula gives for a certain value of the heat quantity Q_1 (which the brine withdraws from the evaporator, within given limits of temperature) the work L_m required by the machine; this work is a minimum; every departure from the perfect cycle, all the separate imperfections in the construction of the machine and the unavoidable, hurtful resistances increase the driving work. The formula contains no quantity that depends on a particular kind of vapor. For the same limits of temperature it is therefore theoretically immaterial whether the machine works with ether, ammonia, or any other vapor; for the same Q_1 the driving work of the perfect machine is always the same.

In the cold-vapor engine we must ever be mindful of the requirement that the driving work of the machine be as small as possible for a prescribed value of Q_1; we see from formula (13) that accordingly the temperature differences $T_2 - T_1$ must be kept as small as possible. With respect to the temperature of the cooling water at our disposal there is prescribed the upper temperature limit, the temperature t_2 or T_2, and consequently we ought not to go lower than is necessary with the temperature t_1 or T_1 in the evaporator. When producing ice we estimate that on the average $t_1 = -15°[+5°]$, when producing a cooling fluid $t_1 = -5°[+23°]$; we will therefore obey the practical rule to avoid the production of ice when the same purpose can be effected by a cooling fluid. Formula (13) is also valid for cold-air engines in so far as they describe the Carnot cycle. It is worthy of note that this same equation is valid for the perfect hot-vapor and hot-air engine, and that this equation embraces all the engines mentioned, just as Fig. 48, given above, represents their constructive and theoretical arrangement. In this engine the pressure and temperature in the boiler, or heating apparatus,

are greater than in the condenser, or cooling apparatus. We therefore simply have $T_1 > T_2$. Accordingly L_m becomes negative, i.e., work is produced instead of being consumed in running the machine and hence the machine is transformed from an operating machine into a driving machine, or a prime mover. If in this machine we reverse the signs in equation (13) we get, in the value L_m, the maximum of work; every departure from the perfect cycle and from the assumptions made above leads to a smaller value of the work produced. Since in the engine the temperature t_2 in the condenser, or cooling apparatus, is pretty well fixed, the practical rule is to keep the temperature t_1 as high as may be in order to increase the work L_m as much as possible with the same heat quantity Q_1. The hot-air engines, therefore, ought to work at high temperature and our ordinary steam engines ought to work with high boiler pressure or highly superheated steam, as has already been emphasized when considering these engines.

Equation (13) gives the machine work per stroke, but in engineering the work is reduced to the second; therefore designating by Q_s the heat quantity which the evaporator withdraws from the brine per second and substituting this value in place of Q_1 in equation (13), L_m gives the work required for the running of the cold-vapor engine per second, or, if the work is expressed in horsepowers and is designated by N_m, we have

$$N_m = \frac{Q_s}{75 \times A T_1}(T_2 - T_1) \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$\left[N_m = \frac{Q_s}{550 \times A T_1}(T_2 - T_1)\right].$$

If the heat quantity by which the cooling action of the machine is to be judged is reduced to the hour and designated by Q_h there obtains

$$Q_h = 3600 Q_s. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Now as regards the dimensions of the cold-vapor engine there arises first of all the question as to the size of the power-cylinder or compressor C for a prescribed performance.

The volume V_1 of this cylinder is directly given by equation (8), but in this formula we can unhesitatingly neglect the specific volume σ of the liquid and then get

$$V_1 = Gx_1u_1.$$

From equation (1), however, $Q_1 = Gr_1(x_1 - x_3)$, hence there follows by division

$$V_1 = \frac{Q_1}{\frac{r_1}{u_1}\left(1 - \frac{x_3}{x_1}\right)}. \quad \ldots \ldots \quad (16)$$

The volume V_2 of the feed-cylinder is determined by the formula

$$V_2 = Gx_3u_1,$$

from which follows

$$\frac{V_2}{V_1} = \frac{x_3}{x_1},$$

provided the cranks of these two pistons make the same number of revolutions.

Let F be the cross-section of the piston, s the piston stroke of the power-cylinder and let the double-acting machine make n revolutions per minute, then $V_1 = Fs$ and the heat quantity Q_e, absorbed by the evaporator per second, is

$$Q_e = \frac{2Q_1n}{60}.$$

Substitution in equation (16) therefore gives, for the calculation of the power-cylinder of the perfect cold-vapor engine. the equation

$$\frac{Fsn}{30} = \frac{Q_e}{\frac{r_1}{u_1}\left(1 - \frac{x_3}{x_1}\right)}, \quad \ldots \ldots \quad (17a)$$

or if we substitute x_2 from equation (5) we get

$$\frac{Fsn}{30} = \frac{Q_e}{\frac{r_1}{u_1}\left(1 - \frac{T_1(\tau_2 - \tau_1)}{x_1 r_1}\right)} \quad \ldots \quad (17b)$$

The right member of this equation contains temperature functions which are different for different vapors, and it is therefore evident that for the same values of Q_e the cylinder dimensions will be different according to the different kind of vapor employed. We will therefore choose for the working vapor not only one whose liquid is easily and cheaply produced but also one which, at the same time, leads to the smallest engine dimensions.

If we combine equation (17*b*) with equation (14), which, however, we omit here, we get the relation between the dimensions of the power-cylinder and the work N_m in horse-powers required by the machine.

The following remark is of practical importance.

In order to produce 1 kg. [lb.] of ice at 0° [32°] from water at 0° [32°] we must withdraw from the water the heat quantity $r_0 = 79$ cal. [142.2 B.t.u.]; if the water has the temperature $t°$ and ice is to be produced of the temperature $-t_0°$, the necessary amount of heat for 1 kg. [lb.] of water is

$$79 + t + ct_0,$$
$$[142.2 + (t-32) + c(t_0 - 32)],$$

where $c = 0.5$ represents the specific heat of ice. If we substitute the average value $t = 15°$ [59°] and, in order that the ice may be as solid as possible, take $t_0 = -8°$ [+17.6°], we get in round numbers 100 calories [180 B.t.u.].

If we wish to produce E_h kg. [lb.] of ice per hour then we must draw, hourly, the heat quantity $Q_h = 100E_h$ [$180E_h$] from the brine.

Example. Suppose that a cold-vapor engine works with ammonia vapor, and suppose the temperature in the evaporator to be $t_1 = -15°$ [+5°], and in the condenser $t_2 = +20°$ [68°], the pressures are, respectively, $p_1 = 2.291$ and $p_2 = 8.509$ atmospheres.

According to Table 8 of the Appendix we have for the assumed temperatures:

$$\frac{r_1}{T_1} = 1.2536, \qquad \frac{r_2}{T_2} = 1.0243,$$
$$\tau_1 = -0.0536, \qquad \tau_2 = +0.0774.$$

Now let the steam quality at the end of the suction in the power-cylinder be $x_1 = 0.90$, then according to equation (4), and utilizing the preceding values, the steam quality at the end of the compression is $x_2 = 0.9735$; therefore during the compression of ammonia evaporation takes place and the vapor will be superheated if it were initially dry saturated, i.e., if $x_1 = 1$. In this case, to which I will return, the above formulas are no longer valid and must be replaced by others.

From equation (5) we get in the assumed case for the steam quality at the end of the expansion in the feed-cylinder $x_3 = 0.1045$, and since according to the table $\frac{r_1}{u_1} = 619.80$ [69.645], equations (17a) and (14) give

$$\frac{Fsn}{30} = 0.001825\, Q_e \quad \text{and} \quad N_m = 0.767\, Q_e$$
$$\left[\frac{Fsn}{30} = 0.016241\, Q_e \quad \text{and} \quad N_m = 0.1906\, Q_e\right].$$

If the machine is to withdraw 36,000 calories [142860 B.t.u.] of heat from the brine, then $Q_e = 10$ [39.683], and hence if the machine makes $n = 50$ revolutions per minute, the required volume of the power-cylinder must be $Fs = 0.0109$ cbm. [0.38494 cu. ft.] and the work necessary is $N_m = 7.67$ horse powers [$= 7.565$].

If it were a case of ice production and E_h kg. [lb.] of ice were produced per hour, then because $Q_1 = 100\, E_h$ [180 E_h], we should have

$$Q_e = \frac{100}{3600} E_h \quad \left[Q_e = \frac{180}{3600} E_h = \frac{E_h}{20}\right].$$

and therefore

$$N_m = 0.0213\, E_h \quad \text{or} \quad E_h = 47\, N_m$$
$$[N_m = 0.009532\, E_h \quad \text{or} \quad E_h = 104.9\, N_m].$$

This perfect machine would therefore deliver per hour per horse power 47 kg. [104.9 lb.] of ice. In actual machines of course the production of ice is much smaller.

§ 60. CYCLE OF A PERFECT COLD-VAPOR ENGINE ACCORDING TO LORENZ.

In the preceding investigations it has been assumed that the temperature T_1 is the same at all points within the evaporator AA (Fig. 48), and that in the interior of the condenser there exists everywhere the same temperature T_2; here T_1 is the lower and T_2 the upper limit of temperature, which temperatures correspond to the saturation pressures p_1 and p_2, and moreover attention is here directed only to the mediating body (ammonia, carbonic acid) and to its changes of state.

If, however, at the same time we think of the changes of state of the body which furnishes the heat to the mediating body (here the brine in the evaporator), and also think of the body, which in another part of the cycle withdraws and absorbs heat (here the cooling water in the condenser), then the problem is broadened; thus under certain conditions there results the cycle which was first announced and investigated by Lorenz[1] (see Vol. I). p. 296).

Let us first think of the condenser EE (Fig. 48), and let

T_3 be the influx temperature,
T_2 the efflux temperature of the cooling water,

of which K kg. [lb.] passes through the apparatus in a unit of time. Let c_2 be the specific heat of this cooling body (with water $c_2=1$); then the heat withdrawn is

$$Q_2=c_2K(T_2-T_3): \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

In the first place let us assume a circulation of the brine in the evaporator. Let the brine at its lowest temperature T_1 be led through a line of pipe to spaces whose cooling is to be effected and then be led back to the evaporator, which it reaches with the higher temperature T, where it is again reduced to the low tempera-

[1] Lorenz, "Beiträge zur Beurteilung der Kühlmaschinen." Zeitschrift des Vereins deutscher Ingenieure, 1894, Vol. 38, pp. 62, 98, and 124.

ture T_1. In Fig. 48 we imagine that the brine flows along the tubes of the evaporator from left to right.

If, in a unit of time, S kg. [lb.] of brine passes through the evaporator, and if c_1 is the specific heat, then the heat quantity Q_1, withdrawn from the brine in a unit of time, is

$$Q_1 = c_1 S(T - T_1). \quad \ldots\ldots \quad (19)$$

Lorenz proceeds on the assumption that the mediating body within the machine possesses everywhere the temperature prevailing outside, so that in the evaporator (where vapor and liquid are present) the change of temperature keeps pace with that of the brine, and on the assumption that in the condenser, also, the change of temperature keeps pace with that of the cooling water.

Under this assumption Fig. 49*a* represents the indicator diagram and Fig. 49*b* represents its transformation, the entropy diagram.

The curves T_2T and T_3T_1 are adiabatic curves appearing as vertical lines in Fig. 49*b*; the two curves T_2T_3 and TT_1 are two different polytropic curves.

Now the polytropic curve is computed from the formula

$$AdP = c\frac{dT}{T},$$

and hence follows, for the curve TT_1,

$$AP_1 - AP_2 = c_1 S \log_e \frac{T}{T_1}, \quad \ldots\ldots \quad (20a)$$

and for the curve T_2T_3,

$$AP_1 - AP_2 = c_2 K \log_e \frac{T_2}{T_3}. \quad \ldots\ldots \quad (20b)$$

Equating we get from these, when the notation

$$\frac{c_1 S}{c_2 K} = m \quad \ldots\ldots \quad (21)$$

is introduced,

$$\frac{T_2}{T_3}=\left(\frac{T}{T_1}\right)^m, \quad \ldots\ldots \quad (22)$$

which equation Lorenz presents as the fundamental equation of the perfect cycle with finite quantities of brine and cooling water.

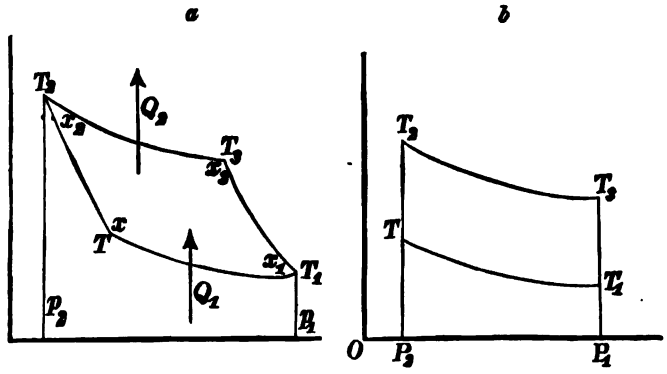

FIG. 49.

Earlier we found for a unit of weight of mixture of vapor and liquid that the entropy was

$$AP=\tau+\frac{xr}{T}.$$

Now let G be the weight of the mediating body reduced to the unit of time, and let the steam quality at the four vertices of the diagram (Fig. 49*a*) be designated by the same subscripts as the corresponding temperatures, then we have

$$\left.\begin{aligned} AP_2&=G\left(\tau_2+\frac{x_2r_2}{T_2}\right)=G\left(\tau+\frac{xr}{T}\right),\\ \text{also}\qquad AP_1&=G\left(\tau_3+\frac{x_3r_3}{T_3}\right)=G\left(\tau_1+\frac{x_1r_1}{T_1}\right)\end{aligned}\right\}\ldots\ldots \quad (23)$$

From this we find

$$AP_1-AP_2=G\left[\tau_1-\tau_2+\frac{x_1r_1}{T_1}-\frac{x_2r_2}{T_2}\right], \quad \ldots \quad (20c)$$

which equation united with equation (20*a*) or (20*b*) gives the relations between S and G or between K and G, where of course it is assumed that the mediating body is not superheated at any of the four vertices.

In so doing we can proceed on the assumption that at the vertex T_2, the beginning of the expansion in the feed-cylinder, there exists complete condensation, and hence that $x_2=0$; the combination of equation (20*c*) with (20*b*) then gives

$$G\left(\tau_1-\tau_2+\frac{x_1 r_1}{T_2}\right)=c_2 K \log_e \frac{T_2}{T_3}, \quad . \quad . \quad . \quad . \quad (24)$$

from which follows the relation between G and K; here we substitute $c_2=1$ because water is employed as the cooling body.

The theoretical driving work L of the machine is found from

$$AL=Q_2-Q_1,$$

or utilizing equations (18), (19), and (21),

$$AL=c_2K[T_2-T_3-m(T-T_1)]. \quad . \quad . \quad . \quad . \quad (25)$$

The foregoing reproduces the main features of the developments by Lorenz[1]; here we first considered, as was emphasized, the case in which the cool brine was led, through a line of pipe, away from the evaporator and then led back to it in the warm condition for cooling; this process in a continuous machine is constantly repeated.

Moreover, it is at the same time assumed that the mediating body (which flows through the pipe system of the evaporator) follows exactly, partly as vapor and partly as

[1] Compare this also with Lorenz on "Die Grenzwerthe der thermodynamischen Energieumwandlung." Inaugural Dissertation. Munich, 1895. Also his "Vergleichende Theorie und Berechnung der Kompressions-Kühlmaschine," Munich, 1897, and his various articles in the "Zeitschrift für die gesammte Kälteindustrie," Munich and Leipsic. Since the appearance of the last edition but one of the present book, the literature of refrigerating machines has experienced an extraordinary extension, so that in this edition we have only had to take up the changes on the purely theoretical side of the questions involved.

liquid, the changes of temperature of the brine. We may well express doubt as to the permissibility of this assumption, particularly when we consider the fact that the mixture of vapor and liquid is everywhere under the same pressure in the pipe system of the evaporator. We have here a case similar to that occurring in the steam boilers of engines in which, in a certain sense, the heat falls from the temperature of the fire gases to the temperature of the boiler charge. It is not possible, at the present time, to exactly follow this procedure, but it is still a question whether the assumption $T = T_1$ (Fig. 49) for the temperature of the mediating body is not nearer the truth.

Of course it is different with the condenser; here the pipe system is filled almost throughout its whole extent with the mediating body in its liquid state, so that the latter may well follow the changes of temperature in the counter-current of cooling water.

If the brine is not conducted away from the evaporator by a pipe line and then brought back in a warm state, but remains standing in the evaporator space, as was conceived to be the case in ice production, and in the representation Fig. 48, p. 463, then the assumption $T = T_1$ is the more to be justified, because here a stirring of the brine occurs, so that we may assume in the total solution an average constant temperature (somewhat greater than $T_1 = T$ of the mediating body).

§ 61. CYCLE OF ACTUAL COLD-VAPOR ENGINES.

Fig. 50 is a sketch of a cold-vapor engine which closely approaches the actual construction; C is the compression-cylinder, the compressor, with its inlet and outlet valves, V is the evaporator and K the condenser; the mediating body circulates in coiled pipes. In the evaporator the coil is surrounded by brine which, as the following developments will assume, is kept in motion by a stirring apparatus, so that we may here assume a constant, mean temperature. Let the same temperature T_1 (lower limit) prevail everywhere in the evaporator coil, and let the corresponding saturation pressure be p_1. In the condenser K the coil

is surrounded by cooling water, which enters the enveloping casing with the temperature T_3 and is discharged and led away with the temperature T_2. The compressor sucks the vapor from the evaporator V, compresses it, and forces it into the condenser coil in which the pressure p_2 every where prevails; above, at the entrance, the mixture of vapor and liquid has a temperature T_2 corresponding to the saturation pressure p_2; the vapor condenses and, under the influence of the opposite current of the cooling water, reaches the lower end with the temperature T_3. As the mediating body, which is here completely liquefied, is still subject to the pressure p_2, but has $T_3 < T_2$, we say that the liquid at this place is "undercooled."

The problem now is to return the liquid to the evaporator. In the theoretically perfect machine (Fig. 48, p. 463) this was accomplished by the feed- or expansion-cylinder, and in so doing it was furthermore assumed that everywhere there prevailed the same temperature t_2, so that $T_3 = T_2$. It was mainly due to this last assumption that in transferring the propositions of § 59, p. 463, to actual machines, particularly to the carbonic acid (cold-vapor) machine, results were obtained which were not in harmony with the observations.

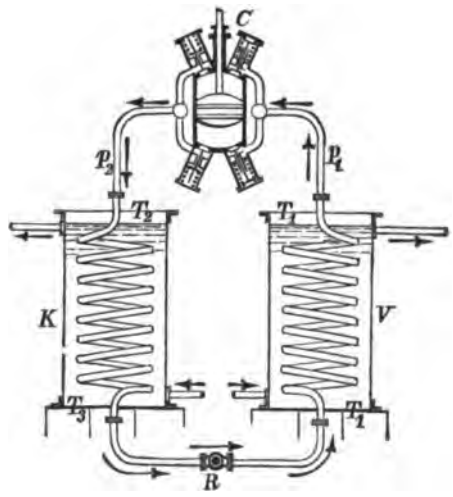

FIG. 50.

The actual machines possess no expansion-cylinder; as the pressure in the condenser is greater than in the evaporator, the two spaces are simply connected by a pipe, provided with a regulating valve R (Fig. 50), through which the liquid can be forced directly back to the evaporator.

Let us now consider this mode of transfer.

Let the liquid flow with the velocity w through the regulating valve, so that the unit of weight possesses the kinetic energy $H=\frac{w^2}{2g}$; ignoring the resistances, this is composed of the following parts.

During the exit from the condenser the liquid takes up the work σp_2, and while going through the valve expands adiabatically to the evaporator pressure p_1 and in so doing x_3' kg. [lb.] of steam is formed; the work of expansion, according to earlier propositions, is therefore

$$\frac{1}{A}(q_3-q_1-x_3'\rho_1),$$

because the liquid on its way reaches the regulating valve with the temperature T_3 or t_3.

During the entrance into the evaporator the constant pressure p_1 must be overcome, which demands the work $p_1(x_3'u_1+\sigma)$; accordingly, collecting and considering the relation $r_1=\rho_1+Ap_1u_1$, we get

$$AH=q_3-q_1-x_3'r_1+A\sigma(p_2-p_1). \quad . \quad . \quad . \quad . \quad (26)$$

In the evaporator the mass spreads out under constant pressure and goes into a state of rest, and in so doing the steam quality grows from x_3' to x_4, so that we also have

$$AH=r_1(x_4-x_3').$$

The combination of the last equations then gives

$$r_1x_4=q_3-q_1+A\sigma(p_2-p_1), \quad . \quad . \quad . \quad . \quad . \quad (27)$$

from which the steam quality x_4 can be calculated.

If the compressor sucks in per stroke a mass G kg. [lb.] of the steam quality x_1, then x_1-x_4 is the increase per stroke in this quality, and accordingly the quantity of heat Q withdrawn from the brine is

$$Q=Gr_1(x_1-x_4), \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

or, with equation (27),

$$Q=G\left[r_1x_1-\left(q_3-q_1+A\sigma(p_2-p_1)\right)\right]. \quad . \quad . \quad . \quad (28a)$$

If x_2 is the steam quality at the end of the adiabatic compression

in the compressor, there will subsist the known relation (equation (4))

$$\tau_2+\frac{x_2r_2}{T_2}=\tau_1+\frac{x_1r_1}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

The combination of the preceding equations gives

$$Q=G\left[\frac{T_1}{T_2}x_2r_2+T_1(\tau_2-\tau_1)+q_1-q_3-A\sigma(p_2-p_1)\right], \quad (30)$$

from which Q can be directly computed.

If we assume that the liquid from the condenser does not undercool, but reaches the regulating valve with the temperature t_2, corresponding to the condenser pressure p_2, then we must substitute in equation (27) q_2 in place of q_3 and x_4' in place of x_4; then we shall have

$$r_1x_4'=q_2-q_1+A\sigma(p_2-p_1),$$

and in combination with equation (27)

$$x_4'=x_4+\frac{q_2-q_3}{r_1},$$

and hence for the heat withdrawn from the brine

$$Q=Gr_1(x_1-x_4')=Gr_1(x_1-x_4)-G(q_2-q_3).$$

Comparison with equation (28) shows that undercooling from t_2 to t_3 h e i g h t e n s the action of the evaporator by the amount $G(q_2-q_3)$; this is a circumstance which needs special consideration in carbonic acid engines working in the vicinity of the critical temperature.

Now it is a question of determining the work L necessary for running the compressor.

Fig. 51 *a* represents the indicator diagram and Fig. 51 *b* its transformation, the entropy diagram. The driving work L of the machine, the necessary compression work, can be found in the same

way as in the derivation of equation (10), p. 468,

$$AL = G[q_2 - q_1 + x_2 r_2 - x_1 r_1 + A\sigma(p_2 - p_1)]. \quad . \quad . \quad (31)$$

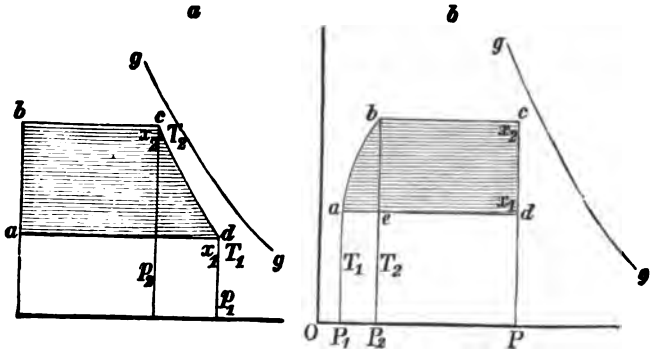

FIG. 51.

In the entropy diagram (Fig. 51 *b*) the expressions for the separate distances are

$$\overline{ae} = \tau_2 - \tau_1; \quad \overline{bc} = \frac{r_2 x_2}{T_2}, \quad \overline{ad} = \frac{x_1 r_1}{T_1}.$$

The area lying under the curve *ab* represents $q_2 - q_1$, the area lying under *bc* and *ad* representing $x_2 r_2$ and $x_1 r_1$ respectively; the hatched area therefore measures the value

$$q_2 - q_1 + x_2 r_2 - x_1 r_1$$

of equation (31). The curve *gg* stands for the limit curve and its transformation respectively.

If a perfect cycle (Carnot) were under consideration, then to the heat quantity Q there would correspond a work L_m, according to equation (13), p. 469,

$$AL_m = \frac{Q}{T_1}(T_2 - T_1); \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

hence the efficiency η of the present machine is

$$\eta = \frac{L_m}{L} = \frac{Q}{AL} \frac{(T_2 - T_1)}{T_1}, \quad . \quad . \quad . \quad . \quad . \quad (33)$$

where Q and AL are to be determined from equations (30) and (31).

If the vapor at the end of the compression is to be just dry saturated, we get from equation (29), with $x_2 = 1$, the steam quality x_1 of the vapor sucked in from the evaporator. In this case the point c of Fig. 51 falls on the limit curve.

Example 1. In an ammonia refrigerating machine let the temperature in the evaporator be $t_1 = -10°$ [$+14°$] and when entering the condenser let the temperature be $t_2 = +20°$ [68°]; according to Table 8 of the Appendix the corresponding pressures are $p_1 = 2.9227$ [41.57] and $p_2 = 8.7923$ kg. per sq. cm. [125.05 lb. per sq. in.].

For $x_2 = 1$ (wet compression) we get from equation (29) the steam quality x_1 during suction,

$$x_1 = 0.933,$$

and from equation (31)

$$AL = 32.701\ G.\ [58.862\ G].$$

Now let the temperature t_3, with which the liquid reaches the regulating valve, be 10° [50°] at one time and at another be 20° [68°], we then get, respectively, for

$$t_3 = 10°\ [50°], \quad t_3 = 20°\ [68°],$$

according to equation (27),

$$x = 0.0638, \quad x_4 = 0.0993,$$

and from equation (28),

$$Q = 278.80\ G\ [501.84\ G], \quad Q = 267.42\ G\ [481.36\ G];$$

finally, from equation (23), the efficiency

$$\eta = 0.973, \quad \eta = 0.933,$$

which shows the influence of the undercooling.

Example 2. In a carbonic acid refrigerating machine let the temperature in the evaporator again be $-10°$ [$+14°$], and hence, according to Table 10 of the Appendix, the pressure there is $p_1 = 27.1$ kg. per sq. cm. [385.45 lb. per sq. in.]. Let us now assume that the temperature corresponding to the condenser pressure is at one time $t_2 = 20°$ [68°], and that at another time it is $t_2 = 30°$ [86°], which in the latter case is very near the critical temperature ($t_k = 31°$) [$t_k = 87.8°$]. For a wet course of compression ($x_2 = 1$) we calculate, in the same way as before, respectively

$$x_1 = 0.813 \quad \text{and} \quad x_1 = 0.668.$$

Now suppose that in the first case we have at one time $t_3 = 10°$ [50°] and again $t_3 = 20°$ [68°], and in the second case we have the two variations $t_3 = 20°$

[68°] and $t_3 = 30°$ [86°]; then we obtain the following numerical results for the efficiency: [1]

$t_1 = -10°$ [14°]	$t_2 = 20°$ [68°]		$t_2 = 30°$ [86°]	
$t_3 =$	10° [50°]	20° [68°]	20° [68°]	30° [86°]
$\eta =$	0.797	0.650	0.612	0.268

§ 62. INFLUENCE OF SUPERHEATING IN THE COMPRESSOR OF ACTUAL COLD-VAPOR ENGINES.

Here we will subject to examination the special case that the compressor draws in dry saturated vapor ($x_1 = 1$) from the evaporator (dry compression). Then we at once get from equation (28*a*) for the heat withdrawn from the brine

$$Q = G[q_1 + r_1 - q_3 - A\sigma(p_2 - p_1)]. \quad . \quad . \quad . \quad . \quad (34)$$

Here the indicator diagram is like Fig. 52*a*, while the corre-

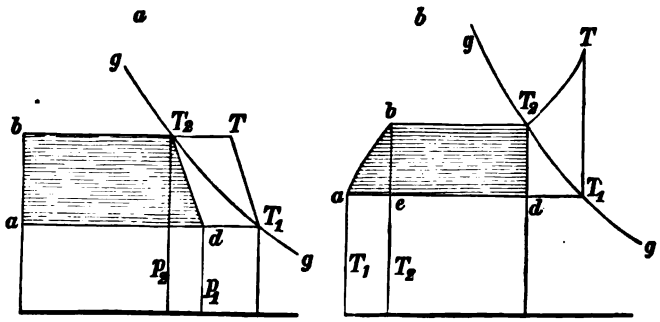

FIG. 52.

sponding entropy diagram is represented by Fig. 52*b*.

[1] According to Linde, "Kälteerzeugungsmaschine" in Lueger's Lexicon der gesammten Technik, Vol. V, p. 353. This article embraces the latest researches in this field and was the basis of the investigations presented in the text above. The article also contains valuable data concerning experiments on machines and concerning different constructive arrangements, and contains also the discussion of absorption machines, of cold-air engines, and of machines for generating the lowest temperatures, so that for further information we refer the reader to this excellent paper.

In both figures the hatched area gives the driving work L according to the combination of equation (31) with equation (39) and with $x_2 = 1$, so that we have

$$AL = G\left[\frac{r_2}{T_2}(T_2 - T_1) + q_2 - q_1 - T_1(\tau_2 - \tau_1) + A\sigma(p_2 - p_1)\right]. \quad (35)$$

On account of the superheating to T there must be added the work L', which in Fig. 52*a* and in Fig. 52*b* is represented by the area of the curved quadrilateral T_2TT_1d. The curves T_2d and TT_1 are adiabatics which, in the entropy diagram, are represented by vertical lines; here the point T_1 lies on the limit curve for $x_1 = 1$.

If c_p is the specific heat of the vapor at constant pressure (which will be assumed as constant), then for the passage from one adiabatic to the other the change of entropy AP is

$$AP = c_p \log_e \frac{T}{T_2};$$

on the other hand for the passage T_2T_1 on the limit curve, we have

$$AP = \left(\frac{r_1}{T_1} + \tau_1\right) - \left(\frac{r_2}{T_2} + \tau_2\right).$$

Equating, we get

$$c_p \log_e \frac{T}{T_2} = \left(\frac{r_1}{T_1} + \tau_1\right) - \left(\frac{r_2}{T_2} + \tau_2\right), \quad . \ . \ . \quad (36)$$

from which equation there now results the temperature T of the superheated vapor at the end of the compression in the compressor.

The whole area lying under the curve T_2T is determined by $c_p(T - T_2)$ and the area lying under T_1d by APT_1, and hence the corresponding work L' follows from

$$AL' = G[c_p(T - T_2) - APT_1],$$

or with equation (36),

$$AL' = G\left[c_p(T - T_2) + \frac{T_1}{T_2}r_2 - r_1 + T_1(\tau_2 - \tau_1)\right]. \quad . \quad (37)$$

If we now add the two equations (35) and (37) we get the work L_c necessary to run the compressor from

$$AL_c = G[c_p(T - T_2) + (r_2 + q_2) - (r_1 + q_1) + A\sigma(p_2 - p_1)], \quad . \quad (38)$$

where the temperature T is determined from equation (36). The efficiency of the machine is then calculated as before (equation 33), and is

$$\eta = \frac{Q}{AL_c} \frac{(T_2 - T_1)}{T_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

where Q is found from equation (34).

Example. In an ammonia machine (cold-vapor engine) let us assume, as in example 1, p. 483, the temperature in the evaporator to be $t_1 = -10°$ [+14°] and the temperature corresponding to the condenser pressure to be $t_2 = +20°$ [68°]. With the help of Table 8 of the Appendix and $c_p = 0.5084$, we get from equation (36) the temperature of the superheated steam at the end of the compression:

$$T = 344° \quad \text{or} \quad t = 71° \text{ C.}$$
$$[T = 619.2° \quad \text{or} \quad t = 159.8° \text{ F.}].$$

The work AL_c for the compressor is found from equation (38):

$$AL_c = 37.078\ G\ [66.74].$$

From equation (34) we calculate

$$Q = (310.892 - q_3)G$$
$$[Q = (559.606 - q_3)G].$$

If we now substitute the temperature t_3 at the regulating valve, namely,

$$t_3 = 10° \quad \text{or} \quad t_3 = 20°$$
$$[t_3 = 50° \quad \text{or} \quad t_3 = 68°],$$

then calculation gives

$$Q = 34.261\ G \quad \text{and} \quad Q = 32.963\ G$$
$$[Q = 61.67\ G \quad \text{and} \quad Q = 59.333\ G],$$

and finally from equation (39)

$$\eta = 0.924 \quad \text{and} \quad \eta = 0.889.$$

We can treat in a similar way the special cases for carbonic acid, particularly those in which the temperature values lie in the vicinity of the critical temperature.

Although the preceding calculations and recent, careful experiments [1] have shown that the carbonic-acid machine effects a certain production of cold in the neighborhood of the critical temperature and even somewhat beyond it (which was formerly doubted by the author), nevertheless when we essay a thorough, theoretical examination of the question we encounter difficulties which cannot as yet be overcome and which further physical investigations must clarify.

In deriving the heat equations for mixtures of vapor and liquid (§ 8, p. 53) and when applying these equations, we and others have always assumed that the specific volume σ of the liquid may be regarded as a constant quantity during the changes of state.

But this assumption is no longer fulfilled by vapors in the neighborhood of the critical point, for here the value σ experiences a comparatively large increment. According to Amagat's experiments this variation (see Table 10 of the Appendix) may be regarded as sufficiently known; but this reacts on the heat of the liquid q and on the heat of evaporation r, so that the tabular data respecting them seem still more uncertain than before. This field is still open for theoretical and experimental investigations.

§ 63. CALCULATION OF THE DIMENSIONS OF THE COMPRESSOR.

In calculating a machine there is usually prescribed the heat quantity Q_h which must be withdrawn hourly from the brine in the evaporator; with its help we get the heat quantity Q per second, and then the corresponding weight G of the vapor which the compressor must suck in per second; in so doing we use, respectively, equations (28a) and (34), where the temperature values t_1, t_2, and t_3 are regarded as known.

If v_1 is the specific volume of the sucked-in vapor, then for wet compression we have $v_1 = x_1 u_1 + \sigma$ for $x_2 = 1$, and for the other

[1] Linde, "Zur Theorie der Kohlensäure- (Kaltdampf-) Maschine." Zeitschrift des Vereins deutscher Ingenieure, Vol. 39, 1895, p. 124.

case of dry compression we have $v_1 = u_1 + \sigma$ for $x_1 = 1$; here we must give σ for carbonic acid the volume found in Table 10 for the temperature t_1, and the volume of vapor that must then be drawn in per second is Gv_1.

Furthermore, if F is the cross-section of the compressor piston, s its stroke, and if the double-acting machine makes n revolutions per minute, then the volume swept through by the piston per second is

$$\frac{Fsn}{30},$$

and if we introduce the coefficient ϕ which in good machines is to be taken from 1.2 to 1.3, we get the relation

$$\frac{Fsn}{30} = \phi G v_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

If d is the piston diameter and if we assume the ratio $s:d$, we can calculate d from equation (40); for the ratio $s:d$ Linde takes 1.5 to 1.75 for ammonia, 3 to 4 for carbonic acid, and 2 to 2.2 for sulphurous acid.

The theoretical value for the driving work per second of the compressor can be found from equation (31) or (38), where G is to be substituted from the preceding formula.

APPENDIX.

Vapor Tables with French Units:

PAGE

Saturated vapor of water.....II
Saturated vapor of ether.....VI
Saturated vapor of alcohol.....VIII
Saturated vapor of acetone.....X
Saturated vapor of chloroform.....XII
Saturated vapor of chloride of carbon.....XIV
Saturated vapor of bisulphide of carbon.....XVI
Saturated vapor of ammonia.....XVIII
Saturated vapor of sulphurous acid.....XX
Saturated vapor of carbonic acid.....XXII
First principal table for water vapor.....XXIV
Second principal table for water vapor.....XXVII

Vapor Tables with English Units:

Saturated vapor of water.....XXX
Saturated vapor of ether.....XXXIV
Saturated vapor of alcohol.....XXXVI
Saturated vapor of acetone.....XXXVIII
Saturated vapor of chloroform.....XL
Saturated vapor of chloride of carbon.....XLII
Saturated vapor of bisulphide of carbon.....XLIV
Saturated vapor of ammonia.....XLVI
Saturated vapor of sulphurous acid.....XLVIII
Saturated vapor of carbonic acid.....L
First principal table for water vapor.....LII
Second principal table for water vapor.....LV

Auxiliary Tables:

English units corresponding to table with French units on pages 62 and 63.....LVIII

Conversion of pounds per square inch into atmospheres.....LX

FRENCH UNITS.

Saturated Vapor

1. Temperature Centigrade t	2. Absolute Temperature T	3. Steam Pressure in Millimeters of Mercury p (Regnault)	4. Differential Coefficient in Millimeters of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
−20	253	0.927	0.07687	0.082926
−15	258	1.400	0.11186	0.079900
−10	263	2.093	0.16114	0.076991
− 5	268	3.113	0.23096	0.074193
0	273	4.600	0.3289	0.071502
5	278	6.534	0.4503	0.068915
10	283	9.165	0.6088	0.066429
15	288	12.699	0.8132	0.064041
20	293	17.391	1.0738	0.061746
25	298	23.550	1.4022	0.059542
30	303	31.548	1.8117	0.057427
35	308	41.827	2.3171	0.055397
40	313	54.906	2.9347	0.053449
45	318	71.390	3.6825	0.051582
50	323	91.980	4.5800	0.049794
55	328	117.475	5.6484	0.048084
60	333	148.786	6.9100	0.046443
65	338	186.938	8.3891	0.044876
70	343	233.082	10.111	0.043380
75	348	288.500	12.104	0.041953
80	353	354.616	14.395	0.040594
85	358	433.002	17.017	0.039300
90	363	525.392	20.002	0.038072
95	368	633.692	23.388	0.036907
100	373	760.000	27.189	0.035775
105	378	906.410	31.454	0.034701
110	383	1075.370	36.212	0.033674
115	388	1269.410	41.499	0.032691
120	393	1491.280	47.348	0.031750
125	398	1743.880	53.795	0.030848
130	403	2030.280	60.873	0.029982
135	408	2353.730	68.617	0.029152
140	413	2717.630	77.060	0.028355
145	418	3125.550	86.234	0.027590
150	423	3581.230	96.171	0.026854
155	428	4088.560	106.901	0.026146
160	433	4651.620	118.455	0.025465
165	438	5274.540	130.858	0.024809
170	443	5961.660	144.138	0.024177
175	448	6717.430	158.319	0.023568
180	453	7546.390	173.423	0.022981
185	458	8453.230	189.470	0.022414
190	463	9442.700	206.478	0.021866
195	468	10519.630	224.462	0.021337
200	473	11688.960	243.438	0.020826

FRENCH UNITS.

of Water. Table 1a.

6. $T.\frac{dp}{pdt}$	7. $\frac{r}{u}=AT\frac{dp}{dt}$	8. $\frac{\rho}{u}=\frac{r}{u}-Ap$	9. $\frac{Apu}{r}=\phi$	1. Temperature Centigrade t
20.980	0.6236	0.5939	0.047664	−20
20.614	0.9255	0.8810	0.048510	−15
20.249	1.3589	1.2918	0.049386	−10
19.884	1.9847	1.8849	0.050293	− 5
19.520	2.879	2.732	0.051229	0
19.158	4.014	3.805	0.052196	5
18.799	5.524	5.230	0.053193	10
18.444	7.510	7.103	0.054219	15
18.091	10.089	9.531	0.055274	20
17.743	13.399	12.644	0.056358	25
17.400	17.602	16.590	0.057470	30
17.062	22.884	21.543	0.058609	35
16.730	29.454	27.693	0.059774	40
16.403	37.550	35.261	0.060964	45
16.083	47.437	44.488	0.062176	50
15.771	59.407	55.641	0.063409	55
15.465	73.785	69.013	0.064661	60
15.168	90.924	84.930	0.065927	65
14.879	111.209	103.736	0.067206	70
14.600	135.064	125.813	0.068494	75
14.329	162.943	151.572	0.069786	80
14.069	195.350	181.464	0.071076	85
13.820	232.829	215.982	0.072359	90
13.582	275.982	255.662	0.073628	95
13.344	325.198	300.827	0.074940	100
13.117	381.249	352.184	0.076236	105
12.897	444.735	410.252	0.077536	110
12.684	516.312	475.607	0.078838	115
12.478	596.678	548.849	0.080143	120
12.277	686.544	630.624	0.081450	125
12.083	786.640	721.537	0.082761	130
11.894	897.716	822.242	0.084075	135
11.711	1020.528	933.385	0.085391	140
11.533	1155.848	1055.624	0.086709	145
11.359	1304.457	1189.621	0.088034	150
11.191	1467.141	1336.037	0.089361	155
11.026	1644.701	1495.541	0.090691	160
10.866	1837.898	1668.765	0.092026	165
10.711	2047.526	1856.359	0.093365	170
10.559	2274.350	2058.948	0.094709	175
10.410	2519.129	2277.146	0.096059	180
10.265	2782.606	2511.544	0.097413	185
10.124	3065.498	2762.707	0.098774	190
9.986	3368.494	3031.171	0.100140	195
9.851	3692.293	3317.474	0.101514	200

FRENCH UNITS.

Saturated Vapor

1. Temperature Centigrade t	2. Total Heat λ (Regnault)	Differences	3. Heat of the Liquid $q=\int_0^t cdt$ (Regnault)	Differences	4. Latent Heat $r=\lambda-q$	Differences
−20	600.400	1.525	−19.994	4.998	620.394	3.473
−15	601.925	"	−14.996	4.998	616.921	3.473
−10	603.450	"	− 9.998	4.999	613.448	3.474
− 5	604.975	"	− 4.999	4.999	609.974	3.474
0	606.500	"	0	5.000	606.500	3.475
5	608.025	"	5.000	5.002	603.025	3.477
10	609.550	"	10.002	5.003	599.548	3.478
15	611.075	"	15.005	5.005	596.070	3.480
20	612.600	"	20.010	5.007	592.590	3.482
25	614.125	"	25.017	5.009	589.108	3.484
30	615.650	"	30.026	5.011	585.624	3.486
35	617.175	"	35.037	5.014	582.138	3.489
40	618.700	"	40.051	5.017	578.649	3.492
45	620.225	"	45.068	5.019	575.157	3.494
50	621.750	"	50.087	5.023	571.663	3.498
55	623.275	"	55.110	5.027	568.165	3.502
60	624.800	"	60.137	5.030	564.663	3.505
65	626.325	"	65.167	5.034	561.158	3.509
70	627.850	"	70.201	5.038	557.649	3.513
75	629.375	"	75.239	5.043	554.136	3.518
80	630.900	"	80.282	5.047	550.618	3.522
85	632.425	"	85.329	5.052	547.096	3.527
90	633.950	"	90.381	5.057	543.569	3.532
95	635.475	"	95.438	5.062	540.037	3.537
100	637.000	"	100 500	5.068	536.500	3.543
105	638.525	"	105.568	5.073	532.957	3.548
110	640.050	"	110.641	5.080	529.409	3.555
115	641.575	"	115.721	5.085	525.854	3.560
120	643.100	"	120.806	5.092	522.294	3.567
125	644.625	"	125.898	5.099	518.727	3.571
130	646.150	"	130.997	5.106	515.153	3.581
135	647.675	"	136.103	5.11[illegible]	511.572	3.587
140	649.200	"	141.215	5.119	507.985	3.594
145	650.725	"	146.334	5.128	504.391	3.603
150	652.250	"	151.462	5.136	500.788	3.611
155	653.775	"	156.598	5.143	497.177	3.618
160	655.300	"	161.741	5.151	493.559	3.626
165	656.825	"	166.892	5.160	489.933	3.635
170	658.350	"	172.052	5.168	486.298	3.643
175	659.875	"	177.220	5.178	482.655	3.653
180	661.400	"	182.398	5.186	479.002	3.661
185	662.925	"	187.584	5.196	475.341	3.671
190	664.450	"	192.780	5.205	471.670	3.680
195	665.975	"	197.985	5.215	467.990	3.690
200	667.500		203.200		464.300	

FRENCH UNITS.

of Water. Table 1b.

5. Outer Latent Heat $Apu = \phi r$	Differences	6. Steam Heat $J = \lambda - Apu$	Differences	7. Inner Latent Heat $\rho = r - Apu$	Differences	8. Values of $u = \frac{r}{\left(\frac{r}{u}\right)}$ $(\sigma = 0.0010)$	1. Temperature Centigrade t
29.570		570.83		590.82		994.78	−20
	0.357		1.17		3.83		
29.927		572.00		586.99		666.59	−15
	0.369		1.15		3.84		
30.296		573.15		583.15		451.42	−10
	0.381		1.15		3.85		
30.677		574.30		579.30		307.33	− 5
	0.394		1.13		3.87		
31.071		575.43		575.43		210.68	0
	0.404		1.12		3.88		
31.475		576.55		571.55		150.24	5
	0.417		1.11		3.89		
31.892		577.66		567.66		108.52	10
	0.426		1.10		3.91		
32.318		578.76		563.75		79.354	15
	0.437		1.08		3.92		
32.755		579.84		559.83		58.726	20
	0.446		1.08		3.92		
33.201		580.92		555.91		43.967	25
	0.455		1.07		3.94		
33.656		581.99		551.97		33.269	30
	0.463		1.07		3.95		
34.119		583.06		548.02		25.438	35
	0.469		1.05		3.96		
34.588		584.11		544.06		19.646	40
	0.476		1.05		3.97		
35.064		585.16		540.09		15.316	45
	0.480		1.05		3.97		
35.544		586.21		536.12		12.050	50
	0.483		1.04		3.98		
36.027		587.25		532.14		9.5622	55
	0.485		1.04		3.99		
36.512		588.29		528.15		7.6538	60
	0.484		1.04		3.99		
36.996		589.33		524.16		6.1717	65
	0.482		1.04		3.99		
37.478		590.37		520.17		5.0144	70
	0.477		1.05		3.99		
37.955		591.42		516.18		4.1028	75
	0.470		1.05		3.99		
38.425		592.47		512.19		3.3792	80
	0.460		1.07		3.98		
38.885		593.54		508.21		2.8006	85
	0.447		1.08		3.97		
39.332		594.62		504.24		2.3346	90
	0.430		1.09		3.97		
39.762		595.71		500.27		1.9568	95
	0.443		1.08		3.98		
40.205		596.79		496.29		1.6498	100
	0.426		1.10		3.96		
40.631		597.89		492.33		1.3979	105
	0.417		1.11		3.97		
41.048		599.00		488.36		1.1904	110
	0.409		1.12		3.96		
41.457		600.12		484.40		1.0185	115
	0.401		1.12		3.96		
41.858		601.24		480.44		0.8753	120
	0.392		1.13		3.96		
42.250		602.37		476.48		0.7556	125
	0.384		1.15		3.96		
42.634		603.52		472.52		0.6549	130
	0.376		1.14		3.96		
43.010		604.66		468.56		0.5699	135
	0.367		1.16		3.95		
43.377		605.82		464.61		0.4977	140
	0.358		1.17		3.95		
43.735		606.99		460.66		0.4363	145
	0.351		1.17		3.96		
44.086		608.16		456.70		0.3839	150
	0.342		1.19		3.95		
44.428		609.35		452.75		0.3388	155
	0.333		1.21		3.95		
44.761		610.54		448.80		0.3001	160
	0.325		1.20		3.95		
45.086		611.74		444.85		0.2665	165
	0.317		1.21		3.96		
45.403		612.95		440.89		0.2375	170
	0.308		1.21		3.95		
45.711		614.16		436.94		0.2122	175
	0.299		1.23		3.95		
46.012		615.39		432.99		0.1901	180
	0.292		1.23		3.95		
46.304		616.62		429.04		0.1703	185
	0.285		1.24		3.96		
46.589		617.86		425.08		0.1538	190
	0.275		1.25		3.95		
46.864		619.11		421.13		0.1389	195
	0.269		1.26		3.96		
47.133		620.37		417.17		0.1257	200

FRENCH UNITS.

Saturated Vapor

1. Temperature Centigrade t	2. Absolute Temperature T	3. Steam Pressure in Millimeters of Mercury p (Regnault)	4. Differential Coefficient in Millimeters of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
0	273	184.39	8.441	0.045778
10	283	286.83	12.224	0.042619
20	293	432.78	17.175	0.039685
30	303	634.80	23.464	0.036963
40	313	907.04	31.242	0.034444
50	323	1264.83	40.622	0.032117
60	333	1725.01	51.709	0.029976
70	343	2304.90	64.576	0.028017
80	353	3022.79	79.327	0.026243
90	363	3898.26	96.127	0.024659
100	373	4953.30	115.313	0.023280
110	383	6214.63	137.523	0.022129
120	393	7719.20	163.979	0.021243

Saturated Vapor

1. Temperature Centigrade t	2. Total Heat λ (Regnault)	Differences	3. Heat of the Liquid $q=\int_0^t cdt$ (Regnault)	Differences	4. Latent Heat $r=\lambda-q$	Differences
0	94.00		0.00		94.00	
		4.44		5.32		0.88
10	98.44		5.32		93.12	
		4.34		5.38		1.04
20	102.78		10.70		92.08	
		4.22		5.44		1.22
30	107.00		16.14		90.86	
		4.11		5.49		1.38
40	111.11		21.63		89.48	
		4.00		5.56		1.56
50	115.11		27.19		87.92	
		3.89		5.61		1.72
60	119.00		32.80		86.20	
		3.78		5.68		1.90
70	122.78		38.48		84.30	
		3.66		5.73		2.07
80	126.44		44.21		82.23	
		3.56		5.79		2.23
90	130.00		50.00		80.00	
		3.44		5.86		2.42
100	133.44		55.86		77.58	
		3.34		5.91		2.57
110	136.78		61.77		75.01	
		3.22		5.97		2.75
120	140.00		67.74		72.26	

FRENCH UNITS.

of Ether. Table 2a.

6. $T.\frac{dp}{pdt}$	7. $\frac{r}{u}=AT\frac{dp}{dt}$	8. $\frac{\rho}{u}=\frac{r}{u}-Ap$	9. $\frac{Apu}{r}=\phi$	1. Temperature Centigrade t
12.497	73.89	67.98	0.08002	0
12.061	110.93	101.73	0.08291	10
11.628	161.36	147.49	0.08600	20
11.200	227.98	207.62	0.08929	30
10.781	313.57	284.48	0.09275	40
10.374	420.74	380.18	0.09640	50
9.982	552.15	496.83	0.10018	60
9.610	710.25	636.34	1.10406	70
9.264	897.93	801.00	0.10794	80
8.951	1118.90	993.90	0.11172	90
8.683	1379.21	1220.37	0.11516	100
8.475	1688.97	1489.69	0.11799	110
8.348	2066.46	1818.93	0.11978	120

of Ether. Table 2b.

5. Outer Latent Heat $Apu=\phi r$	Differences	6. Steam Heat $J=\lambda-Apu$	Differences	7. Inner Latent Heat $\rho=r-Apu$	Differences	8. Values of $u=\frac{r}{\left(\frac{r}{u}\right)}$ $(\sigma=0.0013)$	1. Temperature Centigrade t
7.521		86.48		86.48		1.2721	0
	0.200		4.24		1.08		
7.721		90.72		85.40		0.8394	10
	0.198		4.14		1.24		
7.919		94.86		84.16		0.5707	20
	0.194		4.01		1.41		
8.113		98.89		82.75		0.3985	30
	0.187		3.92		1.57		
8.300		102.81		81.18		0.2853	40
	0.175		3.83		1.73		
8.475		106.64		79.45		0.2089	50
	0.160		3.73		1.88		
8.635		110.37		77.57		0.1561	60
	0.137		3.64		2.04		
8.772		114.01		75.53		0.1186	70
	0.104		3.55		2.18		
8.876		117.56		73.35		0.0916	80
	0.061		3.50		2.29		
8.937		121.06		71.06		0.0715	90
	−0.003		3.45		2.41		
8.934		124.51		68.65		0.0562	100
	−0.084		3.42		2.49		
8.850		127.93		66.16		0.0444	110
	−0.195		3.42		2.55		
8.655		131.35		63.61		0.0350	120

FRENCH UNITS.

Saturated Vapor

1. Temperature Centigrade t	2. Absolute Temperature T	3. Steam Pressure in Millimeters of Mercury p (Regnault)	4. Differential Coefficient in Millimeters of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
0	273	12.70	0.8419	0.066295
10	283	24.23	1.5185	0.062672
20	293	44.46	2.6133	0.058779
30	303	78.52	4.3193	0.055009
40	313	133.69	6.8784	0.051450
50	323	219.90	10.580	0.048114
60	333	350.21	15.757	0.044992
70	343	541.15	22.767	0.042072
80	353	812.91	31.981	0.039342
90	363	1189.30	43.752	0.036788
100	373	1697.55	58.397	0.034401
110	383	2367.64	76.162	0.032168
120	393	3231.73	97.211	0.030080
130	403	4323.00	121.597	0.028128
140	413	5674.59	149.255	0.026302
150	423	7318.40	179.998	0.024595

Saturated Vapor

1. Temperature Centigrade t	2. Total Heat λ (Regnault)	Differences	3. Heat of the Liquid $q=\int_0^t cdt$ (Regnault)	Differences	4. Latent Heat $r=\lambda-q$	Differences
0	236.50		0.00		236.50	
		7.90		5.59		−2.31
10	244.40		5.59		238.81	
		7.60		5.83		−1.77
20	252.00		11.42		240.58	
		6.00		6.07		+0.07
30	258.00		17.49		240.51	
		4.00		6.22		2.22
40	262.00		23.71		238.29	
		2.00		6.50		4.50
50	264.00		30.21		233.79	
		1.00		7.16		6.16
60	265.00		37.37		227.63	
		0.20		7.21		7.01
70	265.20		44.58		220.62	
		0.00		7.53		7.53
80	265.20		52.11		213.09	
		0.80		7.86		7.06
90	266.00		59.97		206.03	
		1.30		8.21		6.91
100	267.30		68.18		199.12	
		2.30		8.56		6.26
110	269.60		76.74		192.86	
		2.90		8.93		6.03
120	272.50		85.67		186.83	
		3.50		9.31		5.81
130	276.00		94.98		181.02	
		4.50		9.72		5.22
140	280.50		104.70		175.80	
		4.80		10.12		5.32
150	285.30		114.82		170.48	

FRENCH UNITS.

of Alcohol. Table 3a.

6. $T.\frac{dp}{pdt}$	7. $\frac{r}{u}=AT\frac{dp}{dt}$	8. $\frac{\rho}{u}=\frac{r}{u}-Ap$	9. $\frac{Apu}{r}=\phi$	1. Temperature Centigrade t
18.099	7.37	6.96	0.05525	0
17.736	13.78	13.00	0.05638	10
17.222	24.55	23.13	0.05806	20
16.668	41.97	39.35	0.05999	30
16.104	69.03	64.74	0.06210	40
15.541	109.58	102.53	0.06435	50
14.982	168.24	157.01	0.06674	60
14.431	250.41	233.06	0.06930	70
13.887	362.01	335.94	0.07201	80
13.354	509.27	471.13	0.07488	90
12.831	698.46	644.03	0.07793	100
12.320	935.37	859.45	0.08117	110
11.821	1225.05	1121.42	0.08459	120
11.335	1571.38	1432.42	0.08822	130
10.863	1976.63	1794.67	0.09206	140
10.404	2441.48	2206.81	0.09612	150

of Alcohol. Table 3b.

5. Outer Latent Heat $Apu=\phi r$	Differences	6. Steam Heat $J=\lambda-Apu$	Differences	7. Inner Latent Heat $\rho=r-Apu$	Differences.	8. Values of $u=\frac{r}{\left(\frac{r}{u}\right)}$ $(\sigma=0.0013)$ r	1. Temperature Centigrade t
13.067		223.43		223.43		32.0875	0
	0.398		7.51		−1.92		
13.465		230.94		225.35		17.3298	10
	0.504		7.09		−1.26		
13.969		238.03		226.61		9.7985	20
	0.460		5.54		+0.53		
14.429		243.57		226.08		5.7309	30
	0.368		3.63		2.59		
14.797		247.20		223.49		3.4516	40
	0.247		1.76		4.74		
15.044		248.96		218.75		2.1334	50
	0.149		0.85		6.31		
15.193		249.81		212.44		1.3529	60
	0.094		+0.10		7.11		
15.287		249.91		205.33		0.8810	70
	0.057		−0.05		7.58		
15.344		249.86		197.75		0.5887	80
	0.084		+0.71		7.15		
15.428		250.57		190.60		0.4045	90
	0.090		1.21		7.00		
15.518		251.78		183.60		0.2850	100
	0.136		2.17		6.39		
15.654		253.95		177.21		0.2062	110
	0.149		2.75		6.18		
15.803		256.70		171.03		0.1525	120
	0.166		3.33		5.98		
15.969		260.03		165.05		0.1152	130
	0.215		4.29		5.43		
16.184		264.32		159.62		0.0889	140
	0.202		4.59		5.53		
16.386		268.91		154.09		0.0698	150

FRENCH UNITS.

Saturated Vapor

1. Temperature Centigrade t	2. Absolute Temperature T	3. Steam Pressure in Millimeters of Mercury p (Regnault)	4. Differential Coefficient in Millimeters of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
0	273	63.33	3.769	0.059516
10	283	110.32	5.726	0.051910
20	293	180.08	8.349	0.046364
30	303	280.05	11.797	0.042125
40	313	419.35	16.242	0.038733
50	323	608.81	21.858	0.035903
60	333	860.96	28.808	0.033460
70	343	1189.90	37.237	0.031294
80	353	1611.05	47.266	0.029338
90	363	2140.82	58.971	0.027546
100	373	2796.20	72.391	0.025889
110	383	3594.33	87.515	0.024348
120	393	4551.95	104.271	0.022917
130	403	5684.90	122.555	0.021558
140	413	7007.64	142.199	0.020292

Saturated Vapor

1. Temperature Centigrade t	2. Total Heat λ (Regnault)	Differences	3. Heat of Liquid $q=\int_0^t cdt$ (Regnault	Differences	4. Latent Heat $r=\lambda-q$	Differences
0	140.50		0.00		140.50	
		3.61		5.10		1.49
10	144.11		5.10		139.01	
		3.51		5.19		1.68
20	147.62		10.29		137.33	
		3.41		5.26		1.85
30	151.03		15.55		135.48	
		3.30		5.34		2.04
40	154.33		20.89		133.44	
		3.20		5.42		2.22
50	157.53		26.31		131.22	
		3.10		5.50		2.40
60	160.63		31.81		128.82	
		2.99		5.58		2.59
70	163.62		37.39		126.23	
		2.89		5.66		2.77
80	166.51		43.05		123.46	
		2.79		5.74		2.95
90	169.30		48.79		120.51	
		2.68		5.82		3.14
100	171.98		54.61		117.37	
		2.58		5.89		3.31
110	174.56		60.50		114.06	
		2.48		5.98		3.50
120	177.04		66.48		110.56	
		2.38		6.06		3.68
130	179.42		72.54		106.88	
		2.27		6.13		3.86
140	181.69		78.67		103.02	

FRENCH UNITS.

of Acetone. Table 4a.

6. $T\cdot\frac{dp}{pdt}$	7. $\frac{r}{u}=AT\frac{dp}{dt}$	8. $\frac{\rho}{u}=\frac{r}{u}-Ap$	9. $\frac{Apu}{r}=\phi$	1. Temperature Centigrade t
16.248	33.00	30.97	0.06155	0
14.690	51.96	48.43	0.06807	10
13.533	78.44	72.67	0.07378	20
12.674	114.62	105.64	0.07834	30
12.123	163.02	149.58	0.08248	40
11.596	226.39	206.87	0.08623	50
11.142	307.61	280.00	0.08975	60
10.733	409.56	371.40	0.09316	70
10.356	535.01	483.35	0.09656	80
9.999	686.42	617.78	0.10001	90
9.657	865.84	776.17	0.10355	100
9.325	1074.80	959.55	0.10723	110
9.002	1314.02	1168.07	0.11108	120
8.688	1583.74	1401.44	0.11510	130
8.380	1883.19	1658.48	0.11932	140

of Acetone. Table 4b.

5. Outer Latent Heat $Apu=\phi r$	Differences	6. Steam Heat $J=\lambda-Apu$	Differences.	7. Inner Latent Heat $\rho=r-Apu$	Differences.	8. Values of $u=\frac{r}{\left(\frac{r}{u}\right)_{(\sigma=0.0012)}}$	1. Temperature Centigrade t
8.647	0.815	131.85	2.80	131.85	2.30	4.2580	0
9.462	0.670	134.65	2.84	129.55	2.35	2.6751	10
10.132	0.481	137.49	2.93	127.20	2.33	1.7507	20
10.613	0.393	140.42	2.90	124.87	2.44	1.1819	30
11.006	0.309	143.32	2.90	122.43	2.52	0.8185	40
11.315	0.246	146.22	2.85	119.91	2.65	0.5797	50
11.561	0.198	149.07	2.79	117.26	2.79	0.4187	60
11.759	0.162	151.86	2.73	114.47	2.93	0.3082	70
11.921	0.131	154.59	2.66	111.54	3.08	0.2307	80
12.052	0.102	157.25	2.58	108.46	3.24	0.1755	90
12.154	0.077	159.83	2.50	105.22	3.39	0.1355	100
12.231	0.050	162.33	2.43	101.83	3.55	0.1061	110
12.281	0.021	164.76	2.36	98.28	3.70	0.0841	120
12.302	0.000	167.12	2.27	94.58	3.86	0.0675	130
12.302		169.39		90.72		0.0547	140

FRENCH UNITS.

Saturated Vapor

1. Temperature Centigrade t	2. Absolute Temperature T	3. Steam Pressure in Millimeters of Mercury p (Regnault)	4. Differential Coefficient in Millimeters of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
0	273	59.72	3.235	0.054178
10	283	100.47	4.953	0.049297
20	293	160.47	7.246	0.045156
30	303	247.51	10.295	0.041593
40	313	369.26	14.211	0.038485
50	323	535.05	19.123	0.035741
60	333	755.44	25.150	0.033292
70	343	1042.11	32.393	0.031084
80	353	1407.64	40.933	0.029079
90	363	1865.22	50.816	0.027244
100	373	2428.54	62.061	0.025555
110	383	3110.99	74.645	0.023994
120	393	3925.74	88.510	0.022546
130	403	4885.10	103.549	0.021197
140	413	6000.16	119.625	0.019937
150	423	7280.62	136.584	0.018760
160	433	8734.20	154.218	0.017658

Saturated Vapor

1. Temperature Centigrade t	2. Total Heat λ (Regnault)	Differences.	3. Heat of the Liquid $q=\int_0^t cdt$ (Regnault)	Differences	4. Latent Heat $r=\lambda-q$	Differences.
0	67.000	1.375	0.00	2.33	67.00	0.96
10	68.375	"	2.33	2.34	66.04	0.96
20	69.75	"	4.67	2.35	65.08	0.98
30	71.12	"	7.02	2.35	64.10	0.97
40	72.50	"	9.37	2.37	63.13	1.00
50	73.87	"	11.74	2.38	62.13	1.00
60	75.25	"	14.12	2.39	61.13	1.02
70	76.62	"	16.51	2.40	60.11	1.02
80	78.00	"	18.91	2.41	59.09	1.04
90	79.37	"	21.32	2.42	58.05	1.04
100	80.75	"	23.74	2.43	57.01	1.06
110	82.12	"	26.17	2.44	55.95	1.06
120	83.50	"	28.61	2.45	54.89	1.08
130	84.87	"	31.06	2.46	53.81	1.08
140	86.25	"	33.52	2.47	52.73	1.10
150	87.62	7	35.99	2.48	51.63	1.10
160	89.00		38.47		50.53	

FRENCH UNITS.

of Chloroform. Table 5a.

6. $T.\frac{dp}{pdt}$	7. $\frac{r}{u}=AT\frac{dp}{dt}$	8. $\frac{\rho}{u}=\frac{r}{u}-Ap$	9. $\frac{Apu}{r}=\phi$	1. Temperature Centigrade t
14.790	28.32	26.41	0.06761	0
13.951	44.95	41.73	0.07168	10
13.231	68.07	62.92	0.07558	20
12.603	100.03	92.09	0.07935	30
12.046	142.63	130.79	0.08302	40
11.544	198.06	180.90	0.08662	50
11.086	268.55	244.33	0.09020	60
10.662	356.28	322.86	0.09379	70
10.265	463.34	418.20	0.09742	80
9.889	591.49	531.68	0.10112	90
9.532	742.29	664.42	0.10491	100
9.190	916.74	816.98	0.10882	110
8.860	1115.40	989.52	0.11286	120
8.542	1338.13	1181.49	0.11706	130
8.234	1584.23	1391.83	0.12145	140
7.935	1852.62	1619.16	0.12602	150
7.646	2141.40	1861.33	0.13079	160

of Chloroform. Table 5b.

5. Outer Latent Heat $Apu=\phi r$	Differences	6. Steam Heat $J=\lambda-Apu$	Differences	7. Inner Latent Heat $\rho=r-Apu$	Differences	8. Values of $u=\frac{r}{\left(\frac{r}{u}\right)}$ $(\sigma=0.0006)$	1. Temperature Centigrade t
4.530	0.204	62.47	1.17	62.47	1.16	2.3660	0
4.734	0.185	63.64	1.19	61.31	1.15	1.4693	10
4.919	0.167	64.83	1.20	60.16	1.15	0.9560	20
5.086	0.155	66.03	1.23	59.01	1.12	0.6408	30
5.241	0.141	67.26	1.23	57.89	1.12	0.4426	40
5.382	0.132	68.49	1.25	56.75	1.13	0.3137	50
5.514	0.124	69.74	1.24	55.62	1.15	0.2276	60
5.638	0.118	70.98	1.26	54.47	1.14	0.1687	70
5.756	0.114	72.24	1.26	53.33	1.15	0.1275	80
5.870	0.111	73.50	1.27	52.18	1.15	0.0981	90
5.981	0.107	74.77	1.26	51.03	1.17	0.0768	100
6.088	0.107	76.03	1.28	49.86	1.16	0.0610	110
6.195	0.104	77.31	1.26	48.70	1.19	0.0492	120
6.299	0.105	78.57	1.28	47.51	1.18	0.0402	130
6.404	0.102	79.85	1.26	46.33	1.21	0.0333	140
6.506	0.103	81.11	1.28	45.12	1.20	0.0279	150
6.609		82.39		43.92		0.0236	160

FRENCH UNITS.

Saturated Vapor of

1. Temperature Centigrade t	2. Absolute Temperature T	3. Steam Pressure in Millimeters of Mercury p (Regnault)	4. Differential Coefficient in Millimeters of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
0	273	32.95	1.82	0.055338
10	283	55.97	2.84	0.050708
20	293	90.99	4.24	0.046574
30	303	142.27	6.10	0.042882
40	313	214.81	8.50	0.039584
50	323	314.38	11.52	0.036637
60	333	447.43	15.21	0.034002
70	343	621.15	19.66	0.031647
80	353	843.29	24.91	0.029539
90	363	1122.26	31.03	0.027653
100	373	1467.09	38.09	0.025964
110	383	1887.44	46.15	0.024451
120	393	2393.67	55.28	0.023094
130	403	2996.88	65.56	0.021876
140	413	3709.04	77.09	0.020784
150	423	4543.13	89.96	0.019801
160	433	5513.14	104.29	0.018917

Saturated Vapor of

1. Temperature Centigrade t	2. Total Heat λ (Regnault)	Differences	3. Heat of the Liquid $q=\int_0^t cdt$ (Regnault)	Differences	4. Latent Heat $r=\lambda-q$	Differences
0	52.00		0.00		52.00	
		1.44		1.99		0.55
10	53.44		1.99		51.45	
		1.42		2.00		0.58
20	54.86		3.99		50.87	
		1.37		2.03		0.66
30	56.23		6.02		50.21	
		1.35		2.04		0.69
40	57.58		8.06		49.52	
		1.30		2.06		0.76
50	58.88		10.12		48.76	
		1.28		2.08		0.80
60	60.16		12.20		47.96	
		1.24		2.10		0.85
70	61.40		14.30		47.10	
		1.20		2.12		0.92
80	62.60		16.42		46.18	
		1.17		2.13		0.96
90	63.77		18.55		45.22	
		1.13		2.15		1.02
100	64.90		20.70		44.20	
		1.11		2.17		1.06
110	66.01		22.87		43.14	
		1.06		2.19		1.13
120	67.07		25.06		42.01	
		1.03		2.21		1.18
130	68.10		27.27		40.83	
		1.00		2.22		1.22
140	69.10		29.49		39.61	
		0.97		2.24		1.27
150	70.07		31.73		38.34	
		0.93		2.27		1.34
160	71.00		34 00		37.00	

FRENCH UNITS.

Chloride of Carbon. Table 6a.

6. $T.\frac{dp}{pdt}$	7. $\frac{r}{u}=AT\frac{dp}{dt}$	8. $\frac{\rho}{u}=\frac{r}{u}-Ap$	9. $\frac{Apu}{r}=\phi$	1. Temperature Centigrade t
15.107	15.96	14.90	0.06619	0
14.350	25.76	23.97	0.06968	10
13.646	39.82	36.90	0.07328	20
12.993	59.27	54.71	0.07696	30
12.390	85.34	78.45	0.08071	40
11.834	119.29	109.21	0.08450	50
11.322	162.44	148.10	0.08832	60
10.855	216.21	196.29	0.09212	70
10.427	281.96	254.93	0.09590	80
10.038	361.24	325.25	0.09962	90
9.685	455.60	408.55	0.10325	100
9.365	566.78	506.25	0.10678	110
9.076	696.63	619.88	0.11018	120
8.816	847.20	751.10	0.11343	130
8.584	1020.90	901.97	0.11650	140
8.376	1220.19	1074.18	0.11939	150
8.191	1448.05	1271.27	0.12208	160

Chloride of Carbon. Table 6b.

5. Outer Latent Heat $Apu=\phi r$	Differences	6. Steam Heat $J=\lambda-Apu$	Differences	7. Inner Latent Heat $\rho=r-Apu$	Differences	8. Values of $u=\frac{r}{\left(\frac{r}{u}\right)}$ $(\sigma=0.0006)$	1. Temperature Centigrade t
3.442		48.56		48.56		3.2584	0
	0.143		1.30		0.69		
3.585		49.86		47.87		1.9971	10
	0.143		1.27		0.73		
3.728		51.13		47.14		1.2776	20
	0.136		1.24		0.79		
3.864		52.37		46.35		0.8471	30
	0.133		1.21		0.83		
3.997		53.58		45.52		0.5803	40
	0.123		1.18		0.88		
4.120		54.76		44.64		0.4087	50
	0.116		1.16		0.92		
4.236		55.92		43.72		0.2952	60
	0.103		1.14		0.96		
4.339		57.06		42.76		0.2178	70
	0.090		1.11		1.01		
4.429		58.17		41.75		0.1638	80
	0.076		1.10		1.03		
4.505		59.27		40.72		0.1252	90
	0.059		1.07		1.08		
4.564		60.34		39.64		0.0970	100
	0.042		1.06		1.11		
4.606		61.40		38.53		0.0761	110
	0.023		1.04		1.15		
4.629		62.44		37.38		0.0603	120
	0.002		1.03		1.18		
4.631		63.47		36.20		0.0482	130
	−0.017		1.02		1.20		
4.614		64.49		35.00		0.0388	140
	−0.037		1.00		1.24		
4 577		65.49		33.76		0.0314	150
	−0.060		0.99		1.28		
4 517		66.48		32.48		0.0255	160

FRENCH UNITS.

Saturated Vapor of

1. Temperature Centigrade t	2. Absolute Temperature T	3. Steam Pressure in Millimeters of Mercury p (Regnault)	4. Differential Coefficient in Millimeters of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
0	273	127.91	5.846	0.045704
10	283	198.46	8.380	0.042226
20	293	298.03	11.664	0.039137
30	303	434.62	15.813	0.036383
40	313	617.53	20.941	0.033911
50	323	857.07	27.157	0.031682
60	333	1164.51	34.540	0.029661
70	343	1552.09	43.181	0.027821
80	353	2032.53	53.124	0.026137
90	363	2619.08	64.406	0.024591
100	373	3325.15	77.030	0.023166
110	383	4164.06	90.972	0.021847
120	393	5148.79	106.173	0.020621
130	403	6291.60	122.573	0.019482
140	413	7603.96	140.057	0.018419
150	423	9095.94	158.497	0.017425

Saturated Vapor of

1. Temperature Centigrade t	2. Total Heat λ (Regnault)	Differences	3. Heat of the Liquid $q=\int_0^t cdt$ (Regnault)	Differences	4. Latent Heat $r=\lambda-q$	Differences
0	90.00		0.00		90.00	
		1.42		2.36		0.94
10	91.42		2.36		89.06	
		1.34		2.38		1.04
20	92.76		4.74		88.02	
		1.25		2.39		1.14
30	94.01		7.13		86.88	
		1.17		2.41		1.24
40	95.18		9.54		85.64	
		1.09		2.42		1.33
50	96.27		11.96		84.31	
		1.01		2.45		1.44
60	97.28		14.41		82.87	
		0.92		2.45		1.53
70	98.20		16.86		81.34	
		0.84		2.48		1.64
80	99.04		19.34		79.70	
		0.76		2.49		1.73
90	99.80		21.83		77.97	
		0.68		2.51		1.83
100	100.48		24.34		76.14	
		0.59		2.52		1.93
110	101.07		26.86		74.21	
		0.51		2.54		2.03
120	101.58		29.40		72.18	
		0.43		2.56		2.13
130	102.01		31.96		70.05	
		0.35		2.57		2.22
140	102.36		34.53		67.83	
		0.26		2.59		2.33
150	102.62		37.12		65.50	

FRENCH UNITS.

Bisulphide of Carbon. Table 7a.

6. $T.\frac{dp}{pdt}$	7. $\frac{r}{u}=AT\frac{dp}{dt}$	8. $\frac{\rho}{u}=\frac{r}{u}-Ap$	9. $\frac{Apu}{r}=\phi$	1. Temperature Centigrade t
12.477	51.18	47.08	0.08015	0
11.950	76.04	69.68	0.08368	10
11.467	109.59	100.03	0.08721	20
11.024	153.64	139.70	0.09071	30
10.614	210.18	190.38	0.09421	40
10.233	281.27	253.80	0.09772	50
9.877	368.81	331.48	0.10124	60
9.543	474.93	425.17	0.10478	70
9.226	601.32	536.15	0.10838	80
8.926	749.69	665.71	0.11202	90
8.641	921.33	814.71	0.11573	100
8.367	1117.25	983.72	0.11951	110
8.104	1337.99	1172.89	0.12339	120
7.851	1583.97	1382.22	0.12737	130
7.607	1854.82	1610.99	0.13146	140
7.371	2149.85	1858.18	0.13567	150

Bisulphide of Carbon. Table 7b.

5. Outer Latent Heat $Apu=\phi r$	Differences	6. Steam Heat $J=\lambda-Apu$	Differences	7. Inner Latent Heat $\rho=r-Apu$	Differences	8. Values of $u=\frac{r}{\left(\frac{r}{u}\right)}$ $(\sigma=0.0008)$	1. Temperature Centigrade t
7.213	0.239	82.79	1.18	82.79	1.18	1.7587	0
7.452	0.224	83.97	1.11	81.61	1.27	1.1712	10
7.676	0.205	85.08	1.05	80.34	1.34	0.8032	20
7.881	0.187	86.13	0.98	79.00	1.43	0.5655	30
8.068	0.171	87.11	0.92	77.57	1.50	0.4074	40
8.239	0.151	88.03	0.86	76.07	1.59	0.2997	50
8.390	0.133	88.89	0.79	74.48	1.66	0.2247	60
8.523	0.115	89.68	0.72	72.82	1.76	0.1712	70
8.638	0.096	90.40	0.67	71.06	1.82	0.1325	80
8.734	0.078	91.67	0.60	69.24	1.91	0.1040	90
8.812	0.057	91.07	0.53	67.33	1.99	0.0826	100
8.869	0.037	92.20	0.47	65.34	2.07	0.0664	110
8.906	0.016	92.67	0.42	63.27	2.14	0.0539	120
8.922	−0.005	93.09	0.35	61.13	2.22	0.0442	130
8.917	−0.031	93.44	0.29	58.91	2.30	0.0366	140
8.886		93.73		56.61		0.0305	150

FRENCH UNITS.

Saturated Vapor

1. Temperature Centigrade t	2. Steam Pressure Atmospheres 1 atmosphere = 10,333 kg.	3. Steam Pressure p Kilogram per sq. m.	4. $\frac{dp}{pdt}$	5. Specific Volume of the Steam s	6. Heat of the Liquid q
					According to the older investigations
					For temperature values
−30	1.153	11890	0.048566	0.9961	−26.60
−25	1.463	15085	0.046647	0.7970	−22.69
−20	1.839	19004	0.044803	0.6434	−18.57
−15	2.291	23670	0.043032	0.5238	−14.24
−10	2.828	29227	0.041332	0.4300	− 9.70
− 5	3.464	35789	0.039699	0.3558	− 4.96
					According to the experimental
					For temperature values
0	4.207	43475	0.038130	0.2940	0.00
+ 5	5.072	52406	0.036623	0.2455	+ 5.62
+10	6.069	62710	0.035175	0.2060	+11.28
+15	7.214	74507	0.033785	0.1740	+17.00
+20	8.509	87923	0.032450	0.1480	+22.80
+25	9.976	103078	0.031168	0.1265	+28.60
+30	11.622	120089	0.029936	0.1085	+34.50
+35	13.458	139064	0.028753	0.0932	+40.40
+40	15.495	160107	0.027617	0.0809	+46.38

FRENCH UNITS.

of Ammonia. Table 8.

7.	8.	9.	10.	11.	1.
	Latent Heat				
Total r	Outer Apu	Inner ρ	$\frac{r}{T}$	$\tau = \int_0^t \frac{cdt}{T}$	Temperature Centigrade t
of the preceding edition.					
below 0° C.					
329.9	27.95	302.0	1.3576	−0.1029	−30
328.0	28.37	299.6	1.3225	−0.0870	−25
325.8	28.77	297.1	1.2879	−0.0706	−20
323.4	29.15	294.3	1.2536	−0.0536	−15
320.8	29.53	291.3	1.2198	−0.0362	−10
318.0	29.89	288.1	1.1864	−0.0183	− 5
results of L. Dieterici.					
above 0° C.					
309.7	29.76	279.9	1.1344	0.0000	0
304.7	29.93	274.8	1.0943	+0.0203	+ 5
298.6	30.01	268.6	1.0544	+0.0405	+10
292.6	30.07	262.5	1.0132	+0.0606	+15
286.6	30.14	256.5	0.9741	+0.0804	+20
279.9	30.12	249.8	0.9359	+0.1001	+25
272.5	30.03	242.5	0.8984	+0.1196	+30
263.8	29.80	234.0	0.8636	+0.1391	+35
256.6	29.69	226.9	0.8252	+0.1583	+40

FRENCH UNITS.

Saturated Vapor of

1.	2.	3.	4.	5.	6.	7.	8.
Temperature Centigrade t	Steam Pressure: Atmospheres 1 Atm. = 10333 kg.	Steam Pressure: p Kilograms per sq. in.	$\frac{dp}{pdt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	Heat of Liquid q	Latent Heat: Outer Apu	Latent Heat: Inner ρ
−30	0.379	3912	0.053646	120.28	− 9.264	7.321	88.566
−25	0.492	5082	0.051019	151.65	− 7.754	7.530	87.998
−20	0.631	6519	0.048599	189.03	− 6.231	7.716	87.284
−15	0.800	8265	0.046364	232.63	− 4.694	7.879	86.424
−10	1.003	10367	0.044295	284.83	− 3.143	8.019	85.418
− 5	1.246	12874	0.042373	344.81	− 1.579	8.136	84.267
0	1.533	15840	0.040583	413.90	0.000	8.230	82.970
+ 5	1.870	19322	0.038011	492.93	+ 1.592	8.301	81.527
+10	2.263	23379	0.037346	382.76	+ 3.199	8.349	79.938
+15	2.717	28074	0.035877	684.18	+ 4.819	8.374	78.204
+20	3.239	33474	0.034495	797.91	+ 6.453	8.376	76.324
+25	3.837	39644	0.033192	924.83	+ 8.101	8.365	74.288
+30	4.515	46655	0.031960	1065.58	+ 9.762	8.311	72.126
+35	5.283	54579	0.030794	1220.88	+11.438	8.244	69.809
+40	6.145	63487	0.029688	1391.45	+13.127	8.154	67.346

FRENCH UNITS.

Sulphurous Acid. Table 9.

9. Latent Heat. Total r	10. $\frac{r}{T}$	11. $\tau=\int_0^t \frac{cdt}{T}$	12. u	13. Specific Volume $v=u+\sigma$ ($\sigma=0.0007$)	14. Specific Weight $\gamma=\frac{1}{v}$ kg. per cbm.	1. Temperature Centigrade t
95.887	0.3946	−0.0359	0.7934	0.7941	1.259	−30
95.528	0.3852	−0.0298	0.6282	0.6289	1.590	−25
95.000	0.3755	−0.0237	0.5019	0.5026	1.990	−20
94.303	0.3655	−0.0177	0.4042	0.4049	2.470	−15
93.437	0.3553	−0.0117	0.3280	0.3287	3.042	−10
92.403	0.3448	−0.0058	0.2680	0.2687	3.722	− 5
91.200	0.3341	0.0000	0.2203	0.2210	4.525	0
89.828	0.3231	+0.0058	0.1822	0.1829	5.467	+ 5
88.287	0.3120	+0.0115	0.1514	0.1521	6.575	+10
86.578	0.3006	+0.0172	0.1265	0.1272	7.862	+15
84.700	0.2891	+0.0228	0.1061	0.1068	9.363	+20
82.653	0.2773	+0.0284	0.0895	0.0902	11.086	+25
80.437	0.2655	+0.0339	0.0755	0.0762	13.123	+30
78.053	0.2534	+0.0394	0.0640	0.0647	15.456	+35
75.500	0.2412	+0.0448	0.0545	0.0552	18.116	+40

FRENCH UNITS.

Saturated Vapor of

According to

1.	2.	3.	4.	5.	6.	7.	8.	9.
Temperature		Steam Pressure				Specific Volume		
t	T	p		$\frac{1}{\frac{dp}{pdt}}$	$\frac{dp}{dt}$	s	σ	$u=s-\sigma$
Centi-grade	Ab-solute	Atm.	Kgs. per sq. cm.		Kgs. per sq. cm.	Vapor	Liquid	
−30	243	14.5	15.0	31.60	0.477	0.0270	0.00097	0.0260
−25	248	16.9	17.5	32.72	0.535	0.0229	0.00098	0.0219
−20	253	19.7	20.3	33.81	0.601	0.0195	0.00100	0.0185
−15	258	22.8	23.5	34.92	0.673	0.0167	0.00102	0.0157
−10	263	26.2	27.1	36.02	0.752	0.0143	0.00104	0.0132
− 5	268	30.0	31.0	37.13	0.836	0.0122	0.00107	0.0111
0	273	34.3	35.4	38.23	0.927	0.0104	0.00110	0.0093
+ 5	278	39.0	40.3	39.34	1.025	0.0089	0.00113	0.0078
+10	283	44.2	45.7	40.44	1.130	0.0075	0.00117	0.0063
+15	288	50.0	51.6	41.55	1.243	0.0063	0.00123	0.0051
+20	293	56.3	58.1	42.65	1.363	0.0052	0.00131	0.0039
+25	298	63.3	65.4	43.76	1.495	0.0042	0.00142	0.0028
+30	303	70.7	73.1	44.86	1.630	0.0030	0.00167	0.0013
+31	304	72.3	74.7	45.08	1.657	0.0026	0.00186	0.0007
+31.35	304.35	72.9	75.3	45.16	1.669	0.0022	0.00216	0.0000

FRENCH UNITS.

Carbonic Acid. Table 10.

Amagat and Mollier.

10.	11.	12.	13.	14.	15.	16.	17.
Heat of Liquid $q=\int_0^t Td\tau$	Latent Heat. Total r	Latent Heat. Outer Apu	Latent Heat. Inner ρ	$\frac{r}{T}$	Entropy. Liquid τ	Entropy. Vapor $\tau+\frac{r}{T}$	Steam Heat $J=\rho+q$
−13.78	70.40	9.15	61.25	0.290	−0.053	0.236	47.47
−11.70	68.47	9.03	59.44	0.276	−0.045	0.231	47.74
− 9.55	66.35	8.87	57.48	0.262	−0.036	0.226	47.93
− 7.32	64.03	8.67	55.36	0.248	−0.028	0.221	48.03
− 5.00	61.47	8.42	53.05	0.234	−0.019	0.215	48.05
− 2.57	58.63	8.12	50.51	0.219	−0.010	0.209	47.94
0.00	55.45	7.76	47.69	0.203	0.000	0.203	47.69
+ 2.74	51.86	5.34	44.52	0.187	+0.010	0.197	47.26
+ 5.71	47.74	6.82	40.92	0.169	+0.021	0.189	46.63
+ 9.01	42.89	6.19	36.70	0.149	+0.032	0.181	45.71
+12.82	36.93	5.37	31.56	0.126	+0.045	0.171	44.38
+17.57	28.98	4.26	24.72	0.097	+0.061	0.159	42.29
+25.25	15.00	2.22	12.78	0.050	+0.087	0.136	38.03
+28.67	8.40	1.25	7.15	0.028	+0.098	0.126	35.82
+32.91	0.00	0.00	0.00	0.000	+0.112	0.112	32.91

FRENCH UNITS.

First Principal Table for the Vapor of Water. Table 11a.

1. Pressure in Atmospheres p (1 atm. = 10,333 kg. per sq. m.)	2. Temperature t	3. Heat of the Liquid q	4. Latent Heat: Inner ρ	5. Latent Heat: Outer Apu	6. Values of u	7. Values of $\frac{\rho}{u}$	8. Specific Weight γ kg. per cbm.
0.02	17.83	17.838	561.296	32.804	67.303	8.3398	0.0149
0.04	29.35	29.375	552.184	33.893	34.769	15.882	0.0288
0.06	36.56	36.601	546.481	34.569	23.641	23.116	0.0423
0.08	41.92	41.977	542.241	35.067	17.987	30.146	0.0556
0.10	46.21	46.282	538.848	35.464	14.552	37.029	0.0687
0.12	49.83	49.917	535.984	35.797	12.241	43.786	0.0817
0.15	54.37	54.477	532.393	36.213	9.9063	53.743	0.1009
0.2	60.45	60.589	527.584	36.764	7.5428	69.945	0.1326
0.3	69.49	69.687	520.433	37.574	5.1393	101.27	0.1945
0.4	76.25	76.499	515.086	38.171	3.9157	131.54	0.2553
0.5	81.71	82.017	510.767	38 637	3.1708	161.08	0.3153
0.6	86.32	86.662	507.121	39 045	2.6703	189.91	0.3743
0.7	90.32	90.704	503.957	39.387	2.3088	218.28	0.4329
0.8	93.88	94.304	501.141	39.688	2.0357	246.18	0.4910
0.9	97.08	97.543	498.610	39.957	1.8218	273.69	0.5486
1.0	100.00	100.500	496.300	40.200	1.6495	300.88	0.6059
1.1	102.68	103.216	494.180	40.421	1.5078	327.75	0.6628
1.2	105.17	105.740	492.210	40.626	1.3892	354.31	0.7193
1.3	107.50	108.104	490.367	40.816	1.2883	380.63	0.7756
1.4	109.68	110.316	488.643	40.993	1.2015	406.69	0.8316
1.5	111.74	112.408	487.014	41.159	1.1259	432.56	0.8874
1.6	113.69	114.389	485.471	41.315	1.0596	458.16	0.9429
1.7	115.54	116.269	484.008	41.463	1.0008	483.62	0.9982
1.8	117.30	118.059	482.616	41.602	0.9484	508.87	1.0533
1.9	118.99	119.779	481.279	41.734	0.9013	533.98	1.1083
2.0	120.60	121.417	480.005	41.861	0.8589	558.86	1.1629
2.1	122.15	122.995	478.779	41.981	0.8203	583.66	1.2176
2.2	123.64	124.513	477.601	42.096	0.7852	608.25	1.2719
2.3	125.07	125.970	476.470	42.207	0.7530	632.76	1.3263
2.4	126.46	127.386	475.370	42.314	0.7235	657.04	1.3803
2.5	127.80	128.753	474.310	42.416	0.6962	681.28	1.4343
2.6	129.10	130.079	473.282	42.515	0.6710	705.34	1.4881
2.7	130.35	131.354	472.293	42.610	0.6476	729.20	1.5418
2.8	131.57	132.599	471.328	42.702	0.6258	753.16	1.5954
2.9	132.76	133.814	470.387	42.791	0.6055	776.86	1.6488

FRENCH UNITS.

First Principal Table for the Vapor of Water. Table 11b.

1. Pressure in Atmospheres p (1 atm. = 10,333 kg. per sq. m.)	2. Temperature t	3. Heat of the Liquid q	4. Latent Heat Inner ρ	5. Latent Heat Outer Apu	6. Values of u	7. Values of $\frac{\rho}{u}$	8. Specific Weight γ kg. per cbm.
3.0	133.91	134.989	469.477	42.876	0.5865	800.47	1.7021
3.1	135.03	136.133	468.591	42.960	0.5686	824.11	1.7556
3.2	136.12	137.247	467.729	43.040	0.5519	847.49	1.8086
3.3	137.19	138.341	466.883	43.119	0.5362	870.73	1.8615
3.4	138.23	139.404	466.060	43.196	0.5213	894.03	1.9147
3.5	139.24	140.438	465.261	43.269	0.5072	917.31	1.9676
3.6	140.23	141.450	464.478	43.342	0.4940	940.24	2.0203
3.7	141.21	142.453	463.703	43.413	0.4814	963.24	2.0729
3.8	142.15	143.416	462.959	43.480	0.4695	986.07	2.1255
3.9	143.08	144.368	462.224	43.548	0.4581	1008.9	2.1780
4.0	144.00	145.310	461.496	43.614	0.4474	1031.6	2.2303
4.1	144.89	146.222	460.792	43.677	0.4371	1054.2	2.2826
4.2	145.76	147.114	460.104	43.739	0.4273	1076.8	2.3349
4.3	146.61	147.985	459.431	43.799	0.4179	1099.3	2.3871
4.4	147.46	148.857	458.759	43.859	0.4090	1121.7	2.4391
4.5	148.29	149.708	458.103	43.918	0.4004	1144.0	2.4911
4.6	149.10	150.539	457.462	43.975	0.3922	1166.3	2.5430
4.7	149.90	151.360	456.829	44.030	0.3844	1188.5	2.5949
4.8	150.69	152.171	456.204	44.085	0.3768	1210.6	2.6467
4.9	151.46	152.961	455.595	44.139	0.3696	1232.7	2.6984
5.0	152.22	153.741	454.994	44.192	0.3626	1254.7	2.7500
5.1	152.97	154.512	454.401	44.243	0.3759	1276.6	2.8016
5.2	153.70	155.262	453.823	44.293	0.3495	1298.5	2.8531
5.3	154.43	156.012	453.246	44.343	0.3433	1320.3	2.9046
5.4	155.14	156.741	452.684	44.392	0.3373	1342.1	2.9560
5.5	155.85	157.471	452.123	44.441	0.3315	1363.8	3.0073
5.6	156.54	158.181	451.577	44.487	0.3259	1385.4	3.0586
5.7	157.22	158.880	451.039	44.533	0.3205	1407.0	3.1098
5.8	157.90	159.579	450.501	44.579	0.3153	1428.5	3.1610
5.9	158.56	160.259	449.979	44.623	0.3103	1450.0	3.2122
6.0	159.22	160.938	449.457	44.667	0.3054	1471.5	3.2632
6.1	159.87	161.607	448.943	44.710	0.3007	1492.9	3.3142
6.2	160.50	162.255	448.444	44.753	0.2962	1514.2	3.3652
6.3	161.14	162.915	447.938	44.794	0.2917	1535.5	3.4161
6.4	161.76	163.553	447.448	44.836	0.2874	1556.7	3.4670

FRENCH UNITS.

First Principal Table for the Vapor of Water. Table 11c.

1. Pressure in Atmospheres p (1 atm. = 10,333 kg. per sq. m.)	2. Temperature t	3. Heat of the Liquid q	4. Latent Heat: Inner ρ	5. Latent Heat: Outer Apu	6. Values of u	7. Values of $\frac{\rho}{u}$	8. Specific Weight γ kg. per cbm.
6.5	162.37	164.181	446.956	44.876	0.2833	1577.9	3.5178
6.6	162.98	164.810	446.483	44.916	0.2792	1599.0	3.5685
6.7	163.58	165.428	446.008	44.956	0.2753	1620.1	3.6192
6.8	164.18	166.047	445.534	44.994	0.2715	1641.2	3.6699
6.9	164.76	166.645	445.075	45.032	0.2678	1662.2	3.7206
7.00	165.34	167.243	444.616	45.070	0.2642	1683.0	3.7711
7.25	166.77	168.718	443.485	45.162	0.2556	1735.2	3.8974
7.50	168.15	170.142	442.393	45.250	0.2475	1787.1	4.0234
7.75	169.50	171.535	441.325	45.337	0.2400	1838.7	4.1490
8.00	170.81	172.888	440.289	45.420	0.2329	1890.1	4.2745
8.25	172.10	174.221	439.269	45.501	0.2263	1941.2	4.3997
8.50	173.35	175.514	438.280	45.578	0.2200	1992.1	4.5248
8.75	174.57	176.775	437.315	45.654	0.2141	2042.8	4.6495
9.00	175.77	178.017	436.366	45.727	0.2085	2093.3	4.7741
9.25	176.94	179.228	435.440	45.798	0.2031	2143.5	4.8985
9.50	178.08	180.408	434.539	45.868	0.1981	2193.5	5.0226
9.75	179.21	181.579	433.645	45.935	0.1933	2243.3	5.1466
10.00	180.31	182.719	432.775	46.001	0.1887	2293.0	5.2704
10.25	181.38	183.828	431.928	46.064	0.1844	2342.5	5.3941
10.50	182.44	184.927	431.090	46.127	0.1802	2391.7	5.5174
10.75	183.48	186.005	430.267	46.189	0.1763	2440.7	5.6405
11.00	184.50	187.065	429.460	46.247	0.1725	2489.5	5.7636
11.25	185.51	188.133	428.661	46.306	0.1689	2538.2	5.8864
11.50	186.49	189.131	427.886	46.362	0.1654	2586.8	6.0092
11.75	187.46	190.139	427.119	46.417	0.1621	2635.2	6.1318
12.00	188.41	191.126	426.368	46.471	0.1589	2683.4	6.2543
12.25	189.35	192.104	425.624	46.524	0.1558	2731.4	6.3765
12.50	190.27	193.060	424.896	46.576	0.1529	2779.3	6.4986
12.75	191.18	194.007	424.177	46.626	0.1500	2827.0	6.6206
13.00	192.08	194.944	423.465	46.676	0.1473	2874.5	6.7424
13.25	192.96	195.860	422.769	46.724	0.1447	2922.0	6.8642
13.50	193.83	196.766	422.080	46.772	0.1421	2969.3	6.9857
13.75	194.69	197.622	421.400	46.818	0.1397	3016.5	7.1072
14.00	195.53	198.537	420.736	46.864	0.1373	3063.4	7.2283

FRENCH UNITS.

Second Principal Table for the Vapor of Water.* Table 12a.

1. Pressure in 1 kg. per sq. cm. (Abs.)	2. Temperature t	3. Heat of the Liquid q	4. Latent Heat Inner ρ	5. Latent Heat Outer Apu	6. Values of u	7. Values of $\frac{\rho}{u}$	8. Specific Weight $\gamma=\frac{1}{v}$	9. $\tau=\int_0^t \frac{cdt}{T}$	10. $\frac{r}{T}$
0.1	45.579	45.649	539.347	35.406	15.0121	35.93	0.0666	0.15463	1.8041
0.2	59.755	59.890	528.134	36.701	7.7806	67.88	0.1285	0.19836	1.6975
0.3	68.742	68.934	521.025	37.507	5.3009	98.29	0.1886	0.22518	1.6344
0.4	75.467	75.710	515.706	38.101	4.0387	127.69	0.2475	0.24482	1.5893
0.5	80.899	81.189	511.409	38.576	3.2712	156.34	0.3056	0.26042	1.5541
0.6	85.484	85.818	507.782	38.972	2.7540	184.38	0.3630	0.27341	1.5252
0.7	89.469	89.844	504.630	39.314	2.3813	211.91	0.4198	0.28458	1.5007
0.8	93.003	93.427	501.835	39.604	2.0990	239.08	0.4762	0.29439	1.4793
0.9	96.187	96.639	499.316	39.882	1.8789	265.75	0.5319	0.30316	1.4605
1.0	99.088	99.576	497.021	40.125	1.7013	292.14	0.5874	0.31108	1.4436
1.1	101.758	102.281	494.909	40.346	1.5552	318.23	0.6426	0.31833	1.4283
1.2	104.235	104.792	492.950	40.550	1.4328	344.05	0.6974	0.32500	1.4142
1.3	106.548	107.138	491.121	40.738	1.3287	369.63	0.7520	0.33120	1.4013
1.4	108.717	109.339	489.405	40.915	1.2391	394.97	0.8064	0.33699	1.3893
1.5	110.763	111.416	487.786	41.081	1.1612	420.07	0.8604	0.34241	1.3781
1.6	112.699	113.382	486.255	41.236	1.0928	444.96	0.9142	0.34752	1.3676
1.7	114.539	115.252	484.800	41.382	1.0321	469.72	0.9679	0.35236	1.3578
1.8	116.290	117.032	483.415	41.521	0.9780	494.29	1.0214	0.35694	1.3484
1.9	117.966	118.737	482.089	41.654	0.9295	518.65	1.0747	0.36131	1.3394
2.0	119.570	120.369	480.820	41.780	0.8857	542.87	1.1278	0.36548	1.3312
2.1	121.109	121.935	479.603	41.900	0.8460	566.91	1.1806	0.36946	1.3232
2.2	122.590	123.443	478.431	42.026	0.8102	590.51	1.2327	0.37328	1.3156
2.3	124.017	124.897	477.303	42.125	0.7766	614.61	1.2860	0.37695	1.3083
2.4	125.395	126.301	476.213	42.231	0.7461	638.27	1.3385	0.38048	1.3013
2.5	126.726	127.658	475.160	42.333	0.7180	661.78	1.3908	0.38388	1.2946
2.6	128.015	128.972	474.140	42.433	0.6920	685.17	1.4430	0.38716	1.2882
2.7	129.264	130.246	473.152	42.528	0.6678	708.52	1.4952	0.39033	1.2819
2.8	130.476	131.483	472.193	42.619	0.6454	731.63	1.5472	0.39340	1.2759
2.9	131.653	132.684	471.262	42.708	0.6244	754.74	1.5989	0.39638	1.2701
3.0	132.798	133.853	470.357	42.793	0.6048	777.71	1.6507	0.39926	1.2645
3.1	133.913	134.992	469.475	42.876	0.5864	800.61	1.7024	0.40206	1.2591
3.2	134.999	136.102	468.616	42.957	0.5692	823.29	1.7537	0.40479	1.2539
3.3	136.057	137.183	467.779	43.035	0.5529	846.05	1.8053	0.40743	1.2488
3.4	137.090	138.239	466.962	43.111	0.5376	868.60	1.8566	0.41001	1.2438
3.5	138.099	139.271	466.164	43.185	0.5232	890.99	1.9076	0.41252	1.2390
3.6	139.085	140.279	465.384	43.285	0.5095	913.41	1.9588	0.41497	1.2343
3.7	140.049	141.265	464.621	43.329	0.4965	935.79	2.0100	0.41737	1.2298
3.8	140.992	142.230	463.875	43.398	0.4842	958.02	2.0609	0.41970	1.2253
3.9	141.915	143.175	463.145	43.464	0.4725	980.20	2.1118	0.42198	1.2210

* Columns 2, 3, 9, and 10 according to Fliegner (Civilingenieur, Vol. 20, 1874, p. 447). Columns 4, 5, 6, 7, and 8 computed by Engineer Connert and according to the formulas:

$$\rho = 575.40 - 0.791\,t; \quad \lambda = 606.5 + 0.305\,t; \quad Apu = \lambda - q - \rho \text{ for } \frac{1}{A} = 424,$$

as were the corresponding values of Table 11.

FRENCH UNITS.

Second Principal Table for Vapor of Water. Table 12b.

1. Pressure in kg. per sq. cm. (Abs.)	2. Temperature t	3. Heat of the Liquid q	4. Latent Heat Inner ρ	5. Latent Heat Outer Apu	6. Values of u	7. Values of $\frac{\rho}{u}$	8. Specific Weight $\gamma=\frac{1}{v}$	9. $=\int_0^t \frac{cdt}{T}$	10. $\frac{r}{T}$
4.0	142.820	144.102	462.429	43.529	0.4614	1002.23	2.1625	0.42421	1.2168
4.1	143.707	145.010	461.728	43.593	0.4508	1024.24	2.2132	0.42639	1.2127
4.2	144.576	145.901	461.040	43.655	0.4407	1046.15	2.2639	0.42853	1.2086
4.3	145.429	146.775	460.366	43.715	0.4311	1067.89	2.3141	0.43062	1.2047
4.4	146.266	147.633	459.704	43.774	0.4218	1089.86	2.3650	0.43267	1.2009
4.5	147.088	148.475	459.053	43.834	0.4130	1111.51	2.4153	0.43467	1.1971
4.6	147.895	149.303	458.415	43.890	0.4046	1133.01	2.4653	0.43664	1.1934
4.7	148.689	150.117	457.787	43.946	0.3964	1154.86	2.5162	0.43858	1.1898
4.8	149.469	150.918	457.170	44.000	0.3887	1176.15	2.5659	0.44047	1.1863
4.9	150.236	151.705	456.563	44.054	0.3812	1197.70	2.6163	0.44233	1.1828
5.0	150.991	152.480	455.966	44.106	0.3740	1219.16	2.6665	0.44416	1.1794
5.1	151.734	153.242	455.378	44.159	0.3671	1240.47	2.7165	0.44596	1.1761
5.2	152.465	153.993	454.800	44.209	0.3605	1261.58	2.7660	0.44773	1.1729
5.3	153.185	154.733	454.231	44.257	0.3541	1282.78	2.8159	0.44946	1.1697
5.4	153.895	155.462	453.669	44.307	0.3479	1304.02	2.8659	0.45117	1.1665
5.5	154.594	156.180	453.116	44.355	0.3419	1325.29	2.9161	0.45285	1.1634
5.6	155.282	156.888	452.572	44.401	0.3362	1346.14	2.9654	0.45451	1.1604
5.7	155.961	157.586	452.035	44.447	0.3306	1367.32	3.0154	0.45613	1.1574
5.8	156.631	158.274	451.505	44.493	0.3253	1387.96	3.0644	0.45774	1.1545
5.9	157.292	158.954	450.982	44.538	0.3201	1408.87	3.1140	0.45932	1.1516
6.0	157.944	159.625	450.466	44.582	0.3150	1430.05	3.1643	0.46088	1.1488
6.1	158.587	160.287	449.958	44.624	0.3102	1450.54	3.2131	0.46241	1.1460
6.2	159.222	160.940	449.455	44.668	0.3055	1471.21	3.2623	0.46392	1.1432
6.3	159.849	161.585	448.959	44.709	0.3009	1492.05	3.3120	0.46542	1.1405
6.4	160.467	162.222	448.471	44.749	0.2965	1512.55	3.3610	0.46689	1.1378
6.5	161.079	162.852	447.987	44.790	0.2922	1533.15	3.4103	0.46834	1.1352
6.6	161.683	163.474	447.509	44.830	0.2880	1553.85	3.4598	0.46977	1.1326
6.7	162.279	164.088	447.037	44.870	0.2840	1574.07	3.5084	0.47118	1.1301
6.8	162.869	164.696	446.571	44.909	0.2800	1594.90	3.5583	0.47258	1.1276
6.9	163.452	165.296	446.109	44.948	0.2762	1615.17	3.6071	0.47395	1.1251
7.0	164.028	165.890	445.654	44.985	0.2725	1635.43	3.6559	0.47531	1.1227
7.1	164.598	166.478	445.203	45.021	0.2689	1655.65	3.7047	0.47666	1.1203
7.2	165.161	167.058	444.758	45.058	0.2653	1676.43	3.7547	0.47798	1.1179
7.3	165.718	167.633	444.317	45.094	0.2619	1696.51	3.8033	0.47929	1.1155
7.4	166.270	168.202	443.880	45.130	0.2586	1716.47	3.8516	0.48059	1.1132
7.5	166.815	168.764	443.449	45.166	0.2553	1736.97	3.9012	0.48187	1.1110
7.6	167.355	169.321	443.022	45.200	0.2522	1756.63	3.9489	0.48314	1.1087
7.7	167.889	169.872	442.600	45.234	0.2491	1776.80	3.9979	0.48439	1.1065
7.8	168.418	170.418	442.181	45.268	0.2461	1796.75	4.0464	0.48562	1.1043
7.9	168.941	170.958	441.768	45.301	0.2431	1817.23	4.0961	0.48685	1.1021
8.0	169.459	171.493	441.358	45.334	0.2403	1836.70	4.1437	0.48806	1.1000
8.1	169.972	172.023	440.952	45.366	0.2375	1856.64	4.1923	0.48925	1.0979
8.2	170.480	172.548	440.550	45.398	0.2347	1877.08	4.2421	0.49044	1.0958
8.3	170.983	173.067	440.152	45.430	0.2321	1896.39	4.2894	0.49161	1.0937
8.4	171.482	173.583	439.758	45.461	0.2295	1916.16	4.3378	0.49277	1.0917

FRENCH UNITS.

Second Principal Table for Vapor of Water. Table 12c.

1. Pressure in kg. per sq. cm. (Abs.)	2. Temperature t	3. Heat of the Liquid q	4. Latent Heat Inner ρ	5. Latent Heat Outer Apu	6. Values of u	7. Values of $\frac{\rho}{u}$	8. Specific Weight $\gamma=\frac{1}{v}$	9. $\tau=\int_0^t \frac{cdt}{T}$	10. $\frac{r}{T}$
8.5	171.976	174.093	439.367	45.492	0.2269	1936.39	4.3872	0.49312	1.0896
8.6	172.465	174.599	438.980	45.523	0.2244	1956.24	4.4359	0.49505	1.0876
8.7	172.950	175.100	438.597	45.553	0.2220	1975.66	4.4836	0.49618	1.0857
8.8	173.430	175.596	438.217	45.583	0.2196	1995.52	4.5324	0.49729	1.0837
8.9	173.906	176.089	437.840	45.612	0.2173	2014.91	4.5801	0.49839	1.0818
9.0	174.379	176.578	437.466	45.642	0.2150	2034.73	4.6289	0.49948	1.0799
9.1	174.846	177.061	437.097	45.670	0.2128	2054.03	4.6765	0.50056	1.0780
9.2	175.310	177.541	436.730	45.699	0.2106	2073.74	4.7251	0.50164	1.0761
9.3	175.770	178.017	436.366	45.727	0.2085	2092.88	4.7725	0.50270	1.0743
9.4	176.226	178.489	436.005	45.755	0.2064	2112.43	4.8208	0.50375	1.0724
9.5	176.679	178.958	435.647	45.782	0.2043	2132.39	4.8701	0.50479	1.0706
9.6	177.127	179.422	435.293	45.809	0.2023	2151.72	4.9180	0.50582	1.0688
9.7	177.572	179.882	434.941	45.836	0.2004	2170.36	4.9644	0.50684	1.0670
9.8	178.014	180.340	434.591	45.863	0.1984	2190.48	5.0141	0.50786	1.0653
9.9	178.451	180.793	434.245	45.890	0.1965	2209.90	5.0624	0.50886	1.0635
10.00	178.886	181.243	433.901	45.916	0.1947	2228.56	5.1089	0.50986	1.0618
10.25	179.957	182.353	433.054	45.980	0.1902	2276.83	5.2291	0.51231	1.0576
10.50	181.008	183.442	432.223	46.042	0.1859	2325.03	5.3494	0.51472	1.0534
10.75	182.040	184.513	431.406	46.103	0.1818	2372.97	5.4694	0.51707	1.0494
11.00	183.053	185.563	430.605	46.163	0.1779	2420.49	5.5885	0.51938	1.0454
11.25	184.049	186.597	429.817	46.221	0.1742	2467.38	5.7065	0.52164	1.0415
11.50	185.027	187.612	429.044	46.277	0.1706	2514.91	5.8262	0.52386	1.0378
11.75	185.989	188.611	428.283	46.333	0.1672	2561.50	5.9439	0.52604	1.0340
12.00	186.985	189.594	427.534	46.387	0.1639	2608.51	6.0629	0.52818	1.0304
12.25	187.866	190.561	426.798	46.440	0.1607	2655.87	6.1828	0.53028	1.0268
12.50	188.782	191.513	426.073	46.493	0.1577	2701.79	6.2996	0.53234	1.0234
12.75	189.685	192.452	425.359	46.543	0.1548	2747.80	6.4168	0.53437	1.0199
13.00	190.573	193.376	424.657	46.592	0.1520	2793.80	6.5342	0.53637	1.0166
13.25	191.449	194.287	423.964	46.641	0.1492	2841.58	6.6560	0.53833	1.0133
13.50	192.311	195.184	423.282	46.689	0.1466	2887.33	6.7732	0.54026	1.0100
13.75	193.162	196.070	422.609	46.735	0.1441	2932.75	6.8898	0.54216	1.0068
14.00	194.001	196.944	421.945	46.781	0.1417	2977.73	7.0057	0.54404	1.0037
14.25	194.828	197.806	421.290	46.826	0.1393	3024.34	7.1255	0.54588	1.0006
14.50	195.644	198.656	420.645	46.870	0.1370	3070.40	7.2442	0.54770	0.9976
14.75	196.449	199.495	420.009	46.913	0.1348	3115.79	7.3615	0.54949	0.9946
15.00	197.24	200.324	419.380	46.955	0.1327	3160.36	7.4771	0.55125	0.9917
16.00	200.32	203.533	416.947	47.118	0.1249	3339.23	7.9428	0.55805	0.9804
17.00	203.26	206.665	414.621	47.268	0.1177	3521.44	8.4175	0.56452	0.9697
18.00	206.07	209.544	412.399	47.408	0.1117	3692.95	8.8652	0.57068	0.9598
19.00	208.75	212.350	410.279	47.540	0.1061	3867.31	9.3284	0.57652	0.9503
20.00	211.34	215.065	408.230	47.664	0.1010	4039.96	9.7943	0.58214	0.9413

ENGLISH UNITS.

Saturated Vapor

1. Temperature Fahrenheit	2. Absolute Temperature T	3. Steam Pressures in Inches of Mercury p (Regnault)	4. Differential Coefficient in Inches of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
− 4	455.4	0.0365	0.0016813	0.046070
+ 5	464.4	0.0551	0.0024466	0.044389
+14	473.4	0.0824	0.0035245	0.042773
23	482.4	0.1226	0.0050516	0.041218
32	491.4	0.1811	0.007194	0.039723
41	500.4	0.2572	0.009849	0.038286
50	509.4	0.3608	0.013316	0.036905
59	518.4	0.5000	0.017787	0.035578
68	527.4	0.6847	0.023486	0.034303
77	536.4	0.9272	0.030669	0.033079
86	545.4	1.2420	0.039626	0.031904
95	554.4	1.6467	0.050680	0.030776
104	563.4	2.1617	0.064188	0.029694
113	572.4	2.8106	0.080545	0.028657
122	581.4	3.6213	0.10017	0.027663
131	590.4	4.6250	0.12354	0.026712
140	599.4	5.8577	0.15114	0.025802
149	608.4	7.3598	0.18349	0.024931
158	617.4	9.1764	0.22115	0.024100
167	626.4	11.3583	0.26474	0.023307
176	635.4	13.9612	0.31485	0.022552
185	644.4	17.0472	0.37220	0.021833
194	653.4	20.6847	0.43749	0.021151
203	662.4	24.9485	0.51155	0.020504
212	671.4	29.9212	0.59468	0.019875
221	680.4	35.6854	0.68797	0.019278
230	689.4	42.3373	0.79204	0.018708
239	698.4	49.9767	0.90768	0.018162
248	707.4	58.7117	1.03561	0.017639
257	716.4	68.6566	1.17662	0.017137
266	725.4	79.9321	1.33143	0.016657
275	734.4	92.6664	1.50081	0.016196
284	743.4	106.9931	1.68547	0.015753
293	752.4	123.0529	1.88613	0.015328
302	761.4	140.9930	2.10347	0.014919
311	770.4	160.9666	2.33816	0.014526
320	779.4	183.1343	2.59087	0.014147
329	788.4	207.6587	2.86216	0.013783
338	797.4	234.7105	3.15262	0.013432
347	806.4	264.4652	3.46279	0.013093
356	815.4	297.1013	3.79315	0.012767
365	824.4	332.8036	4.14413	0.012452
374	833.4	371.7591	4.51613	0.012148
383	842.4	414.1578	4.90948	0.011854
392	851.4	460.1944	5.32453	0.011570

ENGLISH UNITS.

of Water. Table 1a.

6. $T.\frac{dp}{pdt}$	7. $\frac{r}{u}=AT\frac{dp}{dt}$	8. $\frac{\rho}{u}=\frac{r}{u}-Ap$	9. $\frac{Apu}{r}=\phi$	1. Temperature Fahrenheit t
20.980	0.07007	0.06674	0.047664	− 4
20.614	0.10400	0.09900	0.048510	+ 5
20.249	0.15270	0.14516	0.049386	+14
19.884	0.22302	0.21181	0.050293	23
19.520	0.3235	0.30700	0.051229	32
19.158	0.4511	0.42757	0.052196	41
18.799	0.6207	0.58770	0.053193	50
18.444	0.8440	0.79817	0.054219	59
18.091	1.1337	1.0710	0.055274	68
17.743	1.5057	1.4208	0.056358	77
17.400	1.9780	1.8642	0.057470	86
17.062	2.5715	2.4209	0.058609	95
16.730	3.3098	3.1119	0.059774	104
16.403	4.2195	3.9623	0.060964	113
16.083	5.3305	4.9992	0.062176	122
15.771	6.6756	6.2524	0.063409	131
15.465	8.2913	7.7551	0.064661	140
15.168	10.2172	9.5437	0.065927	149
14.879	12.4967	11.6569	0.067206	158
14.600	15.1773	14.1377	0.068494	167
14.329	18.3101	17.0323	0.069786	176
14.069	21.9517	20.3913	0.071076	185
13.820	26.1632	24.2701	0.072359	194
13.582	31.0124	28.7290	0.073628	203
13.344	36.5428	33.8042	0.074940	212
13.117	42.8413	39.5753	0.076236	221
12.897	49.9753	46.1004	0.077536	230
12.684	58.0185	53.4444	0.078838	239
12.478	67.0493	61.6747	0.080143	248
12.277	77.1476	70.8638	0.081450	257
12.083	88.3955	81.0798	0.082761	266
11.894	100.8772	92.3961	0.084075	275
11.711	114.6777	104.8853	0.085391	284
11.533	129.8838	118.6215	0.086709	293
11.359	146.5831	133.6789	0.088034	302
11.191	164.8640	150.1318	0.089361	311
11.026	184.8166	168.0554	0.090691	320
10.866	206.5264	187.5207	0.092026	329
10.711	230.0825	208.6009	0.093365	338
10.559	255.5709	231.3660	0.094709	347
10.410	283.0769	255.8851	0.096059	356
10.265	312.6841	282.2246	0.097413	365
10.124	344.4730	310.4480	0.098774	374
9.986	378.5210	340.6156	0.100140	383
9.851	414.9065	372.7877	0.101514	392

ENGLISH UNITS.

Saturated Vapor

1. Temperature Fahrenheit t ° F.	2. Total Heat λ (Regnault)	Differences	3. Heat of Liquid $q=\int_0^t c\,dt$ (Regnault)	Differences	4. Latent Heat $r=\lambda-q$	Differences
− 4	1080.720	2.745	−35.989	8.996	1116.709	6.251
+ 5	1083.465	"	−26.993	8.997	1110.458	6.252
14	1086.210	"	−17.996	8.998	1104.206	6.253
23	1088.955	"	− 8.998	8.998	1097.953	6.253
32	1091.700	"	0.000	9.000	1091.700	6.255
41	1094.445	"	+ 9.000	9.003	1085.445	6.258
50	1097.190	"	18.003	9.006	1079.187	6.261
59	1099.935	"	27.009	9.009	1072.926	6.264
68	1102.680	"	36.018	9.013	1066.662	6.268
77	1105.425	"	45.031	9.016	1060.394	6.271
86	1108.170	"	54.047	9.020	1054.123	6.275
95	1110.915	"	63.067	9.025	1047.848	6.280
104	1113.660	"	72.092	9.030	1041.568	6.285
113	1116.405	"	81.122	9.035	1035.283	6.290
122	1119.150	"	90.157	9.042	1028.993	6.297
131	1121.895	"	99.199	9.048	1022.696	6.303
140	1124.640	"	108.247	9.054	1016.393	6.309
149	1127.385	"	117.301	9.061	1010.084	6.316
158	1130.130	"	126.362	9.068	1003.768	6.323
167	1132.875	"	135.430	9.077	997.445	6.332
176	1135.620	"	144.507	9.085	991.113	6.340
185	1138.365	"	153.592	9.094	984.773	6.349
194	1141.110	"	162.686	9.102	978.424	6.357
203	1143.855	"	171.788	9.112	972.067	6.367
212	1146.600	"	180.900	9.122	965.700	6.377
221	1149.345	"	190.022	9.132	959.323	6.387
230	1152.090	"	199.154	9.143	952.936	6.398
239	1154.835	"	208.297	9.154	946.538	6.409
248	1157.580	"	217.451	9.166	940.129	6.421
257	1160.325	"	226.617	9.178	933.708	6.433
266	1163.070	"	235.795	9.190	927.275	6.445
275	1165.815	"	244.985	9.202	920.830	6.457
284	1168.560	"	254.187	9.215	914.373	6.470
293	1171.305	"	263.402	9.230	907.903	6.485
302	1174.050	"	272.632	9.244	901.418	6.499
311	1176.795	"	281.876	9.258	894.919	6.513
320	1179.540	"	291.134	9.272	888.406	6.527
329	1182.285	"	300.406	9.288	881.879	6.543
338	1185.030	"	309.694	9.303	875.336	6.558
347	1187.775	"	318.997	9.319	868.778	6.574
356	1190.520	"	328.316	9.335	862.204	6.590
365	1193.265	"	337.651	9.353	855.614	6.608
374	1196.010	"	347.004	9.369	849.006	6.624
383	1198.755	"	356.373	9.387	842.382	6.642
392	1201.500		365.760		835.740	

ENGLISH UNITS.

of Water. Table 1b.

5. Outer Latent Heat $Apu = \phi r$	Differences	6. Steam Heat $J = \lambda - Apu$	Differences	7. Inner Latent Heat $\rho = r - Apu$	Differences	8. Values of $u = \frac{r}{\left(\frac{r}{u}\right)}$ (σ=0.01602)	1. Temperature Fahrenheit t ° F.
53.226	0.643	1027.49	2.11	1063.48	6.89	15934.8	− 4
53.869	0.664	1029.60	2.07	1056.59	6.91	10677.7	+ 5
54.533	0.686	1031.67	2.07	1049.67	6.94	7231.0	14
55.219	0.709	1033.74	2.03	1042.73	6.96	4922.9	23
55.928	0.727	1035.77	2.02	1035.77	6.98	3374.8	32
56.655	0.751	1037.79	2.00	1028.79	7.01	2406.6	41
57.406	0.766	1039.79	1.98	1021.78	7.03	1738.3	50
58.172	0.787	1041.77	1.95	1014.75	7.05	1271.1	59
58.959	0.803	1043.72	1.94	1007.70	7.07	940.70	68
59.762	0.819	1045.66	1.93	1000.63	7.09	704.28	77
60.581	0.833	1047.59	1.91	993.54	7.11	532.92	86
61.414	0.844	1049.50	1.90	986.43	7.12	407.48	95
62.258	0.857	1051.40	1.89	979.31	7.14	314.70	104
63.115	0.864	1053.29	1.89	972.17	7.15	245.34	113
63.979	0.870	1055.18	1.87	965.02	7.17	193.02	122
64.849	0.873	1057.05	1.87	957.85	7.18	153.17	131
65.722	0.871	1058.92	1.87	950.67	7.18	122.60	140
66.593	0.867	1060.79	1.88	943.49	7.18	988.61	149
67.460	0.859	1062.67	1.89	936.31	7.18	80.323	158
68.319	0.846	1064.56	1.90	929.13	7.18	65.720	167
69.165	0.828	1066.46	1.91	921.95	7.17	54.130	176
69.993	0.805	1068.37	1.94	914.78	7.15	44.861	185
70.798	0.774	1070.31	1.97	907.63	7.14	37.397	194
71.572	0.797	1072.28	1.95	900.49	7.16	31.345	203
72.369	0.767	1074.23	1.98	893.33	7.13	26.427	212
73.136	0.750	1076.21	1.99	886.19	7.14	22.392	221
73.886	0.737	1078.20	2.01	879.05	7.13	19.068	230
74.623	0.721	1080.21	2.03	871.92	7.13	16.315	239
75.344	0.706	1082.24	2.04	864.79	7.13	14.021	248
76.050	0.691	1084.28	2.05	857.66	7.13	12.104	257
76.741	0.677	1086.33	2.07	850.53	7.12	10.490	266
77.418	0.661	1088.40	2.08	843.41	7.12	9.129	275
78.079	0.645	1090.48	2.10	836.29	7.11	7.973	284
78.723	0.631	1092.58	2.11	829.18	7.12	6.989	293
79.355	0.615	1094.69	2.13	822.06	7.11	6.150	302
79.970	0.600	1096.82	2.15	814.95	7.11	5.427	311
80.570	0.585	1098.97	2.16	807.84	7.12	4.807	320
81.155	0.570	1101.13	2.17	800.72	7.11	4.269	329
81.725	0.555	1103.30	2.20	793.61	7.11	3.804	338
82.280	0.542	1105.50	2.20	786.50	7.12	3.399	347
82.822	0.525	1107.70	2.22	779.38	7.11	3.045	356
83 347	0.513	1109.92	2.23	772.27	7.12	2.736	365
83.860	0.495	1112.15	2.25	765.15	7.12	2.464	374
84.355	0.484	1114.40	2.27	758.03	7.13	2.225	383
84.839		1116.66		750.90		2.014	392

ENGLISH UNITS.

Saturated Vapor

1. Temperature Fahrenheit t	2. Absolute Temperature T	3. Steam Pressure in Inches of Mercury p (Regnault)	4. Differential Coefficient in Inches of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
32	491.4	7.259	0.18462	0.025432
50	509.4	11.292	0.26737	0.023677
68	527.4	17.039	0.37566	0.022047
86	545.4	24.992	0.51321	0.020536
104	563.4	35.710	0.68333	0.019136
122	581.4	49.796	0.88849	0.017843
140	599.4	67.913	1.13102	0.016653
158	617.4	90.744	1.41242	0.015565
176	635.4	119.007	1.73506	0.014579
194	653.4	153.474	2.10252	0.013699
212	671.4	195.011	2.52215	0.012933
230	689.4	244.670	3.00793	0.012294
248	707.4	303.905	3.58659	0.011802

Saturated Vapor

1. Temperature Fahrenheit t	2. Total Heat λ (Regnault)	Differences	3. Heat of the Liquid $q=\int_0^t cdt$ (Regnault)	Differences	4. Latent Heat $r=\lambda-q$	Differences
32	169.20	8.00	0.00	9.58	169.20	1.58
50	177.20	7.80	9.58	9.68	167.62	1.88
68	185.00	7.60	19.26	9.79	165.74	2.19
86	192.60	7.40	29.05	9.89	163.55	2.49
104	200.00	7.20	38.94	10.00	161.06	2.80
122	207.20	7.00	48.94	10.10	158.26	3.10
140	214.20	6.80	59.04	10.22	155.16	3.42
158	221.00	6.59	69.26	10.32	151.74	3.73
176	227.59	6.41	79.58	10.42	148.01	4.01
194	234.00	6.19	90.00	10.55	144.00	4.36
212	240.19	6.01	100.55	10.64	139.64	4.63
230	246.20	5.80	111.19	10.74	135.01	4.94
248	252.00		121.93		130.07	

ENGLISH UNITS.

of Ether. Table 2a.

6. $T.\frac{dp}{pdt}$	7. $\frac{r}{u}=AT\frac{dp}{dt}$	8. $\frac{\rho}{u}=\frac{r}{u}-Ap$	9. $\frac{Apu}{r}=\phi$	1. Temperature Fahrenheit t
12.497	8.303	7.639	0.08002	32
12.061	12.465	11.432	0.08291	50
11.628	18.132	16.574	0.08600	68
11.200	25.618	23.331	0.08929	86
10.781	35.236	31.967	0.09275	104
10.374	47.279	42.721	0.09640	122
9.982	62.046	55.829	0.10018	140
9.610	79.812	71.506	0.10406	158
9.264	100.901	90.009	0.10794	176
8.951	125.732	111.686	0.11172	194
8.683	154.983	137.134	0.11516	212
8.475	189.791	167.398	0.11799	230
8.348	232.210	204.394	0.11978	248

of Ether. Table 2b.

5. Outer Latent Heat $Apu=\phi r$	Differences.	6. Steam Heat $J=\lambda-Apu$	Differences	7. Inner Latent Heat $\rho=r-Apu$	Differences	8. Values of $u=\frac{r}{\left(\frac{r}{u}\right)}$ ($\sigma=0.0208$)	1. Temperture Fahrenheit t
13.538	0.360	155.66	7.64	155.66	1.94	20.377	32
13.898	0.356	163.30	7.45	153.72	2.23	13.446	50
14.254	0.349	170.75	7.25	151.49	2.54	9.142	68
14.603	0.337	178.00	7.06	148.95	2.83	6.383	86
14.940	0.315	185.06	6.89	146.12	3.11	4.570	104
15.255	0.288	191.95	6.72	143.01	3.39	3.346	122
15.543	0.247	198.67	6.54	139.62	3.67	2.501	140
15.790	0.187	205.21	6.40	135.95	3.92	1.900	158
15.977	0.110	211.61	6.30	132.03	4.12	1.467	176
16.087	−0.006	217.91	6.20	127.91	4.35	1.145	194
16.081	−0.151	224.11	6.16	123.56	4.48	0.900	212
15.930	−0.351	230.27	6.15	119.08	4.59	1.711	230
15.579		236.42		114.49		0.561	248

ENGLISH UNITS.

Saturated Vapor

1. Temperature Fahrenheit t	2. Absolute Temperature T	3. Steam Pressure in Inches of Mercury p (Regnault)	4. Differential Coefficient in Inches of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
32	491.4	0.500	0.018414	0.036831
50	509.4	0.954	0.033213	0.034818
68	527.4	1.750	0.057159	0.032655
86	545.4	3.091	0.094473	0.030561
104	563.4	5.263	0.15045	0.028583
122	581.4	8.658	0.23141	0.026730
140	599.4	13.788	0.34464	0.024996
158	617.4	21.305	0.49797	0.023373
176	635.4	32.005	0.69950	0.021857
194	653.4	46.822	0.95695	0.020438
212	671.4	66.831	1.27727	0.019112
230	689.4	93.214	1.66583	0.017871
248	707.4	127.234	2.12622	0.016711
266	725.4	170.196	2.65959	0.015627
284	743.4	223.408	3.26454	0.014612
302	761.4	288.125	3.93696	0.013664

Saturated Vapor

1. Temperature Fahrenheit t	2. Total Heat λ (Regnault)	Differences	3. Heat of Liquid $q=\int_0^t cdt$ (Regnault)	Differences	4. Latent Heat $r=\lambda-q$	Differences
32	425.70		0.00		425.70	
		14.22		10.06		−4.16
50	439.92		10.06		429.86	
		13.68		10.50		−3.18
68	453.60		20.56		433.04	
		10.80		10.92		+0.12
86	464.40		31.48		432.92	
		7.20		11.20		4.00
104	471.60		42.68		428.92	
		3.60		11.70		8.10
122	475.20		54.38		420.82	
		1.80		12.89		11.09
140	477.00		67.27		409.73	
		0.36		12.97		12.61
158	477.36		80.24		397.12	
		0.00		13.56		13.56
176	477.36		93.80		383.56	
		1.44		14.15		12.71
194	478.80		107.95		370.85	
		2.34		14.77		12.43
212	481.14		122.72		358.42	
		4.14		15.41		11.27
230	485.28		138.13		347.15	
		5.22		16.08		10.86
248	490.50		154.21		336.29	
		6.30		16.75		10.45
266	496.80		170.96		325.84	
		8.10		17.50		9.40
284	504.90		188.46		316.44	
		8.64		18.22		9.58
302	513.54		206.68		306.86	

ENGLISH UNITS.

of Alcohol. Table 3a.

6. $T \cdot \frac{dp}{pdt}$	7. $\frac{r}{u} = AT\frac{dp}{dt}$	8. $\frac{\rho}{u} = \frac{r}{u} - Ap$	9. $\frac{Apu}{r} = \phi$	1. Temperature Fahrenheit t
18.099	0.828	0.782	0.05525	32
17.736	1.548	1.461	0.05638	50
17.222	2.759	2.599	0.05806	68
16.668	4.716	4.422	0.05999	86
16.104	7.757	7.275	0.06210	104
15.541	12.314	11.521	0.06435	122
14.982	18.905	17.643	0.06674	140
14.431	28.139	26.189	0.06930	158
13.887	40.679	37.750	0.07201	176
13.354	57.227	52.941	0.07488	194
12.831	78.487	72.370	0.07793	212
12.320	105.108	96.577	0.08117	230
11.821	137.660	126.015	0.08459	248
11.335	176.578	160.962	0.08822	266
10.863	222.116	201.669	0.09206	284
10.404	274.351	247.981	0.09612	302

of Alcohol. Table 3b.

5. Outer Latent Heat $Apu = \phi r$	Differences.	6. Steam Heat $J = \lambda - Apu$	Differences.	7. Inner Latent Heat $\rho = r - Apu$	Differences.	8. Values of $u = \frac{r}{\left(\frac{r}{u}\right)}$ $(\sigma = 0.0208)$	1. Temperature Fahrenheit t
23.521	0.716	402.18	13.50	402.18	−3.44	513.989	32
24.237	0.907	415.68	12.78	405.62	−2.28	277.595	50
25.144	0.828	428.46	9.97	407.90	+0.95	156.956	68
25.972	0.662	438.43	6.54	406.95	4.66	91.800	86
26.634	0.445	444.97	3.15	402.29	8.55	55.289	104
27.079	0.268	448.12	1.53	393.74	11.36	34.174	122
27.347	0.170	449.65	+0.19	382.38	12.78	21.671	140
27.517	0.102	449.84	−0.10	369.60	13.66	14.112	158
27.619	0.151	449.74	+1.29	355.94	12.86	9.430	176
27.770	0.162	451.03	2.18	343.08	12.59	6.479	194
27.932	0.245	453.21	3.89	330.49	11.52	4.564	212
28.177	0.267	457.10	4.96	318.97	11.12	3.305	230
28.445	0.299	462.06	6.00	307.85	10.75	2.443	248
28.744	0.387	468.06	7.71	297.10	9.79	1.845	266
29.131	0.364	475.77	8.27	287.31	9.95	1.424	284
29.495		484.04		277.36		1.118	302

ENGLISH UNITS.

Saturated Vapor

1. Temperature Fahrenheit t	2. Absolute Temperature T	3. Steam Pressure in Inches of Mercury p (Regnault)	4. Differential Coefficient in Inches of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
32	491.4	2.493	0.08244	0.033064
50	509.4	4.343	0.12524	0.028839
68	527.4	7.090	0.18261	0.025758
86	545.4	11.026	0.25802	0.023403
104	563.4	16.510	0.35525	0.021518
122	581.4	23.969	0.47808	0.019946
140	599.4	33.896	0.63009	0.018589
158	617.4	46.846	0.81446	0.017386
176	635.4	63.427	1.03381	0.016299
194	653.4	84.284	1.28983	0.015303
212	671.4	110.086	1.58335	0.014383
230	689.4	141.509	1.91415	0.013527
248	707.4	179.210	2.28064	0.012726
266	725.4	223.815	2.68055	0.011977
284	743.4	275.891	3.11021	0.011273

Saturated Vapor

1. Temperature Fahrenheit t	2. Total Heat λ (Regnault)	Differences	3. Heat of Liquid $q=\int_0^t c\,dt$ (Regnault)	Differences	4. Latent Heat $r=\lambda-q$	Differences
32	252.90	6.50	0.00	9.18	252.90	2.68
50	259.40	6.32	9.18	9.34	250.22	3.02
68	265.72	6.13	18.52	9.47	247.20	3.34
86	271.85	5.94	27.99	9.61	243.86	3.67
104	277.79	5.76	37.60	9.76	240.19	4.00
122	283.55	5.58	47.36	9.90	236.19	4.32
140	289.13	5.39	57.26	10.04	231.87	4.65
158	294.52	5.20	67.30	10.19	227.22	4.99
176	299.72	5.02	77.49	10.33	222.23	5.31
194	304.74	4.82	87.82	10.48	216.92	5.66
212	309.56	4.65	98.30	10.60	211.26	5.95
230	314.21	4.46	108.90	10.76	205.31	6.30
248	318.67	4.29	119.66	10.91	199.01	6.62
266	322.96	4.08	130.57	11.04	192.39	6.96
284	327.04		141.61		185.43	

ENGLISH UNITS.

of Acetone. Table 4a.

6. $T\frac{dp}{pdt}$	7. $\frac{r}{u}=AT\frac{dp}{dt}$	8. $\frac{\rho}{u}=\frac{r}{u}-Ap$	9. $\frac{Apu}{r}=\phi$	1. Temperature Fahrenheit t
16.248	3.708	3.480	0.06155	32
14.690	5.839	5.442	0.06807	50
13.553	8.814	8.166	0.07378	68
12.674	12.880	11.871	0.07834	86
12.123	18.319	16.807	0.08248	104
11.596	25.440	23.246	0.08623	122
11.142	34.566	31.464	0.08975	140
10.733	46.023	41.735	0.09316	158
10.356	60.120	54.315	0.09656	176
9.999	77.134	69.421	0.10001	194
9.657	97.295	87.219	0.10355	212
9.325	120.776	107.826	0.10723	230
9.002	147.658	131.257	0.11108	248
8.688	177.966	157.481	0.11510	266
8.380	211.616	186.365	0.11932	284

of Acetone. Table 4b.

5. Outer Latent Heat $Apu=\phi r$	Differences	6. Steam Heat $J=\lambda-Apu$	Differences.	7. Inner Latent Heat $\rho=r-Apu$	Differences.	8. Values of $u=\frac{r}{\left(\frac{r}{u}\right)}$ $(\sigma=0.0192)$	1. Temperature Fahrenheit t
15.564		237.34		237.34		68.206	32
	1.468		5.03		4.15		
17.032		242.37		233.19		42.851	50
	1.206		5.11		4.23		
18.238		247.48		228.96		28.043	68
	0.865		5.27		4.20		
19.103		252.75		224.76		18.932	86
	0.708		5.23		4.38		
19.811		257.98		220.38		13.111	104
	0.556		5.20		4.56		
20.367		263.18		215.82		9.286	122
	0.443		5.14		4.76		
20.810		268.32		211.06		6.707	140
	0.356		5.03		5.01		
21.166		273.35		206.05		4.937	158
	0.292		4.91		5.28		
21.458		283.26		200.77		3.696	176
	0.236		4.79		5.54		
21.694		287.05		195.23		2.811	194
	0.183		4.63		5.85		
21.877		287.68		189.38		2.171	212
	0.139		4.51		6.09		
22.016		292.19		183.29		1.700	230
	0.090		4.37		6.39		
22.106		296.56		176.90		1.347	248
	0.038		4.24		6.65		
22.144		300.82		170.25		1.081	266
	0.000		4.08		6.96		
22.144		304.90		163.29		0.876	284

ENGLISH UNITS.

Saturated Vapor

1. Temperature Fahrenheit t	2. Absolute Temperature T	3. Steam Pressure in Inches of Mercury p (Regnault)	4. Differential Coefficient in Inches of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
32	491.4	2.351	0.07076	0.030099
50	509.4	3.956	0.10633	0.027387
68	527.4	6.318	0.15849	0.025087
86	545.4	9.745	0.22518	0.023107
104	563.4	14.538	0.31083	0.021381
122	581.4	21.065	0.41826	0.019856
140	599.4	29.742	0.55009	0.018496
158	617.4	41.028	0.70851	0.017269
176	635.4	55.418	0.89530	0.016155
194	653.4	73.435	1.11146	0.015136
212	671.4	95.612	1.35741	0.014197
230	689.4	122.480	1.63265	0.013330
248	707.4	154.556	1.93591	0.012526
266	725.4	192.326	2.26485	0.011776
284	743.4	236.226	2.61647	0.011076
302	761.4	286.638	2.98740	0.010422
320	779.4	343.865	3.37309	0.009810

Saturated Vapor

1. Temperature Fahrenheit t	2. Total Heat λ (Regnault)	Differences	3. Heat of Liquid $q=\int_0^t cdt$ (Regnault)	Differences	4. Latent Heat $r=\lambda-q$	Differences
32	120.60		0.00		120.60	
		2.475		4.19		1.72
50	123.07	"	4.19	4.22	118.88	1.74
68	125.55	"	8.41	4.23	117.14	1.76
86	128.02	"	12.64	4.23	115.38	1.75
104	130.50	"	16.87	4.26	113.63	1.79
122	132.97	"	21.13	4.29	111.84	1.81
140	135.45	"	25.42	4.30	110.03	1.83
158	137.92	"	29.72	4.32	108.20	1.84
176	140.40	"	34.04	4.34	106.36	1.87
194	142.87	"	38.38	4.35	104.49	1.87
212	145.35	"	42.73	4.38	102.62	1.91
230	147.82	"	47.11	4.39	100.71	1.91
248	150.30	"	51.50	4.41	98.80	1.94
266	152.77	"	55.91	4.43	96.86	1.95
284	155.25	"	60.34	4.44	94.91	1.97
302	157.72	"	64.78	4.47	92.94	1.99
320	160.20		69.25		90.95	

ENGLISH UNITS.

of Chloroform. Table 5a.

6. $T\frac{dp}{pdt}$	7. $\frac{r}{u}=AT\frac{dp}{dt}$	8. $\frac{\rho}{u}=\frac{r}{u}-Ap$	9. $\frac{Apu}{r}=\phi$	1. Temperature Fahrenheit t
14.790	3.182	2.968	0.06761	32
13.951	5.051	4.689	0.07168	50
13.231	7.649	7.070	0.07558	68
12.603	11.240	10.348	0.07935	86
12.046	16.028	14.697	0.08302	104
11.544	22.256	20.328	0.08662	122
11.086	30.177	27.456	0.09020	140
10.662	40.036	36.280	0.09379	158
10.265	52.066	46.994	0.09742	176
9.889	66.466	59.745	0.10112	194
9.532	83.412	74.662	0.10491	212
9.190	103.015	91.805	0.10882	230
8.860	125.339	111.193	0.11286	248
8.542	150.367	132.765	0.11706	266
8.234	178.021	156.401	0.12145	284
7.935	208.181	181.947	0.12602	302
7.646	240.631	209.160	0.13079	320

of Chloroform. Table 5b.

5. Outer Latent Heat $Apu=\phi r$	Differences	6. Steam Heat $J=\lambda-Apu$	Differences	7. Inner Latent Heat $\rho=r-Apu$	Differences	8. Values of $u=\frac{r}{\left(\frac{r}{u}\right)_{\sigma=(0.0096}}$	1. Temperature Fahrenheit t
8.154	0.367	112.45	2.10	112.45	2.09	37.899	32
8.521	0.333	114.55	2.15	110.36	2.07	23.536	50
8.854	0.300	116.70	2.16	108.29	2.06	15.314	68
9.154	0.279	118.86	2.21	106.23	2.03	10.265	86
9.433	0.253	121.07	2.21	104.20	2.05	7.090	104
9.687	0.238	123.28	2.25	102.15	2.04	5.025	122
9.925	0.223	125.53	2.24	100.11	2.06	3.646	140
10.148	0.213	127.77	2.27	98.05	2.05	2.702	158
10.361	0.205	130.04	2.26	96.00	2.08	2.042	176
10.566	0.200	132.30	2.28	93.92	2.07	1.571	194
10.766	0.192	134.58	2.28	91.85	2.07	1.230	212
10.958	0.193	136.86	2.29	89.75	2.10	0.977	230
11.151	0.187	139.15	2.28	87.65	2.13	0.788	248
11.338	0.189	141.43	2.29	85.52	2.14	0.644	266
11.527	0.184	143.72	2.29	83.38	2.15	0.533	284
11.711	0.185	146.01	2.29	81.23	2.18	0.447	302
11.896		148.30		79.05		0.378	320

ENGLISH UNITS.

Saturated Vapor of

1. Temperature Fahrenheit t	2. Absolute Temperature T	3. Steam Pressure in Inches of Mercury p (Regnault)	4. Differential Coefficient in Inches of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
32	491.4	1.297	0.0398	0.030743
50	509.4	2.204	0.0621	0.028171
68	527.4	3.582	0.0927	0.025874
86	545.4	5.601	0.1334	0.023823
104	563.4	8.457	0.1859	0.021991
122	581.4	12.377	0.2520	0.020354
140	599.4	17.615	0.3328	0.018890
158	617.4	24.455	0.4300	0.017582
176	635.4	33.200	0.5448	0.016411
194	653.4	44.184	0.6787	0.015363
212	671.4	57.760	0.8331	0.014424
230	689.4	74.310	1.0094	0.013584
248	707.4	94.239	1.2093	0.012830
266	725.4	117.987	1.4339	0.012153
284	743.4	146.025	1.6861	0.011547
302	761.4	178.863	1.9676	0.011001
320	779.4	217.052	2.2811	0.010509

Saturated Vapor of

1. Temperature Fahrenheit t	2. Total Heat λ (Regnault)	Differences	3. Heat of the Liquid $q=\int_0^t cdt$ (Regnault)	Differences	4. Latent Heat $r=\lambda-q$	Differences
32	93.60		0.00		93.60	
		2.59		3.58		0.99
50	96.19		3.58		92.61	
		2.56		3.60		1.04
68	98.75		7.18		91.57	
		2.46		3.66		1.20
86	101.21		10.84		90.37	
		2.43		3.67		1.24
104	103.64		14.51		89.13	
		2.34		3.71		1.37
122	105.98		18.22		87.76	
		2.31		3.74		1.43
140	108.29		21.96		86.33	
		2.23		3.78		1.55
158	110.52		25.74		84.78	
		2.16		3.82		1.66
176	112.68		29.56		83.12	
		2.11		3.83		1.72
194	114.79		33.39		81.40	
		2.03		3.87		1.84
212	116.82		37.26		79.56	
		2.00		3.91		1.91
230	118.82		41.17		77.65	
		1.91		3.94		2.03
248	120.73		45.11		75.62	
		1.85		3.98		2.13
266	122.58		49.09		73.49	
		1.80		3.99		2.19
284	124.38		53.08		71.30	
		1.75		4.03		2.28
320	126.13		57.11		69.02	
		1.67		4.09		2.42
302	127.80		61.20		66.60	

ENGLISH UNITS.

Chloride of Carbon. Table 6a.

6. $T\cdot\frac{dp}{pdt}$	7. $\frac{r}{u}=AT\frac{dp}{dt}$	8. $\frac{\rho}{u}=\frac{r}{u}-Ap$	9. $\frac{Apu}{r}=\phi$	1. Temperature Fahrenheit t
15.107	1.793	1.674	0.06619	32
14.350	2.895	2.694	0.06968	50
13.646	4.475	4.147	0.07328	68
12.993	6.660	6.148	0.07696	86
12.390	9.590	8.816	0.08071	104
11.834	13.405	12.272	0.08450	122
11.322	18.254	16.642	0.08832	140
10.855	24.296	22.057	0.09212	158
10.427	31.684	28.647	0.09590	176
10.038	40.593	36.549	0.09962	194
9.685	51.196	45.909	0.10325	212
9.365	63.690	56.888	0.10678	230
9.076	78.281	69.657	0.11018	248
8.816	95.201	84.402	0.11343	266
8.584	114.720	101.355	0.11650	284
8.376	137.114	120.707	0.11939	302
8.191	162.719	142.854	0.12208	320

Chloride of Carbon. Table 6b.

5. Outer Latent Heat $Apu=\phi r$	Differences	6. Steam Heat $J=\lambda-Apu$	Differences	7. Inner Latent Heat $\rho=r-Apu$	Differences	8. Values of $u=\frac{r}{\left(\frac{r}{u}\right)}$ $(\sigma=0.0069)$	1. Temperature Fahrenheit t
6.196	0.257	87.40	2.34	87.40	1.24	52.196	32
6.453	0.257	89.74	2.30	86.16	1.30	31.990	50
6.710	0.245	92.04	2.22	84.86	1.44	20.465	68
6.955	0.240	94.26	2.18	83.42	1.48	13.569	86
7.195	0.221	96.44	2.12	81.94	1.60	9.295	104
7.416	0.209	98.56	2.10	80.34	1.64	6.547	122
7.625	0.185	100.66	2.05	78.70	1.73	4.729	140
7.810	0.162	102.71	2.00	76.97	1.82	3.489	158
7.972	0.137	104.71	1.97	75.15	1.86	2.624	176
8.109	0.106	106.68	1.93	73.29	1.94	2.006	194
8.215	0.076	108.61	1.92	71.35	1.99	1.554	212
8.291	0.041	110.53	1.87	69.36	2.07	1.219	230
8.332	0.004	112.40	1.84	67.29	2.14	0.966	248
8.336	0.031	114.24	1.84	35.15	2.15	0.772	266
8.305	0.066	116.08	1.81	63.00	2.22	0.622	284
8.239	0.108	117.89	1.78	60.78	2.31	0.503	302
8.131		119.67		58.47		0.408	320

ENGLISH UNITS.

Saturated Vapor of

1. Temperature Fahrenheit t	2. Absolute Temperature T	3. Steam Pressure in Inches of Mercury p (Regnault)	4. Differential Coefficient in Inches of Mercury $\frac{dp}{dt}$	5. $\frac{dp}{pdt}$
32	491.4	5.036	0.12787	0.025391
50	509.4	7.813	0.18329	0.023459
68	527.4	11.733	0.25512	0.021743
86	545.4	17.111	0.34587	0.020213
104	563.4	24.312	0.45803	0.018839
122	581.4	33.743	0.59398	0.017601
140	599.4	45.847	0.75547	0.016478
158	617.4	61.106	0.94446	0.015456
176	635.4	80.021	1.16194	0.014521
194	653.4	103.113	1.40870	0.013662
212	671.4	130.911	1.68485	0.012870
230	689.4	163.939	1.98976	0.012137
248	707.4	202.708	2.32224	0.011456
266	725.4	247.700	2.68095	0.010823
284	743.4	299.368	3.06336	0.010233
302	761.4	358.107	3.46673	0.009681

Saturated Vapor of

1. Temperature Fahrenheit t	2. Total Heat λ (Regnault)	Differences	3. Heat of Liquid $q=\int_0^t cdt$ (Regnault)	Differences	4. Latent Heat $r=\lambda-q$	Differences
32	162.00		0.00		162.00	
		2.56		4.25		1.69
50	164.56		4.25		160.31	
		2.41		4.28		1.87
68	166.97		8.53		158.44	
		2.25		4.30		2.05
86	169.22		12.83		156.39	
		2.10		4.34		2.24
104	171.32		17.17		154.15	
		1.97		4.36		2.39
122	173.29		21.53		151.76	
		1.81		4.41		2.60
140	175.10		25.94		149.16	
		1.66		4.41		2.75
158	176.76		30.35		146.41	
		1.51		4.46		2.95
176	178.27		34.81		143.46	
		1.37		4.48		3.11
194	179.64		39.29		140.35	
		1.22		4.52		3.30
212	180.86		43.81		137.05	
		1.07		4.54		3.47
230	181.93		48.35		133.58	
		0.91		4.57		3.66
248	182.84		52.92		129.92	
		0.78		4.61		3.83
266	183.62		57.53		126.09	
		0.63		4.62		3.99
284	184.25		62.15		122.10	
		0.47		4.67		4.20
302	184.72		66.82		117.90	

ENGLISH UNITS.

Bisulphide of Carbon. Table 7a.

6. $T.\frac{dp}{pdt}$	7. $\frac{r}{u}=AT\frac{dp}{dt}$	8. $\frac{\rho}{u}=\frac{r}{u}-Ap$	9. $\frac{Apu}{r}=\phi$	1. Temperature Fahrenheit t
12.477	5.751	5.290	0.08015	32
11.950	8.545	7.830	0.08368	50
11.467	12.315	11.241	0.08721	68
11.024	17.265	15.698	0.09071	86
10.614	23.618	21.393	0.09421	104
10.233	31.607	28.520	0.09772	122
9.877	41.444	37.249	0.10124	140
9.543	53.368	47.777	0.10478	158
9.226	67.571	60.248	0.10838	176
8.926	84.243	74.807	0.11202	194
8.641	103.531	91.550	0.11573	212
8.367	125.546	110.542	0.11951	230
8.104	150.351	131.799	0.12339	248
7.851	177.992	155.321	0.12737	266
7.607	208.428	181.029	0.13146	284
7.371	241.581	208.805	0.13567	302

Bisulphide of Carbon. Table 7b.

5. Outer Latent Heat $Apu=\phi r$	Differences	6. Steam Heat $J=\lambda-Apu$	Differences	7. Inner Latent Heat $\rho=r-Apu$	Differences	8. Values of $u=\frac{r}{\left(\frac{r}{u}\right)_{\sigma=0.013}}$	1. Temperature Fahrenheit t
12.983	0.431	149.02	2.13	149.02	2.12	28.172	32
13.414	0.403	151.15	2.00	146.90	2.28	18.762	50
13.817	0.369	153.15	1.88	144.62	2.42	12.866	68
14.186	0.336	155.03	1.77	142.20	2.57	9.058	86
14.522	0.308	156.80	1.66	139.63	2.70	6.526	104
14.830	0.272	158.46	1.54	136.93	2.87	4.801	122
15.102	0.239	160.00	1.42	134.06	2.99	3.599	140
15.341	0.207	161.42	1.30	131.07	3.16	2.742	158
15.548	0.173	162.72	1.20	127.91	3.28	2.123	176
15.721	0.141	163.92	1.08	124.63	3.44	1.666	194
15.862	0.102	165.00	0.97	121.19	3.57	1.323	212
15.964	0.067	165.97	0.84	117.62	3.73	1.064	230
16.031	0.029	166.81	0.75	113.89	3.86	0.863	248
16.060	−0.009	167.56	0.64	110.03	3.98	0.708	266
16.051	−0.056	168.20	0.52	106.05	4.15	0.586	284
15.995		168.72		101.90		0.489	302

ENGLISH UNITS.

Saturated Vapor

1. Temperature Fahrenheit t	2. Vapor Pressure Atmospheres 1 Atmosphere = 1.0333 kg/cm²	3. Vapor Pressure Pounds per sq. in. p	4. $\frac{dp}{pdt}$	5. Specific Volume of the Vapor s	6. Heat of the Liquid q
					According to the older For temperature values
−22	1.153	16.912	0.026981	15.956	−47.88
−13	1.463	21.456	0.025915	12.767	−40.84
− 4	1.839	27.030	0.024891	10.306	−33.43
5	2.291	33.667	0.023907	8.390	−25.63
14	2.828	41.571	0.022962	6.888	−17.46
23	3.464	50.904	0.022055	5.699	− 8.93
					According to the experimental For temperature values
32	4.207	61.836	0.021183	4.709	0.00
41	5.072	74.539	0.020346	3.933	10.12
50	6.069	89.195	0.019542	3.300	20.30
59	7.214	105.974	0.018769	2.787	30.60
68	8.509	125.056	0.018028	2.371	41.04
77	9.976	146.612	0.017316	2.026	51.48
86	11.622	170.807	0.016631	1.738	62.10
95	13.458	197.796	0.015974	1.493	72.72
104	15.495	227.727	0.015343	1.296	83.48

ENGLISH UNITS.

of Ammonia. Table 8.

7.	8.	9.	10.	11.	1.
	Latent Heat				
Total r	Outer Apu	Inner ρ	$\frac{r}{T}$	$\tau = \int_0^t \frac{cdt}{T}$	Temperature Fahrenheit t
investigations of the preceding edition.					
below 32° F.					
593.8	50.31	543.6	1.3576	−0.1029	−22
590.4	51.07	539.3	1.3225	−0.0870	−13
586.4	51.79	534.8	1.2879	−0.0706	− 4
582.1	52.47	529.7	1.2536	−0.0536	5
577.4	53.15	524.3	1.2198	−0.0362	14
572.4	53.80	518.6	1.1864	−0.0183	23
results of L. Dieterici.					
above 32° F.					
557.5	53.57	503.8	1.1344	0.0000	32
548.5	53.87	494.6	1.0943	+0.0203	41
537.5	54.02	483.5	1.0544	+0.0405	50
526.7	54.13	472.5	1.0132	+0.0660	59
515.9	54.25	461.7	0.9741	+0.0804	68
503.8	54.22	449.6	0.9359	+0.1001	77
490.5	54.05	436.5	0.8984	+0.1196	86
474.8	53.64	421.2	0.8636	+0.1391	95
461.9	53.44	408.4	0.8252	+0.1583	104

ENGLISH UNITS.

Saturated Vapor of

1.	2.	3.	4.	5.	6.	7.	8.
Temperature Fahrenheit t ° F.	Steam Pressure Atmospheres	Steam Pressure Pounds per sq. in. p	$\frac{dp}{pdt}$	$\frac{r}{u}A = T\frac{dp}{dt}$	Heat of Liquid q	Latent Heat Outer Apu	Latent Heat Inner ρ
−22	0.379	5.564	0.029803	13.516	−16.675	13.178	159.419
−13	0.492	7.228	0.028344	17.041	−13.957	13.554	158.396
− 4	0.631	9.272	0.026999	21.242	−11.216	13.889	157.111
+ 5	0.800	11.756	0.025758	26.141	− 8.449	14.182	155.563
+14	1.003	14.745	0.024608	32.007	− 5.657	14.434	153.753
+23	1.246	18.311	0.023541	38.747	− 2.842	14.645	151.680
+32	1.533	22.530	0.022546	46.510	0.000	14.814	149.346
+41	1.870	27.483	0.021117	55.391	+ 2.866	14.942	146.748
+50	2.263	33.253	0.020748	65.485	+ 5.758	15.028	143.889
+59	2.717	39.931	0.019932	76.882	+ 8.674	15.073	140.767
+68	3.239	47.611	0.019164	89.662	+11.615	15.077	137.383
+77	3.837	56.386	0.018440	103.924	+14.582	15.057	133.718
+86	4.515	66.359	0.017756	119.740	+17.572	14.960	129.827
+95	5.283	77.630	0.017108	137.192	+20.588	14.839	125.656
+104	6.145	90.300	0.016493	156.358	+23.629	14.677	121.223

ENGLISH UNITS.

Sulphurous Acid. Table 9.

9. Total Latent Heat r	10. $\frac{r}{T}$	11. $\tau=\int_0^t \frac{cdt}{T}$	12. u	13. Specific Volume $v=u+\sigma$ $\sigma=0.0112$	14. Specific Weight $\gamma=\frac{1}{v}$	1. Temperature Fahrenheit t
172.597	0.3946	−0.0359	12.709	12.720	0.0786	− 22
171 950	0.3852	−0.0298	10.063	10.074	0.0993	− 13
171.000	0.3755	−0.0237	8.040	8.051	0.1242	− 4
169.745	0.3655	−0.0177	6.475	6.486	0.1542	+ 5
168.187	0.3553	−0.0117	5.254	5.265	0.1899	+ 14
166.325	0.3448	−0.0058	4.293	4.304	0.2324	+ 23
164.160	0.3341	0.0000	3.529	3.540	0.2825	+ 32
161.690	0.3231	+0.0058	2.919	2.930	0.3413	+ 41
158.917	0.3120	+0.0115	2.425	2.436	0.4105	+ 50
155.840	0.3006	+0.0172	2.026	2.037	0.4908	+ 59
152.460	0.2891	+0.0228	1.700	1.711	0.5845	+ 68
148.775	0.2773	+0.0284	1.434	1.445	0.6921	+ 77
144.787	0.2655	+0.0339	1.209	1.220	0.8192	+ 86
140.495	0.2534	+0.0394	1.025	1.036	0.9649	+ 95
135.900	0.2412	+0.0448	0.873	0.884	1.1310	+104

ENGLISH UNITS.

Saturated Vapor of

According to

1.	2.	3.	4.	5.	6.	7.	8.	9.
Temperature		Steam Pressure p				Specific Volume		
Fahrenheit t ° F.	Absolute T	Atmospheres	Pounds per sq. in.	$\frac{1}{\frac{dp}{pdt}}$	Pounds per sq. in. $\frac{dp}{dt}$	Vapor s	Liquid σ	$u = s - \sigma$
−22	437.4	14.5	213	56.88	3.77	0.432	0.0155	0.416
−13	446.4	16.9	249	58.90	4.23	0.367	0.0157	0.351
− 4	455.4	19.7	289	60.86	4.75	0.312	0.0160	0.296
+ 5	464.4	22.8	334	62.86	5.32	0.267	0.0163	0.251
+14	473.4	26.2	385	64.84	5.94	0.228	0.0167	0.211
+23	482.4	30.0	441	66.83	6.61	0.195	0.0171	0.178
+32	491.4	34.3	504	68.81	7.33	0.167	0.0176	0.149
+41	500.4	39.0	573	70.81	8.10	0.143	0.0181	0.125
+50	509.4	44.2	650	72.79	8.93	0.120	0.0187	0.101
+59	518.4	50.0	734	74.79	9.82	0.101	0.0197	0.081
+68	527.4	56.3	826	76.77	10.77	0.083	0.0210	0.062
+77	536.4	63.3	930	78.77	11.81	0.067	0.0227	0.044
+86	545.4	70.7	1040	80.75	12.88	0.048	0.0268	0.021
+87.8	547.2	72.3	1062	81.14	13.09	0.042	0.0298	0.012
+88.43	547.83	72.9	1071	81.29	13.19	0.035	0.0346	0.000

ENGLISH UNITS.

Carbonic Acid. Table 10.

Amagat and Mollier.

10.	11.	12.	13.	14.	15.	16.	17.
Heat of Liquid	Latent Heat				Entropy		
$q=\int_0^t Td\tau$	Total r	Outer Apu	Inner ρ	$\frac{r}{T}$	Liquid τ	Vapor $\tau+\frac{r}{T}$	Steam Heat $J=\rho+q$
−24.80	126.72	16.47	110.25	0.290	−0.053	0.236	85.45
−21.06	123.25	16.25	107.00	0.276	−0.045	0.231	85 94
−17.19	119.43	15.97	103.46	0.262	−0.03	0.226	86.27
−13.18	115.25	15.61	99.64	0.248	−0.028	0.221	86.46
− 9.00	110.65	15.16	95.49	0.234	−0.019	0.215	86.49
− 4.63	105.53	14.62	90.91	0. 19	−0.010	0.209	86.28
0.00	99.81	13.97	85.84	0.203	0.000	0.203	85.84
+ 4.93	93.35	13.21	80.14	0.187	+0.010	0.197	85.07
+10.28	85.93	12.28	73.65	0.169	+0 0 1	0.189	83.93
+16.22	77.20	11.14	66.06	0.149	+0.032	0.181	82.28
+23.08	66.47	9.67	56.80	0.126	+0.045	0.171	79.88
+31.63	52.16	7.67	44.49	0.097	+0.061	0.159	76.12
+45.45	27.00	4.00	23.00	0.050	+0.087	0.136	68.45
+51.61	15.12	2.25	12.87	0.028	+0.098	0.126	64.48
+59.24	0.00	0.00	0.00	0.000	+0.112	0.112	59.24

ENGLISH UNITS.

First Principal Table for Vapor of Water. Table 11a.

1. Pressure in Atmospheres p 1 atm. = 14.6967 lb. per sq. in.	2. Temperature Fahrenheit t	3. Heat of Liquid q	4. Latent Heat Inner ρ	5. Latent Heat Outer Apu	6. Values of u	7. Values of $\frac{\rho}{u}$	8. Specific Weight lb. per cu. ft. γ
0.02	64.09	32.108	1010.333	59.047	1078.08	0.93715	0.00093
0.04	84.83	52.875	993.931	61.007	556.94	1.7847	0.00180
0.06	97.81	65.882	983.666	62.224	378.69	2.5976	0.00264
0.08	107.46	75.559	976.034	63.121	288.12	3.3875	0.00347
0.10	115.18	83.308	969.926	63.835	233.10	4.1610	0.00429
0.12	121.69	89.851	964.771	64.435	196.08	4.9203	0.00510
0.15	129.87	98.059	958.307	65.183	158.68	6.0392	0.00630
0.2	140.81	109.060	949.651	66.175	120.82	7.8598	0.00828
0.3	157.08	125.437	936.779	67.633	82.323	11.380	0.01214
0.4	169.25	137.698	927.155	68.708	62.723	14.781	0.01594
0.5	179.08	147.631	919.381	69.547	50.791	18.101	0.01968
0.6	187.38	155.992	912.818	70.281	42.774	21.340	0.02337
0.7	194.58	163.267	907.123	70.897	36.983	24.528	0.02703
0.8	200.98	169.747	902.054	71.438	32.609	27.664	0.03065
0.9	206.74	175.577	897.498	71.923	29.182	30.755	0.03425
1.0	212.00	180.900	893.340	72.360	26.422	33.810	0.03782
1.1	216.82	185.789	889.524	72.758	24.153	36.830	0.04138
1.2	221.31	190.332	885.978	73.127	22.253	39.814	0.04491
1.3	225.50	194.587	882.661	73.469	20.637	42.772	0.04842
1.4	229.42	198.569	879.557	73.787	19.246	45.700	0.05192
1.5	233.13	202.334	876.625	74.086	18.035	48.607	0.05540
1.6	236.64	205.900	873.848	74.367	16.973	51.484	0.05887
1.7	239.97	209.284	871.214	74.633	16.031	54.345	0.06232
1.8	243.14	212.506	868.709	74.884	15.192	57.182	0.06576
1.9	246.18	215.602	866.302	75.121	14.438	60.004	0.06919
2.0	249.08	218.551	864.009	75.350	13.758	62.800	0.07260
2.1	251.87	221.391	861.802	75.566	13.140	65.586	0.07601
2.2	254.55	224.123	859.682	75.773	12.578	68.350	0.07940
2.3	257.13	226.746	857.646	75.973	12.062	71.104	0.08280
2.4	259.63	229.295	855.666	76.165	11.589	73.832	0.08617
2.5	262.04	231.755	853.758	76.349	11.152	76.556	0.08954
2.8	264.38	234.142	851.908	76.527	10.748	79.260	0.09290
2.7	266.63	236.437	850.127	76.698	10.374	81.952	0.09625
2.8	268.83	238.678	848.390	76.864	10.024	84.633	0.09960
2.9	270.97	240.865	846.697	77.024	9.699	87.297	0.10293

ENGLISH UNITS.

First Principal Table for Vapor of Water. Table 11b.

1. Pressure in Atmospheres p 1 atm. = 14.6967 lb. per sq. in.	2. Temperature Fahrenheit t	3. Heat of Liquid q	4. Latent Heat Inner ρ	5. Latent Heat Outer Apu	6. Values of u	7. Values of $\frac{\rho}{u}$	8. Specific Weight lb. per cu. ft. γ
3.0	273.04	242.980	845.059	77.177	9.395	89.950	0.10626
3.1	275.05	245.039	843.464	77.328	9.108	92.606	0.10959
3.2	277.02	247.045	841.912	77.472	8.840	95.233	0.11292
3.3	278.94	249.014	840.389	77.614	8.589	97.845	0.11622
3.4	280.81	250.927	838.908	77.753	8.350	100.463	0.11953
3.5	282.63	252.788	837.470	77.884	8.125	103.079	0.12283
3.6	284.41	254.610	836.060	78.016	7.913	105.66	0.12612
3.7	286.18	256.415	834.665	78.143	7.712	108.24	0.12940
3.8	287.87	258.149	833.326	78.264	7.521	110.81	0.13268
3.9	289.54	259.862	832.003	78.386	7.338	113.37	0.13596
4.0	291.20	261.558	830.693	78.505	7.167	115.92	0.13923
4.1	292.80	263.200	829.426	78.619	7.002	118.46	0.14249
4.2	294.37	264.805	828.187	78.730	6.845	120.99	0.14576
4.3	295.90	266.373	826.976	78.838	6.694	123.53	0.14902
4.4	297.43	267.943	825.766	78.946	6.552	126.04	0.15226
4.5	298.92	269.474	824.585	79.052	6.414	128.55	0.15551
4.6	300.38	270.970	823.432	79.155	5.283	131.05	0.15875
4.7	301.82	272.448	822.292	79.254	6.158	133.55	0.16199
4.8	303.24	273.908	821.167	79.353	6.036	136.03	0.16522
4.9	304.63	275.330	820.071	79.450	5.921	138.52	0.16846
5.0	306.00	276.734	818.989	79.546	5.808	140.99	0.17168
5.1	307.35	278.122	817.922	79.637	5.701	143.45	0.17490
5.2	308.66	279.472	816.881	79.727	5.599	145.91	0.17812
5.3	309.97	280.822	815.843	79.817	5.499	148.36	0.18134
5.4	311.25	282.134	814.831	79.906	5.403	150.81	0.18454
5.5	312.53	283.448	813.821	79.994	5.310	153.25	0.18774
5.6	313.77	284.726	812.839	80.077	5.221	155.67	0.19094
5.7	315.00	285.984	811.870	80.159	5.134	158.10	0.19414
5.8	316.22	287.242	810.902	80.242	5.051	160.52	0.19734
5.9	317.41	288.466	809.962	80.321	4.971	162.93	0.20053
6.0	318.60	289.688	809.023	80.401	4.892	165.35	0.20373
6.1	319.77	290.893	808.097	80.478	4.817	167.75	0.20690
6.2	320.90	292.059	807.199	80.555	4.744	170.15	0.21008
6.3	322.05	293.247	806.288	80.629	4.671	172.54	0.21326
6.4	323.17	294.395	805.406	80.705	4.604	174.93	0.21644

ENGLISH UNITS.

First Principal Table for Vapor of Water. Table 11c.

1. Pressures in Atmospheres p 1 atm. = 14.6967 lb. per sq. in.	2. Temperature Fahrenheit t	3. Heat of Liquid q	4. Latent Heat Inner ρ	5. Latent Heat Outer Apu	6. Values of u	7. Values of $\frac{\rho}{u}$	8. Specific Weight lb. per cu. ft. γ
3.5	324.27	295.526	804.537	80.777	4.538	177.31	0.21961
6.6	325.36	296.658	803.669	80.849	4.472	179.69	0.22278
6.7	326.44	297.770	802.814	80.921	4.410	182.06	0.22594
6.8	327.52	298.885	801.961	80.989	4.349	184.43	0.22910
6.9	328.57	299.961	801.135	81.058	4.290	186.79	0.23227
7.00	329.61	301.037	800.309	81.126	4.232	189.12	0.23542
7.25	332.19	303.692	798.273	81.292	4.094	194.99	0.24330
7.50	334.67	306.256	796.307	81.450	3.965	200.82	0.25117
7.75	337.10	308.763	794.385	81.607	3.845	206.62	0.25901
8.00	339.46	311.198	792.520	81.756	3.731	212.39	0.26685
8.25	341.78	313.598	790.684	81.902	3.625	218.14	0.27466
8.50	344.03	315.925	788.904	82.040	3.524	223.86	0.28247
8.75	346.23	318.195	787.167	82.177	3.430	229.55	0.29026
9.00	348.39	320.431	785.459	82.309	3.340	235.23	0.29804
9.25	350.49	322.610	783.792	82.436	3.253	240.87	0.30580
9.50	352.54	324.734	782.170	82.562	3.173	246.49	0.31355
9.75	354.58	326.842	780.561	82.683	3.096	252.08	0.32129
10.00	356.56	328.894	778.995	82.802	3.023	257.67	0.32902
10.25	358.48	330.890	777.470	82.915	2.954	263.23	0.33674
10.50	360.39	332.869	775.962	83.029	2.886	268.76	0.34444
10.75	362.26	334.809	774.481	83.140	2.824	274.26	0.35212
11.00	364.10	336.717	773.028	83.245	2.763	279.75	0.35980
11.25	365.92	338.639	771.590	83.351	2.706	285.22	0.36747
11.50	367.68	340.436	770.195	83.452	2.650	290.68	0.37514
11.75	369.43	342.250	768.814	83.551	2.597	296.12	0.38279
12.00	371.14	344.027	767.462	83.648	2.545	301.54	0.39045
12.25	372.83	345.787	766.123	83.743	2.496	306.93	0.39807
12.50	374.49	347.508	764.813	83.837	2.449	312.31	0.40569
12.75	376.12	349.213	763.519	83.927	2.403	317.68	0.41330
13.00	377.74	350.899	762.237	84.017	2.360	323.01	0.42091
13.25	379.33	352.548	760.984	84.103	2.318	328.35	0.42851
13.50	380.89	354.179	759.744	84.190	2.276	333.66	0.43610
13.75	382.44	355.720	758.520	84.272	2.238	338.97	0.44368
14.00	383.75	357.367	757.325	84.355	2.199	344.24	0.45125

ENGLISH UNITS.

Second Principal Table for Vapor of Water. Table 12a.

1. Press. in kg. per sq. cm. (Abs.)	1′. Press. in lb. per sq. in.	2. Temp. Fahrenheit t	3. Heat of Liquid q	4. Latent Heat Inner ρ	5. Latent Heat Outer Apu	6. Values of u	7. Values of $\frac{\rho}{u}$	8. Specific Weight $\gamma=\frac{1}{v}$	9. $\tau=\int_0^t \frac{cdt}{T}$	10. $\frac{r}{T}$
0.1	1.4223	114.042	82.168	970.825	63.731	240.469	4.038	0.00416	0.15463	1.8041
0.2	2.8447	139.559	107.802	950.641	66.062	124.633	7.628	0.00802	0.19836	1.6975
0.3	4.2670	155.736	124.081	937.845	67.513	84.912	11.045	0.01177	0.22518	1.6344
0.4	5.6894	167.841	136.278	928.271	68.582	64.693	14.349	0.01545	0.24482	1.5893
0.5	7.1117	177.618	146.140	920.536	69.437	52.399	17.568	0.01908	0.26042	1.5541
0.6	8.5340	185.871	154.472	914.008	70.150	44.115	20.719	0.02266	0.27341	1.5252
0.7	9.9564	193.044	161.719	908.334	70.765	38.145	23.813	0.02621	0.28458	1.5007
0.8	11.3787	199.405	168.169	903.303	71.287	33.623	26.866	0.02973	0.29439	1.4793
0.9	12.8011	205.137	173.950	898.769	71.788	30.097	29.863	0.03321	0.30316	1.4605
1.0	14.2234	210.358	179.237	894.638	72.225	27.252	32.828	0.03667	0.31108	1.4436
1.1	15.6457	215.164	184.106	890.836	72.623	24.912	35.760	0.04012	0.31833	1.4283
1.2	17.0681	219.623	188.626	887.310	72.990	22.951	38.661	0.04354	0.32500	1.4142
1.3	18.4904	223.786	192.848	884.018	73.328	21.284	41.536	0.04695	0.33120	1.4013
1.4	19.9128	227.691	196.810	880.929	73.647	19.848	44.383	0.05034	0.33699	1.3893
1.5	21.3351	231.373	200.549	878.015	73.946	18.601	47.204	0.05371	0.34241	1.3781
1.6	22.7574	234.858	204.088	875.259	74.225	17.505	50.001	0.05707	0.34752	1.3676
1.7	24.1798	238.170	207.454	872.640	74.488	16.533	52.783	0.06042	0.35236	1.3578
1.8	25.6021	241.322	210.658	870.147	74.738	15.666	55.544	0.06376	0.35694	1.3484
1.9	27.0245	244.339	213.727	867.760	74.977	14.889	58.281	0.06709	0.36131	1.3394
2.0	28.4468	247.226	216.664	865.476	75.204	14.188	61.003	0.07041	0.36548	1.3312
2.1	29.8691	249.996	219.483	863.285	75.420	13.552	63.704	0.07370	0.36946	1.3232
2.2	31.2915	252.662	222.197	861.176	75.647	12.978	66.356	0.07696	0.37328	1.3156
2.3	32.7138	255.231	224.815	859.145	75.825	12.440	69.064	0.08028	0.37695	1.3083
2.4	34.1362	257.711	228.342	857.183	76.016	11.951	71.723	0.08356	0.38048	1.3013
2.5	35.5585	260.107	229.784	855.288	76.199	11.501	74.365	0.08683	0.38388	1.2946
2.6	36.9808	262.427	232.150	853.452	76.379	11.085	76.993	0.09008	0.38716	1.2882
2.7	38.4032	264.675	234.443	851.674	76.550	10.697	79.617	0.09334	0.39033	1.2819
2.8	39.8255	266.857	236.669	849.947	76.714	10.338	82.214	0.09658	0.39340	1.2759
2.9	41.2479	268.975	238.831	848.272	76.874	10.002	84.811	0.09982	0.39638	1.2701
3.0	42.6702	271.036	240.935	846.643	77.027	9.688	87.392	0.10305	0.39926	1.2645
3.1	44.0925	273.043	242.986	845.055	77.177	9.393	89.965	0.10628	0.40206	1.2591
3.2	45.5149	274.998	244.984	843.509	77.323	9.118	92.514	0.10948	0.40479	1.2539
3.3	46.9372	276.903	246.929	842.002	77.463	8.857	95.071	0.11270	0.40743	1.2488
3.4	48.3596	278.762	248.830	840.532	77.600	8.610	97.605	0.11590	0.41001	1.2438
3.5	49.7819	280.578	250.688	839.095	77.733	8.381	100.121	0.11909	0.41252	1.2390
3.6	51.2042	282.353	252.502	837.691	77.913	8.161	102.641	0.12227	0.41497	1.2343
3.7	52.6266	284.088	254.277	836.318	77.992	7.953	105.155	0.12548	0.41737	1.2298
3.8	54.0489	285.786	256.014	834.975	78.116	7.756	107.654	0.12866	0.41970	1.2253
3.9	55.4713	287.447	257.715	833.661	78.235	7.569	110.146	0.13184	0.42198	1.2210

The formulas underlying these tabular values are given at bottom of the corresponding metric table 12*a*, page XXVII.

ENGLISH UNITS.

Second Principal Table for Vapor of Water. Table 12b.

1. Press. in kg. per sq. cm. (Abs.)	1'. Press. in lb. per sq. in.	2. Temp. Fahrenheit t	3. Heat of Liquid q	4. Latent Heat Inner ρ	5. Latent Heat Outer Apu	6. Values of u	7. Values of $\frac{\rho}{u}$	8. Specific Weight $\gamma=\frac{1}{v}$	9. $\tau=\int_0^t \frac{cdt}{T}$	10. $\frac{r}{T}$
4.0	56.8936	289.076	259.384	832.372	78.352	7.391	112.622	0.13500	0.42421	1.2168
4.1	58.3159	290.673	261.018	831.110	78.467	7.221	115.095	0.13817	0.42639	1.2127
4.2	59.7383	292.237	262.622	829.872	78.579	7.059	117.557	0.14133	0.42853	1.2086
4.3	61.1606	293.772	264.195	828.659	78.687	6.906	119.999	0.14447	0.43062	1.2047
4.4	62.5830	295.279	265.739	827.467	78.793	6.757	122.469	0.14764	0.43267	1.2009
4.5	64.0053	296.758	267.255	826.295	78.901	6.616	124.901	0.15078	0.43467	1.1971
4.6	65.4276	298.211	268.745	825.147	79.002	6.481	127.317	0.15391	0.43664	1.1934
4.7	66.8500	299.640	270.211	824.017	79.103	6.350	129.773	0.15708	0.43858	1.1898
4.8	68.2723	301.044	271.652	822.906	79.200	6.226	132.165	0.16019	0.44047	1.1863
4.9	69.6947	302.425	273.069	821.813	79.297	6.106	134.587	0.16333	0.44233	1.1828
5.0	71.1170	303.784	274.464	820.739	79.391	5.991	136.998	0.16647	0.44416	1.1794
5.1	72.5393	305.121	275.836	819.680	79.486	5.880	139.393	0.16959	0.44596	1.1761
5.2	73.9617	306.437	277.187	818.640	79.576	5.775	141.765	0.17268	0.44773	1.1729
5.3	75.3840	307.733	278.519	817.616	79.662	5.672	144.147	0.17579	0.44946	1.1697
5.4	76.8064	309.011	279.832	816.604	79.753	5.573	146.532	0.17891	0.45117	1.1665
5.5	78.2287	310.269	281.124	815.608	79.839	5.477	148.924	0.18204	0.45285	1.1634
5.6	79.6510	311.508	282.398	814.630	79.922	5.385	151.267	0.18513	0.45451	1.1604
5.7	81.0734	312.730	283.655	813.663	80.005	5.296	153.647	0.18825	0.45613	1.1574
5.8	82.4957	313.936	284.893	812.709	80.087	5.211	155.966	0.19131	0.45774	1.1545
5.9	83.9181	315.126	286.117	811.768	80.168	5.127	158.316	0.19440	0.45932	1.1516
6.0	85.3404	316.299	287.325	810.839	80.248	5.046	160 696	0.19754	0.46088	1.1488
6.1	86.7627	317.457	288.517	809.924	80.323	4.969	162.999	0.20059	0.46241	1.1460
6.2	88.1851	318.600	289.692	809.019	80.402	4.894	165.321	0.20366	0.46392	1.1432
6.3	89.6074	319.728	290.853	808.126	80.476	4.820	167.663	0.20676	0.46542	1.1405
6.4	91.0298	320.841	292.000	807.248	80.548	4.749	169.967	0.20982	0.46689	1.1378
6.5	92.4521	321.942	293.134	806.377	80.622	4.681	172.282	0.21290	0.46834	1.1352
6.6	93.8744	323.029	294.253	805.516	80.694	4.613	174.608	0.21599	0.46977	1.1326
6.7	95.2968	324.102	295.358	804.667	80.766	4.549	176.880	0.21902	0.47118	1.1301
6.8	96.7191	325.164	296.453	803.828	80.836	4.485	179.220	0.22214	0.47258	1.1276
6.9	98.1415	326.214	297.533	802.996	80.906	4.424	181.498	0.22519	0.47395	1.1251
7.0	99.5638	327.250	298.602	802.177	80.973	4.365	183.775	0.22823	0.47531	1.1227
7.1	100.9861	328.276	299.660	801.365	81.038	4.307	186.047	0.23128	0.47666	1.1203
7.2	102.4085	329.290	300.704	800.564	81.104	4.250	188.382	0.23440	0.47798	1.1179
7.3	103.8308	330.292	301.739	799.771	81.169	4.195	190.638	0.23743	0.47929	1.1155
7.4	105.2532	331.286	302.764	798.984	81.234	4.142	192.881	0.24045	0.48059	1.1132
7.5	106.6755	332.267	303.775	798.208	81.299	4.089	195.185	0.24355	0.48187	1.1110
7.6	108.0978	333.239	304.778	797.440	81.360	4.040	197.394	0.24652	0.48314	1.1087
7.7	109.5202	334.200	307.770	796.680	81.421	3.990	199.661	0.24958	0.48439	1.1065
7.8	110.9425	335.152	306.752	795.926	81.482	3.942	201.903	0.25261	0.48562	1.1043
7.9	112.3649	336.094	307.724	795.182	81.542	3.894	204.203	0.25571	0.48685	1.1021
8.0	113.7872	337.026	308.687	794.444	81.601	3.849	206.392	0.25868	0.48806	1.1000
8.1	115.2095	337.950	309.641	793.714	81.659	3.804	208.632	0.26172	0.48925	1.0979
8.2	116.6319	338.864	310.586	792.990	81.716	3.760	210.929	0.26483	0.49044	1.0958
8.3	118.0542	339.769	311.521	792.274	81.774	3.718	213.099	0.26778	0.49161	1.0937
8.4	119.4766	340.668	312.449	791.564	81.830	3.676	215.321	0.27080	0.49277	1.0917

ENGLISH UNITS.

Second Principal Table for Vapor of Water. Table 12c.

1. Press. in kg. per sq. cm. (Abs.)	1'. Press in lb. per sq. in.	2. Temp. Fahrenheit t	3. Heat of Liquid q	4. Latent Heat Inner ρ	5. Latent Heat Outer Apu	6. Values of u	7. Values of $\frac{\rho}{u}$	8. Specific Weight $\gamma=\frac{1}{v}$	9. $\tau=\int_0^t \frac{cdt}{T}$	10. $\frac{r}{T}$
8.5	120.899	341.557	313.367	790.861	81.886	3.634	217.594	0.27389	0.49392	1.0896
8.6	122.321	342.437	314.278	790.164	81.941	3.594	219.825	0.27693	0.49505	1.0876
8.7	123.744	343.310	315.180	789.475	81.995	3.556	222.007	0.27990	0.49618	1.0857
8.8	125.166	344.174	316.073	788.791	82.049	3.518	224.239	0.28295	0.49729	1.0837
8.9	126.588	345.031	316.960	788.112	82.102	3.481	226.417	0.28593	0.49839	1.0818
9.0	128.011	345.882	317.840	787.439	82.156	3.444	228.645	0.28897	0.49948	1.0799
9.1	129.433	346.723	318.710	786.775	82.206	3.409	230.813	0.29195	0.50056	1.0780
9.2	130.855	347.558	319.574	786.114	82.258	3.375	233.028	0.29498	0.50164	1.0761
9.3	132.278	348.386	320.431	785.459	82.309	3.340	235.179	0.29794	0.50270	1.0743
9.4	133.700	349.207	321.280	784.809	82.359	3.306	237.376	0.30095	0.50375	1.0724
9.5	135.122	350.022	322.124	784.165	82.408	3.273	239.619	0.30403	0.50479	1.0706
9.6	136.545	350.829	322.960	783.527	82.456	3.241	241.791	0.30702	0.50582	1.0688
9.7	137.967	351.630	323.788	782.893	82.505	3.210	243.885	0.30992	0.50684	1.0670
9.8	139.389	352.425	324.612	782.264	82.554	3.178	246.146	0.31302	0.50786	1.0653
9.9	140.812	353.212	325.427	781.461	82.602	3.148	248.329	0.31604	0.50886	1.0635
10.00	142.234	353.995	326.237	781.022	82.649	3.119	250.425	0.31894	0.50986	1.0618
10.25	145.790	355.923	328.235	779.497	82.764	3.047	255.850	0.32644	0.51231	1.0576
10.50	149.346	357.814	330.196	778.001	82.876	2.978	261.266	0.33395	0.51472	1.0534
10.75	152.902	359.672	332.123	776.531	82.985	2.912	266.653	0.34144	0.51707	1.0494
11.00	156.457	361.495	334.013	775.089	83.093	2.850	271.993	0.34888	0.51938	1.0454
11.25	160.013	363.288	335.875	773.671	83.198	2.790	277.262	0.35625	0.52164	1.0415
11.50	163.569	365.049	337.702	772.279	83.299	2.733	282.603	0.36372	0.52386	1.0378
11.75	167.125	366.780	339.500	770.909	83.399	2.678	287.838	0.37107	0.52604	1.0340
12.00	170.681	368.573	341.269	769.561	83.497	2.625	293.121	0.37850	0.52818	1.0304
12.25	174.237	370.159	343.010	768.236	83.592	2.574	298.443	0.38598	0.53028	1.0268
12.50	177.793	371.808	344.723	766.931	83.687	2.526	303.603	0.39327	0.53234	1.0234
12.75	181.348	373.433	346.414	765.646	83.777	2.480	308.773	0.40059	0.53437	1.0199
13.00	184.904	375.031	348.077	764.383	83.866	2.435	313.942	0.40792	0.53637	1.0166
13.25	188.460	376.608	349.717	763.135	83.954	2.390	319.311	0.41552	0.53833	1.0133
13.50	192.016	378.160	351.331	761.908	84.040	2.348	324.452	0.42284	0.54026	1.0100
13.75	195.572	379.692	352.926	760.696	84.123	2.308	329.556	0.43012	0.54216	1.0068
14.00	199.128	381.202	354.499	759.501	84.206	2.269	334.610	0.43735	0.54404	1.0037
14.25	202.684	382.690	356.051	758.322	84.287	2.231	339.848	0.44483	0.54588	1.0006
14.50	206.239	384.159	357.581	757.161	84.366	2.195	345.024	0.45224	0.54770	0.9976
14.75	209.795	385.608	359.091	756.016	84.443	2.159	350.124	0.45957	0.54949	0.9946
15.00	213.351	387.032	360.583	754.884	84.519	2.126	355.133	0.46678	0.55125	0.9917
16.00	227.574	392.576	366.359	750.505	84.812	2.001	375.232	0.49586	0.55805	0.9804
17.00	241.798	397.868	371.997	746.318	85.082	1.885	395.708	0.52549	0.56452	0.9697
18.00	256.021	402.926	377.179	742.318	85.334	1.789	414.980	0.55344	0.57068	0.9598
19.00	270.245	407.750	382.230	738.502	85.572	1.700	434.573	0.58236	0.57652	0.9503
20.00	284.468	412.412	387.117	734.814	85.795	1.618	453.974	0.61144	0.58214	0.9413

ENGLISH UNITS.

Auxiliary Table for

1. Pressure in Atmospheres p 1 Atmosphere = 2116.3 ft.	2. T	3. $\frac{dp}{dt}$ Pounds per sq. ft.	4. $\frac{1}{A}\log_e T$	5. $\frac{\tau}{A}$	6 $\frac{r}{AT}$
0.5	638.48	23.60	4991.76	203.044	1197.016
1	671.40	42.06	5030.53	242.336	1111.588
2	708.48	74.40	5072.16	284.524	1024.675
3	732.44	103.46	5097.86	310.731	973.089
4	750.60	130.46	5116.80	330.097	936.118
5	765.40	156.00	5131.88	345.554	907.261
6	778.00	180.40	5144.51	358.522	883.511
7	789.01	203.82	5155.36	369.728	863.356
8	798.86	226.43	5164.96	379.628	845.790
9	807.79	248.42	5173.43	388.515	830.209
10	815.96	269.78	5181.33	396.568	816.244
11	823.50	290.60	5188.44	403.933	803.585
12	830.54	310.93	5195.01	410.772	791.969
13	837.14	330.82	5201.18	417.141	781.235
14	843.35	350.31	5206.82	423.092	771.296

* For French Units see

ENGLISH UNITS.

Saturated Steam. Table 13.*

7. $\frac{1}{A}\left(\tau+\frac{r}{T}\right)$	8. $\int_0^t h dt$	9. $\frac{1}{A}\frac{dq}{dp}$	10. $\frac{1}{A}\frac{d}{dp}\left(\frac{\rho}{u}\right)$	11. $\frac{d}{dt}\left(\frac{\rho}{u}\right)$	1. Pressure in Atmospheres p
1400.060	−228.145	33.0486	11.976	0.366	0.5
1353.924	−267.246	18.6108	11.088	0.604	1
1309.199	−307.150	10.5725	10.194	0.982	2
1283.820	−330.800	7.6298	9.765	1.295	3
1266.215	−347.693	6.0680	9.299	1.570	4
1252.815	−360.823	5.0860	9.014	1.820	5
1242.033	−371.509	4.4080	8.774	2.048	6
1233.084	−380.666	3.9086	8.557	2.257	7
1225.418	−388.552	3.5258	8.409	2.464	8
1218.724	−395.507	3.2182	8.264	2.657	9
1212.812	−401.720	2.9683	8.143	2.843	10
1207.518	−407.32	2.7601	8.004	3.010	11
1202.741	−412.441	2.5839	7.896	3.177	12
1198.373	−417.154	2.4317	7.794	3.337	13
1194.388	−421.497	2.2987	7.699	3.490	14

pp. 62 and 63 of Vol. II.

Auxiliary Table for Converting Pounds per Square Inch into Atmospheres. Table 14.

Lb. per sq. in.	Atmospheres	Lb. per sq. in.	Atmospheres	Lb. per sq. in.	Atmospheres	Lb. per sq. in.	Atmospheres	Lb. per sq. in.	Atmospheres	Lb. per sq. in.	Atmospheres	Lb. per sq. in.	Atmospheres
0.02	0.00136	8.0	0.54433	20	1.3608	80	5.4433	140	9.5257	200	13.608	260	17.691
0.03	0.00204	8.2	0.55794	21	1.4289	81	5.5113	141	9.5938	201	13.676	261	17.759
0.04	0.00272	8.4	0.57154	22	1.4969	82	5.5794	142	9.6618	202	13.744	262	17.827
0.06	0.00408	8.6	0.58515	23	1.5649	83	5.6474	143	9.7299	203	13.812	263	17.895
0.08	0.00544	8.8	0.59876	24	1.6330	84	5.7154	144	9.7979	204	13.880	264	17.963
0.10	0.00680	9.0	0.61237	25	1.7010	85	5.7835	145	9.8659	205	13.948	265	18.031
0.15	0.01021	9.2	0.62598	26	1.7691	86	5.8515	146	9.9340	206	14.016	266	18.099
0.20	0.01361	9.4	0.63958	27	1.8371	87	5.9196	147	10.0020	207	14.084	267	18.167
0.30	0.02041	9.6	0.65319	28	1.9051	88	5.9876	148	10.0701	208	14.153	268	18.235
0.40	0.02722	9.8	0.66680	29	1.9732	89	6.0556	149	10.1381	209	14.221	269	18.303
0.50	0.03402	10.0	0.68041	30	2.0412	90	6.1237	150	10.206	210	14.289	270	18.371
0.60	0.04082	10.2	0.69402	31	2.1093	91	6.1917	151	10.274	211	14.357	271	18.439
0.70	0.04763	10.4	0.70763	32	2.1773	92	6.2598	152	10.342	212	14.425	272	18.507
0.80	0.05443	10.6	0.72123	33	2.2454	93	6.3278	153	10.410	213	14.493	273	18.575
0.90	0.06124	10.8	0.73484	34	2.3134	94	6.3958	154	10.478	214	14.561	274	18.643
1.00	0.06804	11.0	0.74845	35	2.3814	95	6.4639	155	10.546	215	14.629	275	18.711
1.10	0.07484	11.2	0.76206	36	2.4495	96	6.5319	156	10.614	216	14.691	276	18.779
1.20	0.08165	11.4	0.77567	37	2.5175	97	6.6000	157	10.682	217	14.765	277	18.847
1.30	0.08845	11.6	0.78928	38	2.5856	98	6.6680	158	10.750	218	14.833	278	18.915
1.40	0.09526	11.8	0.80288	39	2.6536	99	6.7360	159	10.819	219	14.901	279	18.983
1.50	0.10206	12.0	0.81649	40	2.7216	100	6.8041	160	10.887	220	14.969	280	19.051
1.60	0.10887	12.2	0.83010	41	2.7897	101	6.8721	161	10.955	221	15.037	281	19.119
1.70	0.11567	12.4	0.84371	42	2.8577	102	6.9402	162	11.023	222	15.105	282	19.188
1.80	0.12247	12.6	0.85732	43	2.9258	103	7.0082	163	11.091	223	15.173	283	19.256
1.90	0.12928	12.8	0.87092	44	2.9938	104	7.0763	164	11.159	224	15.241	284	19.324
2.00	0.13608	13.0	0.88453	45	3.0618	105	7.1443	165	11.227	225	15.309	285	19.392
2.10	0.14289	13.2	0.89814	46	3.1299	106	7.2123	166	11.295	226	15.377	286	19.460
2.20	0.14969	13.4	0.91175	47	3.1979	107	7.2804	167	11.363	227	15.445	287	19.528
2.30	0.15649	13.6	0.92536	48	3.2660	108	7.3484	168	11.431	228	15.513	288	19.596
2.40	0.16330	13.8	0.93897	49	3.3340	109	7.4165	169	11.499	229	15.581	289	19.664
2.50	0.17010	14.0	0.95257	50	3.4020	110	7.4845	170	11.567	230	15.649	290	19.732
2.60	0.17691	14.2	0.96618	51	3.4701	111	7.5525	171	11.635	231	15.717	291	19.800
2.70	0.18371	14.4	0.97979	52	3.5381	112	7.6206	172	11.703	232	15.785	292	19.868
2.80	0.19051	14.6	0.99340	53	3.6062	113	7.6886	173	11.771	233	15.854	293	19.936
2.90	0.19732	14.8	1.00701	54	3.6742	114	7.7567	174	11.839	234	15.922	294	20.004
3.00	0.20412	15.0	1.02061	55	3.7422	115	7.8247	175	11.907	235	15.990	295	20.072
3.20	0.21773	15.2	1.03422	56	3.8103	116	7.8928	176	11.975	236	16.058	296	20.140
3.40	0.23134	15.4	1.04783	57	3.8783	117	7.9608	177	12.043	237	16.126	297	20.208
3.60	0.24495	15.6	1.06144	58	3.9464	118	8.0288	178	12.111	238	16.194	298	20.276
3.80	0.25856	15.8	1.07505	59	4.0144	119	8.0969	179	12.179	239	16.262	299	20.344
4.00	0.27216	16.0	1.0887	60	4.0825	120	8.1649	180	12.247	240	16.330		
4.20	0.28577	16.2	1.1023	61	4.1505	121	8.2330	181	12.315	241	16.398		
4.40	0.29938	16.4	1.1159	62	4.2185	122	8.3010	182	12.383	242	16.466		
4.60	0.31299	16.6	1.1295	63	4.2866	123	8.3690	183	12.451	243	16.534		
4.80	0.32660	16.8	1.1431	64	4.3546	124	8.4371	184	12.520	244	16.602		
5.00	0.34020	17.0	1.1567	65	4.4227	125	8.5051	185	12.588	245	16.670		
5.20	0.35381	17.2	1.1703	66	4.4907	126	8.5732	186	12.656	246	16.738		
5.40	0.36742	17.4	1.1839	67	4.5587	127	8.6412	187	12.724	247	16.806		
5.60	0.38103	17.6	1.1975	68	4.6268	128	8.7092	188	12.792	248	16.874		
5.80	0.39464	17.8	1.2111	69	4.6948	129	8.7773	189	12.860	249	16.942		
6.0	0.40825	18.0	1.2247	70	4.7629	130	8.8453	190	12.928	250	17.010		
6.2	0.42185	18.2	1.2383	71	4.8309	131	8.9134	191	12.996	251	17.078		
6.4	0.43546	18.4	1.2520	72	4.8989	132	8.9814	192	13.064	252	17.146		
6.6	0.44907	18.6	1.2656	73	4.9670	133	9.0494	193	13.132	253	17.214		
6.8	0.46268	18.8	1.2792	74	5.0350	134	9.1175	194	13.200	254	17.282		
7.0	0.47629	19.0	1.2928	75	5.1031	135	9.1855	195	13.268	255	17.350		
7.2	0.48989	19.2	1.3064	76	5.1711	136	9.2536	196	13.336	256	17.418		
7.4	0.50350	19.4	1.3200	77	5.2391	137	9.3216	197	13.404	257	17.487		
7.6	0.51711	19.6	1.3336	78	5.3072	138	9.3897	198	13.472	258	17.555		
7.8	0.53072	19.8	1.3472	79	5.3752	139	9.4577	199	13.540	259	17.623		

INDEX.

VOLUME II.

A

Actual cold-vapor engines, Cycle of, 478
Actual cold-vapor engines, Superheating in compressor of, 484
Actual steam-engine cycle, 396
Adiabatic change and equality of steam weight, 79
Adiabatic change in mixture of air and wet steam, 333
Adiabatic curve for wet steam a polytropic, 81, 83
Adiabatic curve of wet vapor, 75
Adiabatic expansion, Non-reversible, 124
Adiabatic expansion of steam in cylinder, 116
Adiabatic for superheated steam, 231, 234, 274
Adiabatic production of work, 84
Air and steam mixtures, Behavior of, 320
Air as a liquid and vapor, 303
Air liquefaction, Linde's machine for, 303, 308
Air's equation of condition extended, 313
Air-volume in jet condensers, 141
Amagat's experiments on carbonic acid, 263
Ammonia-vapor, 252
Ammonia-vapor data from Dieterici, 255
Ammonia-vapor, equation of condition for, 253
Andrew's experiments, 209
Apparatus for superheating, 281
Auxiliary table (English) for saturated steam, App.
Auxiliary table (French) for saturated steam, 62, 63

B

Battelli's experiments on volumes of superheated steam, 240, 242
Battelli's investigations of the vapor of water, 48
Behavior of mixture of air and steam, 320
Behavior of mixture of air and unsaturated vapor, 338
Behavior of mixture of air, steam and ice, 350
Behavior of saturated and superheated steam, Recent researches on, 357
Behavior of saturated vapors, 9
Behavior of superheated vapors, 202
Behavior of unsaturated vapor, 202
Boiler-explosion problem, 94
Boiler pressure, Velocity of rise of, 96

C

Cailletet's experiments on sulphurous acid, 257
Calorimetric investigation of engines, 419
Calorimetric investigation of injector, 142
Carbonic acid, Amagat's experiments on, 263
Carbonic acid, Equation of condition of, 263

Carbonic acid, Formulas for Entropy and Energy, 271
Carbonic acid, Heat of the liquid by Mollier, 269
Carbonic acid, Mollier's formula for, 262
Carbonic acid, Total latent heat by Mollier, 269
Carbonic acid vapor, 262
Carnot's cycle of the steam engine, 371
Clapeyron's equation, Discussion of, 38
Clapeyron's equation for wet steam, 29
Clausius equation of condition, 215
Clearance, Loss of work due to, 410
Cold-vapor engine, 459
Cold-vapor engine, Actual cycle of, 478
Cold-vapor engine-compressors, Superheating in, 484
Cold-vapor engine, Lorenz cycle of perfect, 474
Cold-vapor engine, Theoretically perfect, 463
Comparison of injector with feed pump, 150
Comparison of observed and computed volumes, 50, 51
Comparison of temperatures of saturated vapors, 18
Comparison of vapors at ordinary boiling points, 36
Compression, Action of, 410
Compressor of cold-vapor engine, Dimensions of the, 487
Condenser, Theory of jet, 138
Condenser, Theory of surface, 135
Constant steam weight, Curve of, 99
Constants for derivatives of Regnault's vapor-formulas, 17
Constants for Regnault's formulas for vapors, 14
Constants of Regnault's formula, Determination of, 12
Constant volume, Heating or cooling at, 89
Cooling at constant volume, 89
Critical temperature and the limit curve, 205
Critical temperature, pressure and volume, 205, 207
Curve of constant steam weight, 99
Cycle of actual cold-vapor engines, 478
Cycle of the actual steam engine, 396
Cycle, standard, corresponding efficiency, 441
Cylinder walls, Loss due to the action of, 419

D

Definition of vapors, 3
Definition of zero curve, 105
De Laval nozzle, 196
De Laval's steam-turbine nozzle, 180
Derivatives of Regnault's vapor-formulas, Constants for, 17
De Saint Venant and Wantzel's hypothesis, 171
Despretz, Heat for a cubic unit of vapor, 40, 42
Determination of constants of Regnault's formula, 12
Dieterici's data for ammonia vapor, 255
Dieterici's investigation of heat of water, 366
Dimensions of the compressor of cold-vapor engine, 487
Dimensions of the perfect steam engine, 381
Discharge of dry saturated vapor, Tables, 176, 177
Discussion of Clapeyron's equation for wet steam, 38
Discussion of heat exchanges in cylinder, 428
Distinction between gases and vapors, 3
"Dry saturated" steam, 6
Dry saturated vapor, Efflux of (First hypothesis), 172
Dry saturated vapor Efflux of (Second hypothesis), 173
Dry saturated vapors, Total heat of, 21, 22

E

Efficiency based on standard cycle, 441
Efflux and flow of wet vapors, 153
Efflux experiments with steam jet pump, 189
Efflux of dry saturated vapor, First hypothesis, 172
Efflux of dry saturated vapor, Second hypothesis, 173
Efflux of dry saturated vapor under constant pressure, 167
Efflux of highly heated liquids, 160
Efflux of saturated steam, Napier's experiments on, 183, 185, 186, 188
Efflux of steam, Experiments on, 181

Efflux resistances to flow of vapor, 171
Empirical formulas for saturated vapor, 9
Engines, Calorimetric investigation of, 419
Engines, Cold-vapor, 459
Engines for saturated vapors, 371
Engines for superheated steam, 441
Entropy and energy for carbonic acid, 271
Entropy changes of vapors with temperature, 68
Entropy of wet vapors, 61
Equality of steam weight during adiabatic change, 79
Equation of condition and its technical requirements, 224
Equation of condition by Clausius, 215
Equation of condition by Hirn, 227
Equation of condition by Vander Waals, 211
Equation of condition by Weyrauch, 228
Equation of condition for air extended, 313
Equation of condition for ammonia vapor, 253
Equation of condition for carbonic acid, 263
Equation of condition for saturated steam, 242
Equation of condition of superheated vapor, 208
Equation of condition of wet steam, 28
Equation of condition, Older and approximate form of, 223
Equation of condition, Testing of, 360
Equation of the zero curve, 103
Ether, Entropy changes of, 67
Ether, Exceptional behavior of, 67
Evaporation, Heat of, 21
Evaporation, Spontaneous, 127
Expansion in cylinder, Hypotheses concerning, 106
Expansion, Loss due to incomplete, 406
Expansion of steam as per Poncelet, 114
Expansion of vapors under varied conditions, 120
Expansion when heat supply varies with temperature, 117
Experiments of Andrew's, 209
Experiments on efflux with steam jet pump, 189
Experiments on the efflux of steam, 181
Explosion problem with boilers, 94
External latent heat, Formulas for, 32

F

Fairbairn and Tate's experiments, 35
Fairbairn's experiments and rise of the boiler pressure, 95
Feed pump, Comparison with injector, 150
Fireless steam engine, Theory of, 449
Flow and efflux of wet vapors, 153
Flow of hot liquids under constant pressure, 160
Flow of steam from one vessel to another, 293
Flow of vapor, Lorenz on, 201
Flow of vapor, Ratio of orifice- to efflux-section, 180
Flow of vapors, Recent investigations on, 196
Flow of vapors, Stodola on, 200
Fundamental equations for superheated vapor, 243

G

Generation of superheated steam, 281
Giffard injector, Calorimetric investigation of, 142

H

Heat contents of superheated vapor, 251
Heat equation of superheated vapors, 243
Heat equation of wet vapors, 53
Heat equation of wet vapor, First transformation of, 55
Heat equation of wet vapor, Fourth transformation of, 60
Heat equation of wet vapor, Second transformation of, 57
Heat equation of wet vapor, Third transformation of, 58
Heat exchange between steam and cylinder walls, 428
Heat for a cubic unit of vapor per Despretz, 40, 42
Heating at constant volume, 89
Heat in water according to Dieterici, 366
Heat of evaporation, 21
Heat of the liquid, 20
Heat of the liquid of saturated vapor, 22
Heat supply varies with temperature, Expansion when, 117
Heaviness and pressure of steam, Formula for, 37

Hirn's equation of condition. 227
Hirn's experiments on volume of superheated steam. 239
Hot-vapor engines, 371
Hypotheses concerning expansion in cylinder, 106

I

Imperfection of cycle, Loss of work due to, 396
Indicated work of steam engine, 441
Injector, Calorimetric investigation of, 142
Injector, Comparison with feed pump, 150
Inner latent heat, 26
Inner latent heat, Formulas for, 31
Investigations of flow of vapors, Recent, 196
Isodynamic curve of wet vapors, 71
Isodynamic expansion, Non-reversible, 123
Isodynamic for superheated steam, 274
Isothermal change in mixture of air, steam, water, and ice, 329
Isothermal change of mixture of air and wet steam, 323
Isothermal curve of wet vapors, 71
Isothermal expansion, Non-reversible, 122
Isothermal for superheated steam, 274

J

Jet condensers, Air volume in, 141
Jet condenser, Theory of, 138

L

Latent heat, Formulas for external, 32
Latent heat, Formulas for inner, 31
Latent heat, Inner and outer, 26
Latent heat of saturated vapors, 23
Latent heat of steam, 20
Limit curve and the critical temperature, 205
Limit curves, Lower and upper, 7, 8
Limit curves, Transformation of, 64
Linde's machine for liquefaction of air, 303, 308
Liquid air, 303
Liquid air and its applications, 310
Liquid at constant pressure, Specific heat of, 21
Liquids, highly heated, Flow under constant pressure of, 160
Liquid weight, Specific, 5
Lorenz. Cycle of a perfect cold-vapor engine, 474
Lorenz on the flow of vapors, 201
Loss due to action of cylinder walls, 419
Loss due to clearance, 410
Loss due to imperfection of engine's cycle, 396
Loss due to incomplete expansion, 406

M

Mathias' experiments on sulphurous acid, 257
Mixture of air and unsaturated vapor, Behavior of, 338
Mixture of air and wet steam, Adiabatic change in, 333
Mixture of air and wet steam, Isothermal change of, 323
Mixture of air, steam, and ice, Behavior of, 350
Mixture of air, steam, water, and ice, Isothermal change in, 329
Mixtures of air and steam, Behavior of, 320
Mixtures of different kinds of liquids, 353
Mixtures of different kinds of liquids and their vapors, 353
Mixtures of different kinds of vapors, 353
Mixtures of wet vapors of the same kind, 129
Mollier's formula for carbonic acid, 267

N

Napier's experiments on the efflux of saturated steam, 183, 185, 186, 188
Non-reversible adiabatic expansion, 124
Non-reversible change of wet vapors, 120
Non-reversible changes of superheated vapor, 274
Non-reversible isodynamic expansion, 123
Non-reversible isothermal expansion, 122
Nozzle of De Laval, 196

O

Outer Latent Heat, 26

P

Pambour's hypothesis as to expansion in cylinder, 112

Pambour's theory of expansion in cylinder, 66
Perfect cold-vapor engine as per Lorenz, 474
Perfect steam engine and its dimensions, 381
Poncelet's hypothesis as to expansion in cylinder, 114
Pressure and temperature of saturated steam, 5
Pressure of saturated vapors, 9
Pressure-temperature formula by Rankine, 42
Pressure-temperature formula by Raoult Pictet, 44
Pressure-temperature formula by Unwin, 43
Pressure-temperature formula used by Regnault, 10
Pressure-temperature-volume formula for steam by Tumlirz, 49

Q

Quality of moist steam, 5

R

Range and reliability of Regnault's formulas, 23
Rankine's pressure-temperature formula, 42
Raoult Pictet's pressure-temperature formula, 44
Refrigerating machines in general, 459
Regnault's empirical formula, Range and reliability of, 23
Regnault's pressure-temperature formula, 10
Relation between heaviness and pressure of steam, 37
Relative weight of saturated steam to air, 35
Reliability and range of Regnault's formula, 23
Reversible changes in wet vapors, 89
Reversible changes of superheated vapor, 274

S

Saturated and superheated steam, Recent researches on, 357
Saturated steam, Auxiliary table (English), App.
Saturated steam, Auxiliary table (French), 62, 63
Saturated steam, Equation of condition of, 242
Saturated vapor, 4
Saturated vapor, Behavior of, 9
Saturated vapor, Empirical formulas for, 9
Saturated vapor, Engines for, 371
Saturated vapor, Heat of the liquid of, 22
Saturated vapor, Latent heat of, 23
Saturated vapor of water, Specific weight of, 34
Saturated vapor, Pressure of, 9
Saturated vapor, Specific weight of, 23
Specific heat of superheated steam, 364
Specific heat of the liquid at constant pressure, 21
Specific liquid weight, 5
Specific steam weight, 5
Specific volume for superheated steam, 239
Specific volume of vapors, 358
Specific volume of water, Formula for, 24
Specific weight and pressure of steam, Formula for, 37
Specific weight of saturated vapor, 33
Specific weight of saturated vapor of water, 74
Spontaneous evaporation, 127
Standard cycle and corresponding efficiency, 441
Steam, Battelli's investigation of, 48
Steam, Dry saturated, 6
Steam engine, Carnot cycle of, 371
Steam engine, Indicated work of, 441
Steam engines, 371
Steam engine, Theory of, 368
Steam engine, Theory of fireless, 449
Steam engine, Thermal efficiency of, 441
Steam expansion as per Pambour, 112
Steam expansion by polytropic curve, 108
Steam flow from one vessel to another, 293
Steam heat, 26, 27
Steam heat of superheated vapor, 249
Steam quality, 5
Steam quality, Charpentier's formula for, 85
Steam, Superheated, 6
Steam, Throttling, 293
Steam-turbine nozzle, de Laval, 180
Steam weight, Specific, 5

Stodola on the flow of vapors, 200
Sulphurous acid, Mathias' experiments on, 257
Sulphurous acid vapor, 256
Superheated and saturated steam, Recent researches on, 357
Superheated steam, 6, 229
Superheated steam, Adiabatic for, 231–234
Superheated steam by mixing of vapors, 281
Superheated steam, Engines for, 441
Superheated steam, Generation of, 281
Superheated steam, Heat needed for production of, 289
Superheated steam, Isothermal, isodynamic and adiabatic for, 274
Superheated steam produced by waste fire gases, 317
Superheated steam, Specific heat of, 364
Superheated steam, Specific volume for, 239
Superheated steam, Tumlirz formula, 49, 241
Superheated-steam volume, Battelli's experiments on, 240, 242
Superheated-steam volume, Hirn's experiments on, 239
Superheated steam, Wethered's experiments on, 283, 290
Superheated vapor, 4
Superheated vapor, Behavior of, 202
Superheated vapor, Equation of condition of, 208
Superheated vapor, Fundamental equation for, 243
Superheated vapor, Heat contents of, 251
Superheated vapor, Heat equations of, 243
Superheated vapor, Reversible and non-reversible changes, 274
Superheated vapor, Steam heat of, 249
Superheating apparatus, 281
Superheating in compressor of actual cold-vapor engines, 484
Supply of heat, Equation for, 25
Surface condenser, Theory of, 135
Surface condenser, Watt's formula for, 137, 141

T

Tables of the Appendix (French units):
1a. Water, saturated vapor of; p, T, and ratios, II, III
1b. Water, saturated vapor of; heat quantities, IV, V
2a. Ether, saturated vapor of; p, T, and ratios, VI, VII
2b. Ether, saturated vapor of; heat quantities, VI, VII
3a. Alcohol, saturated vapor of; p, T, and ratios, VIII, IX
3b. Alcohol, saturated vapor of; heat quantities, VIII, IX
4a. Acetone, saturated vapor of; p, T, and ratios, X, XI
4b. Acetone, saturated vapor of; heat quantities, X, XI
5a. Chloroform, saturated vapor of; p, T, and ratios, XII, XIII
5b. Chloroform, saturated vapor of; heat quantities, XII, XIII
6a. Chloride of carbon, saturated vapor of; p, T, and ratios, XIV, XV
6b. Chloride of carbon, saturated vapor of; heat quantities, XIV, XV
7a. Bisulphide of carbon, saturated vapor of; p, T, and ratios, XVI, XVII
7b. Bisulphide of carbon, saturated vapor of; heat quantities, XVI, XVII
8. Ammonia, saturated vapor of; p, t, u, and heat elements, XVIII, XIX
9. Sulphurous acid, saturated vapor of; p, t, u, and heat elements, XX, XXI
10. Carbonic acid, saturated vapor of; p, t, u, and heat elements, XXII, XXIII
11a. First principal table for water vapor, XXIV
11b. First principal table for water vapor, XXV
11c. First principal table for water vapor, XXVI
12a. Second principal table for water vapor, XXVII
12b. Second principal table for water vapor, XXVIII
12c. Second principal table for water vapor, XXIX
Tables of the Appendix (English units):
1a. Water, saturated vapor of; p, T, u, and ratios, XXX, XXXI
1b. Water, saturated vapor of; heat quantities, XXXII, XXXIII
2a. Ether, saturated vapor of; heat quantities, XXXIV, XXXV
2b. Ether, saturated vapor of; heat quantities, XXXIV, XXXV

Tables of the Appendix (English units):
3a. Alcohol, saturated vapor of; p, T, u, and ratios, XXXVI, XXXVII
3b. Alcohol, saturated vapor of; heat quantities, XXXVI, XXXVII
4a. Acetone, saturated vapor of; p, T, u, and ratios, XXXVIII, XXXIX
4b. Acetone, saturated vapor of; heat quantities, XXXVIII, XXXIX
5a. Chloroform, saturated vapor of; $p\ T, u$, and ratios, XL, XLI
5b. Chloroform, saturated vapor of; heat quantities, XL, XLI
6a. Chloride of carbon, saturated vapor of; p, T, u, and ratios, XLII, XLIII
6b. Chloride of carbon, saturated vapor of; heat quantities, XLII, XLIII
7a. Bisulphide of carbon, saturated vapor of; p, T, u, and ratios, XLIV, XLV
7b Bisulphide of carbon, saturated vapor of; heat quantities, XLIV, XLV
8. Ammonia, saturated vapor of; p, t, u, and heat elements, XLVI, XLVII
9. Sulphurous acid, saturated vapor of; p, t, u, and heat elements, XLVIII, XLIX
10. Carbonic acid, saturated vapor of; p, t, u, and heat elements, L, LI
11a. First principal table for water vapor, LII
11b. First principal table for water vapor, LIII
11c. First principal table for water vapor, LIV
12a. Second principal table for water vapor, LV
12b. Second principal table for water vapor, LVI
12c. Second principal table for water vapor, LVII
13. Auxiliary table (English units) corresponding to pp. 62 and 63, LVIII, LIX
14. Auxiliary table for conversion of pounds per square inch into atmospheres, LX
Temperature and pressure of saturated steam, 51
Temperature of saturated vapor, Comparison of, 18
Temperature-pressure formula used by Regnault, 9
Theoretically perfect cold-vapor engine, 463
Theory of fireless steam engine, 449
Theory of steam engine, 368
Theory of surface condenser, 135
Theory of the jet condenser, 138
Theory of vapors, Preliminary remarks, 3
Thermal efficiency of steam engines, 441
Throttling of steam, 293
Time of rise of boiler pressure, 95
Total heat of dry saturated vapor, 21, 22
Total heat of steam, 20
Transformation (1) of heat equation of wet vapor, 55
Transformation (2), 57
Transformation (3), 58
Transformation (4), 60
Transformation of limit curves, 64
Transformation of wet-vapor diagrams, 61
Tumlirz empirical formula for steam, 49
Tumlirz formula for superheated steam, 241

U

Unsaturated vapor, Behavior of, 202
Unwin's pressure-temperature formula, 43

V

Van der Waals' equation of condition, 211
Vapor constants for Regnault's pressure-temperature formulas, 14
Vapor, Definition of, 3
Vapor, Dry saturated, Discharge of, Tables, 176, 177
Vapor, Dry saturated, Efflux under constant pressure of, 167
Vapor-flow, Efflux resistances to, 171
Vapor-flow formula, First transformation, 155
Vapor-flow formula, Second transformation, 156
Vapor-flow, Ratio of orifice- to efflux-section, 180
Vapor of ammonia, 252
Vapor of carbonic acid, 262
Vapor of sulphurous acid, 256
Vapor, Entropy changes of, 68
Vapor, Fundamental formulas for flow of, 153

Vapor, Specific volume of, 258
Vapor, Two essentially different states of, 3
Varied conditions of vapor expansion, 120
Velocity of rise in boiler pressure, 96
Volume of vapor, Specific, 358
Volumes of steam, Comparison of observed and computed, 50, 51

W

Walls of cylinder, Heat exchange between steam and, 228
Water, Battelli's investigation of, 48
Watt's formula for surface condensers, 137, 141
Wetherred's experiments on superheated steam, 283, 290
"Wetness" of mixture of steam and water, 5
Wet steam, Clapeyron's equation for. 29
Wet steam, Equation of condition of 28
Wet steam, Zero curve of, 99. 101
Wet-vapor diagrams, Transformation of, 61
Wet vapor, Adiabatic curve of, 75
Wet vapor, Efflux and flow of, 153
Wet vapor, Entropy of, 61
Wet vapor, Fundamental formulas for flow of, 53
Wet vapor, Heat equations of, 53
Wet vapor, Isothermal and isodynamic curve of, 71
Wet vapor, non-reversible changes, 120
Wet vapor of the same kind, Mixtures of, 129
Wet vapor, Reversible changes in, 89
Weyrauch's equation of condition, 228

Z

Zero curve, Definition of, 105
Zero curve, Equation of, 105
Zero curve of wet steam, 99, 101

Catalogue of Scientific Publications

and Importations of the D. Van Nostrand Company, 23 Murray Street and 27 Warren Street, New York.

A B C CODE. (See Clausen-Thue.)

ABBOT, H. L., Gen'l. The Defence of the Seacoast of the United States. Lectures delivered before the U. S. Naval War College. 8vo, red cloth..... $2.00

ABBOTT, A. V. The Electrical Transmission of Energy. A Manual for the Design of Electrical Circuits. *New edition, revised, and entirely rewritten.* Fully illustrated. 8vo, cloth, *net*, $5.00

ADAM, P. Practical Bookbinding. With illustrations and figures. Translated from the German by Thomas E. Maw. 8vo, cloth, illustrated.....*net*, $2.50

ADAMS, J. W. Sewers and Drains for Populous Districts. Embracing Rules and Formulas for the dimensions and construction of works of Sanitary Engineers. 8vo, cloth.... $2.50

ADDYMAN, F. T. Practical X-Ray Work. Part I, Historical. Part II, Apparatus and its Management. Part III, Practical X-Ray Work. Illustrated with twelve plates from photographs. 8vo, cloth, illustrated.....*net*, $4.00

A 1 CODE. (See Clausen-Thue.)

AIKMAN, C. M., Prof. Manures and the Principles of Manuring. 8vo, cloth..... $2.50

ALEXANDER, J. H. Universal Dictionary of Weights and Measures, Ancient and Modern, reduced to the Standards of the United States of America. *New Edition, enlarged.* 8vo, cloth..... $3.50

ALEXANDER, S. A. Broke Down: What Should I Do? A Ready Reference and Key to Locomotive Engineers and Firemen, Round-house Machinists, Conductors, Train Hands and Inspectors. With 5 folding plates. 12mo, cloth..... $1.50

ANDERSON, G. L., A.M. (Captain of U. S. Artillery). Handbook for the use of Electricians in the operation and care of Electrical Machinery and Apparatus of the United States Seacoast Defenses. Prepared under the direction of the Lieutenant-General Commanding the Army. With tables, diagrams and illustrations. 8vo, cloth, illustrated..... $3.00

ANDERSON, J. W. Prospector's Handbook. A Guide for the Prospector and Traveller in search of Metal-bearing or other Valuable Minerals. *Eighth Edition, revised.* 8vo, cloth....$1.50

ANDERSON, W. On the Conversion of Heat into Work. A Practical Handbook on Heat-engines. *Third Edition.* Illustrated. 12mo, cloth..... $2.25

ANDÉS, L. Vegetable Fats and Oils: Their Practical Preparation, Purification and Employment for Various Purposes. Their Properties, Adulteration and Examination. A Handbook for Oil Manufacturers and Refiners, Candle, Soap and Lubricating-oil Manufacturers, and the Oil and Fat Industry in general. Translated from the German. With 94 illus. 8vo, cloth....*net*, $4.00

——**Animal Fats and Oils. Their Practical Production,** Purification and Uses for a great variety of purposes; their Properties, Falsification and Examination. A Handbook for Manufacturers of Oil and Fat Products, Soap and Candle Makers, Agriculturists, Tanners, etc. Translated by Charles Salter. With 62 illustrations. 8vo, cloth..... *net*, $4.00

——**Drying Oils, Boiled Oil, and Solid and Liquid Driers.** A practical work for manufacturers of Oils, Varnishes, Printing Inks, Oilcloth and Linoleum, Oil-cakes, Paints, etc. 8vo, cloth, illustrated..... *net*, $5.00

——**Iron Corrosion, Anti-fouling and Anti-corrosive** Paints. Translated from the German by Charles Salter. Illustrated with engravings and half-tone cuts. 8vo, cloth..*net*, $4.00

——**Oil Colors, and Printers' Ink. A Practical Hand-**book treating of Linseed-oil, Boiled Oil, Paints, Artists' Colors, Lampblack, and Printers' Inks (black and colored). Translated from the German by Arthur Morris and Herbert Robson. With 56 figures and diagrams. 8vo, cloth, 212 pages.....*net*, $2.50

ANNUAL REPORTS on the Progress of Chemistry for 1904. Vol. I. Issued by the Chemical Society. 8vo, cloth...*net*, $2.00
Vol. II (1905). 8vo, cloth.....*net*, $2.00

ARNOLD, E. Armature Windings of Direct-Current Dynamos. Extension and Application of a General Winding Rule. Translated from the original German by Francis B. DeGress, M.E. With numerous illustrations. 8vo, cloth... $2.00

ARNOLD, R., Dr. Ammonia and Ammonium Compounds. A Practical Manual for Manufacturers, Chemists, Gas Engineers and Drysalters. *Second Edition.* 12mo, cloth.... $2.00

Art of Dyeing Wool, Silk and Cotton. Translated from the French of M. Hellott, M. Macquer and M. Le Pileur D'Apligny. First published in English in 1789. 8vo, cloth, illustrated, *net*, $2.00

ASHE, S. W., and KEILEY, J. D. Electric Railways, Theoretically and Practically Treated; Rolling Stock. With numerous figures, diagrams, and folding plates. 12mo, cloth, illustrated. ..... *net*, $2.50

—— Vol. 2. **Sub-stations and the Distributing System**..... *In Press.*

ATKINSON, A. A., Prof. (Ohio University). Electrical and Magnetic Calculations, for the use of Electrical Engineers and Artisans, Teachers, Students and all others interested in the Theory and Application of Electricity and Magnetism. *Second Edition, revised.* 8vo, cloth, illustrated. ..... *net*, $1.50

ATKINSON, P. The Elements of Electric Lighting. including Electric Generation, Measurement, Storage and Distribution. *Tenth Edition, fully revised and new matter added.* Illustrated. 12mo, cloth..... $1.50

—— **The Elements of Dynamic Electricity and Magnetism.** *Fourth Edition.* 120 illustrations. 12mo, cloth.. $2.00

—— **Power Transmitted by Electricity and its Application** by the Electric Motor, including Electric Railway Construction. *Fourth Edition, fully revised, new matter added.* 12mo, cloth, illustrated..... $2.00

AUCHINCLOSS, W. S. Link and Valve Motions Simplified. Illustrated with 29 woodcuts and 20 lithographic plates, together with a Travel Scale, and numerous useful tables. *Fourteenth Edition, revised.* 8vo, cloth..... $2.00

AYRTON, H. The Electrical Arc. With numerous figures, diagrams and plates. 8vo, cloth, illustrated..... $5.00

AYRTON, W. E., M.I.C.E. Practical Electricity. A Laboratory and Lecture Course for the first-year students of Electrical Engineering, based on the International Definitions of the Electrical Units. Vol. I, Current, Pressure, Resistance, Energy, Power, and Cells. Completely rewritten and containing many figures and diagrams. 12mo, cloth..... $2.00

BACON, F. W. A Treatise on the Richards Steam-engine Indicator, with directions for its use. By Charles T. Porter. Revised, with notes and large additions as developed by American practice; with an appendix containing useful formulæ and rules for engineers. Illustrated. *Fourth Edition.* 12mo, cloth. . $1.00

BAKER, Arthur L., Prof. (Univ. of Rochester). Quaternions ..... *In Press.*

BAKER, M. N. Potable Water and Methods of Detecting Impurities. *New Edition, revised and largely rewritten.* 16mo, cloth. (*Van Nostrand's Science Series*)..... $0.50

BALCH, G. T., Col. Methods of Teaching Patriotism in the Public Schools. 8vo, cloth..... $1.00

BALE, M. P. Pumps and Pumping. A Handbook for Pump Users. 12mo, cloth..... $1.50

BALL, S. R. Popular Guide to the Heavens. A series of eighty-three plates, many of which are colored and lithographed, with explanatory text and index. Small 4to, cloth, illustrated. *net*, $4.50

BARBA, J. The Use of Steel for Constructive Purposes. Method of Working, Applying and Testing Plates and Bars. With a Preface by A. L. Holley, C.E. 12mo, cloth..... $1.50

BARKER, A. H. Graphic Methods of Engine Design. Including a Graphical Treatment of the Balancing of Engines. 12mo, cloth..... $1.50

BARNARD, F. A. P. Report on Machinery and Processes of the Industrial Arts and Apparatus of the Exact Sciences at the Paris Universal Exposition, 1867. 152 illustrations and 8 folding plates. 8vo, cloth..... $5.00

BARNARD, J. H. The Naval Militiaman's Guide. Full leather, pocket size..... $1.25

BARRUS, G. H. Boiler Tests: Embracing the Results of one hundred and thirty-seven evaporative tests, made on seventy-one boilers, conducted by the author. 8vo, cloth. $3.00

——**Engine Tests: Embracing the Results of over one** hundred feed-water tests and other investigations of various kinds of steam-engines, conducted by the author. With numerous figures, tables, and diagrams. 8vo, cloth, illustrated.. $4.00
The above two purchased together..... $6.00

BARWISE, S., M.D. (London). The Purification of Sewage. Being a brief account of the Scientific Principles of Sewage Purification and their Practical Application. 12mo, cloth, illustrated. *New Edition*.....*net*, $3.50

BEAUMONT, R. Color in Woven Design. With 32 colored plates and numerous original illustrations. Large, 12mo..... $7.50

—— W. W. Practical Treatise on the Steam-engine Indicator, and Indicator Diagrams. With notes on Engine Performances, Expansion of Steam, Behavior of Steam in Steam-engine Cylinders, and on Gas- and Oil-engine Diagrams. *Second Edition, revised and enlarged.* 8vo, cloth, illustrated...*net*, $2.50

BEECH, F. Dyeing of Cotton Fabrics. A Practical Handbook for the Dyer and Student. Containing numerous recipes for the production of Cotton Fabrics of all kinds, of a great range of colors, thus making it of great service in the dye-house, while to the student it is of value in that the scientific principles which underlie the operations of dyeing are clearly laid down. With 44 illustrations of Bleaching and Dyeing Machinery. 8vo, cloth, illustrated..... *net*, $3.00

—— Dyeing of Woolen Fabrics. With diagrams and figures. 8vo, cloth, illustrated..... *net*, $3.50

BECKWITH, A. Pottery. Observations on the Materials and Manufacture of Terra-cotta, Stoneware, Firebrick, Porcelain, Earthenware, Brick, Majolica, and Encaustic Tiles. *Second Edition.* 8vo, paper..... .60

BEGTRUP, J., M.E. The Slide Valve and its Functions. With Special Reference to Modern Practice in the United States. With numerous diagrams and figures. 8vo, cloth..... $2.00

BERNTHASEN, A. A Text-book of Organic Chemistry. Translated by George M'Gowan, Ph.D. *Fifth English Edition*, revised and extended by author and translator. Illustrated. 12mo, cloth..... *In Press.*

BERRY, W. J. Differential Equations of the First Species. 12mo, cloth, illustrated.....*In Press.*

BERSCH, J., Dr. Manufacture of Mineral and Lake Pigments. Containing directions for the manufacture of all artificial artists' and painters' colors, enamel colors, soot and metallic pigments. A text-book for Manufacturers, Merchants, Artists and Painters. *Translated from the second revised edition* by Arthur C. Wright, M.A. 8vo, cloth, illustrated.....*net*, $5.00

BERTIN, L. E. Marine Boilers: Their Construction and Working, dealing more especially with Tubulous Boilers. Translated by Leslie S. Robertson, Assoc. M. Inst. C. E., M. I. Mech. E., M.I.N.A., containing upward of 250 illustrations. Preface by Sir William White, K.C.B., F.R.S., Director of Naval Construction to the Admiralty, and Assistant Controller of the Navy. *Second Edition, revised and enlarged.* 8vo, cloth, illustrated. *net*, $5.00

BIGGS, C. H. W. First Principles of Electricity and Magnetism. A book for beginners in practical work, containing a good deal of useful information not usually to be found in similar books. With numerous tables and 343 diagrams and figures. 12mo, cloth, illustrated..... $2.00

BINNS, C. F. Ceramic Technology. Being Some Aspects of Technical Science as applied to Pottery Manufacture. 8vo, cloth.....*net*, $5.00

—— Manual of Practical Potting. Compiled by Experts. *Third Edition, revised and enlarged.* 8vo, cloth.....*net*, $7.50

BIRCHMORE, W. H., Dr. How to Use a Gas Analysis. 12mo, cloth, illustrated.....*net*, $1.25

BLAKE, W. H. Brewer's Vade Mecum. With Tables and marginal reference notes. 8vo, cloth.....*net*, $4.00

—— W. P. Report upon the Precious Metals. Being Statistical Notices of the Principal Gold and Silver producing regions of the world, represented at the Paris Universal Exposition. 8vo, cloth..... $2.00

BLAKESLEY, T. H. Alternating Currents of Electricity. For the use of Students and Engineers. *Third Edition, enlarged.* 12mo, cloth..... $1.50

BLYTH, A. W., M.R.C.S., F.C.S. Foods: Their Composition and Analysis. A Manual for the use of Analytical Chemists, with an Introductory Essay on the History of Adulterations. With numerous tables and illustrations. *Fifth Edition, thoroughly revised, enlarged and rewritten.* 8vo, cloth..... $7.50

BLYTH, A. W., M.R.C.S., F.C.S., Poisons: Their Effects and Detection. A Manual for the use of Analytical Chemists and Experts, with an Introductory Essay on the Growth of Modern Toxicology. *New Edition*.....*In Press.*

BODMER, G. R. Hydraulic Motors and Turbines. For the use of Engineers, Manufacturers and Students. *Third Edition, revised and enlarged.* With 192 illustrations. 12mo, cloth..... $5.00

BOILEAU, J. T. A New and Complete Set of Traverse Tables, showing the Difference of Latitude and Departure of every minute of the Quadrant and to five places of decimals. 8vo, cloth..... $5.00

BONNEY, G. E. The Electro-platers' Handbook. A Manual for Amateurs and Young Students of Electro-metallurgy. 60 illustrations. 12mo, cloth..... $1.20

BOOTH, W. H. Water Softening and Treatment, Con- densing Plant, Feed Pumps, and Heaters for Steam Users and Manufacturers. 8vo, cloth, illustrated.....*net*, $2.50

BOURRY, E. Treatise on Ceramic Industries. A Complete Manual for Pottery, Tile and Brick Works. Translated from the French by Wilton P. Rix. With 323 figures and illustrations. 8vo, cloth, illustrated..... *net*, $8.50

BOW, R. H. A Treatise on Bracing. With its applica- tion to Bridges and other Structures of Wood or Iron. 156 illustrations. 8vo, cloth..... $1.50

BOWIE, AUG. J., Jr., M.E. A Practical Treatise on Hydraulic Mining in California. With Description of the Use and Construction of Ditches, Flumes, Wrought-iron Pipes and Dams; Flow of Water on Heavy Grades, and its Applicability, under High Pressure, to Mining. *Ninth Edition.* Small quarto, cloth. Illustrated..... $5.00

BOWKER, Wm. R. Dynamo, Motor and Switchboard Circuits. For Electrical Engineers. A practical book, dealing with the subject of Direct, Alternating, and Polyphase Currents. With over 100 diagrams and engravings. 8vo, cloth.. *net*, $2.25

BOWSER, E. A., Prof. An Elementary Treatise on Analytic Geometry. Embracing Plane Geometry, and an Introduction to Geometry of three Dimensions. *Twenty-first Edition.* 12mo, cloth.....*net*, $1.75

BOWSER, E. A., Prof. An Elementary Treatise on the Differential and Integral Calculus. With numerous examples. *Twenty-first Edition.* Enlarged by 640 additional examples. 12mo, cloth.....*net*, $2.25

—— **An Elementary Treatise on Analytic Mechanics. With** numerous examples. *Sixteenth Edition.* 12mo, cloth..*net*, $3.00

—— **An Elementary Treatise on Hydro-mechanics. With** numerous examples. *Fifth Edition.* 12mo, cloth.....*net*, $2.50

—— **A Treatise on Roofs and Bridges. With Numerous** Exercises, especially adapted for school use. 12mo, cloth. Illustrated.....*net*, $2.25

BRASSEY'S Naval Annual for 1905. Edited by T. A. Brassey. With numerous full-page diagrams, half-tone illustrations and tables. Nineteenth year of publication. 8vo, cloth, illustrated.....*net*, $6.00

BRAUN, E. The Baker's Book: A Practical Handbook of the Baking Industry in all Countries. Profusely illustrated with diagrams, engravings, and full-page colored plates. Translated into English and edited by Emil Braun. Vol. I., 8vo, cloth, illustrated, 308 pages..... $2.50
Vol. II. 363 pages, illustrated..... $2.50

British Standard Sections. Issued by the Engineering Standards Committee, Supported by The Institution of Civil Engineers, The Institution of Mechanical Engineers, The Institution of Naval Architects, The Iron and Steel Institute, and The Institution of Electrical Engineers. Comprising 9 plates of diagrams, with letter-press and tables. Oblong pamphlet, $8\frac{3}{4}\times15$..... $1.00

BROWN, WM. N. The Art of Enamelling on Metal. With figures and illustrations. 12mo, cloth, illustrated. ....*net*, $1.00

—— **Handbook on Japanning and Enamelling, for Cycles,** Bedsteads, Tinware, etc. 12mo, cloth, illustrated.....*net*, $1.50

—— **House Decorating and Painting. With Numerous** illustrations. 12mo, cloth.....*net*, $1.50

—— **History of Decorative Art. With Designs and Illus-** trations. 12mo, cloth.....*net*, $1.25

BROWN, WM. N. Principle and Practice of Dipping, Burnishing, Lacquering and Bronzing Brass Ware. 12mo, cloth. *net*, $1.00

—— Workshop Wrinkles for Decorators, Painters, Paper-Hangers and Others. 8vo, cloth..... *net*, $1.00

BUCHANAN, E. E. Tables of Squares. Containing the square of every foot, inch, and sixteenth of an inch, between one sixteenth of an inch and fifty feet. For Engineers and Calculators. 16mo, oblong, cloth..... $1.00

BULMAN, H. F., and REDMAYNE, R. S. A. Colliery Working and Management; comprising the duties of a colliery manager, the superintendence and arrangement of labor and wages, and the different systems of working coal-seams. With engravings, diagrams, and tables. *Second Edition, revised and enlarged.* 8vo, cloth, illustrated..... *net*, $6.00

BURGH, N. P. Modern Marine Engineering, Applied to Paddle and Screw Propulsion. Consisting of 36 colored plates, 259 practical woodcut illustrations and 403 pages of descriptive matter. The whole being an exposition of the present practice of James Watt & Co., J. & G. Rennie, R. Napier & Sons, and other celebrated firms. Thick quarto, half morocco..... $10.00

BURT, W. A. Key to the Solar Compass, and Surveyor's Companion. Comprising all the rules necessary for use in the field; also description of the Linear Surveys and Public Land System of the United States, Notes on the Barometer, Suggestions for an Outfit for a Survey of Four Months, etc. *Seventh Edition.* Pocket size, full leather..... $2.50

BUSKETT, E. W. Fire Assaying. 12mo, cloth, illus- trated..... *In Press.*

CAIN, W., Prof. Brief Course in the Calculus. With figures and diagrams. 8vo, cloth, illustrated..... *net*, $1.75

—— Theory of Steel-concrete Arches and of Vaulted Structures. *New Edition, revised and enlarged.* 16mo, cloth, illustrated. (*Van Nostrand Science Series*)..... $0.50

CAMPIN, F. On the Construction of Iron Roofs. A Theoretical and Practical Treatise, with woodcuts and plates of roofs recently executed. 8vo, cloth..... $2.00

CARPENTER, Prof. R. C., and DIEDERICHS, Prof. H. Internal Combustion Motors. With figures and diagrams. 8vo. cloth, illustrated.....*In Press*,

CARTER, E. T. Motive Power and Gearing for Electrical Machinery. A treatise on the Theory and Practice of the Mechanical Equipment of Power Stations for Electrical Supply and for Electric Traction. *Second Edition*, revised in part by G. Thomas-Davies. 8vo, cloth, illustrated..... $5.00

CATHCART, WM. L., Prof. Machine Design. Part I. Fastenings. 8vo, cloth, illustrated.....*net*, $3.00

—— **Machine Elements; Shrinkage and Pressure Joints.** With tables and diagrams.....*In Press.*

—— **Marine-Engine Design**.....*In Press.*

—— **and CHAFFEE, J. I. Course of Graphic Statics Applied** to Mechanical Engineering.....*In Press*

CHAMBER'S MATHEMATICAL TABLES, consisting of Logarithms of Numbers 1 to 108,000, Trigonometrical, Nautical and other Tables. *New Edition.* 8vo, cloth..... $1.75

CHARPENTIER, P. Timber. A Comprehensive Study of Wood in all its Aspects, Commercial and Botanical. Showing the Different Applications and Uses of Timber in Various Trades, etc. Translated into English. 8vo, cloth, illus...*net*, $6.00

CHAUVENET, W., Prof. New Method of Correcting Lunar Distances, and Improved Method of Finding the Error and Rate of a Chronometer, by Equal Altitudes. 8vo, cloth. $2.00

CHILD, C. T. The How and Why of Electricity. A Book of Information for non-technical readers, treating of the Properties of Electricity, and how it is generated, handled, controlled, measured and set to work. Also explaining the operation of Electrical Apparatus. 8vo, cloth, illustrated..... $1.00

CHRISTIE, W. W. Boiler-waters, Scale, Corrosion, Foaming. 8vo, cloth, illustrated.....*net*, $3.00

—— **Chimney Design and Theory. A Book for Engineers** and Architects, with numerous half-tone illustrations and plates of famous chimneys. *Second Edition, revised.* 8vo, cloth. $3.00

CHRISTIE, W. W. Furnace Draft: its Production by Mechanical Methods. A Handy Reference Book, with figures and tables. 16mo, cloth, illustrated. (*Van Nostrand's Science Series*). $0.50

CLAPPERTON, G. Practical Paper-making. A Manual for Paper-makers and Owners and Managers of Paper Mills, to which is appended useful tables, calculations, data, etc., with illustrations reproduced from micro-photographs. 12mo, cloth, illustrated..... $2.50

CLARK, D. K., C.E. A Manual of Rules, Tables and Data for Mechanical Engineers. Based on the most recent investigations. Illustrated with numerous diagrams. 1012 pages. 8vo, cloth. *Sixth Edition*..... $5.00

—— **Fuel: its Combustion and Economy; consisting of** abridgments of Treatise on the Combustion of Coal. By C. W. Williams, and the Economy of Fuel, by T. S. Prideaux. With extensive additions in recent practice in the Combustion and Economy of Fuel, Coal, Coke, Wood, Peat, Petroleum, etc. *Fourth Edition.* 12mo, cloth..... $1.50

—— **The Mechanical Engineer's Pocket-book of Tables,** Formulæ, Rules and Data. A Handy Book of Reference for Daily Use in Engineering Practice. 16mo, morocco. *Fifth Edition, carefully revised throughout*..... $3.00

—— **Tramways: Their Construction and Working. Em**bracing a comprehensive history of the system, with accounts of the various modes of traction, a description of the varieties of rolling stock, and ample details of Cost and Working Expenses. *Second Edition, rewritten and greatly enlarged, with upwards of 400 illustrations.* Thick 8vo, cloth.....$89.00

CLARK, J. M. New System of Laying Out Railway Turnouts instantly, by inspection from tables. 12mo, cloth... $1.00

CLAUSEN-THUE, W. The A B C Universal Commercial Electric Telegraphic Code; specially adapted for the use of Financiers, Merchants, Ship-owners, Brokers, Agents, etc. *Fourth Edition.* 8vo, cloth..... $5.00
Fifth Edition of same..... $7.00

—— **The A 1 Universal Commercial Electric Telegraphic** Code. Over 1240 pages and nearly 90,000 variations. 8vo, cloth..... $7.50

CLEEMANN, T. M. The Railroad Engineer's Practice. Being a Short but Complete Description of the Duties of the Young Engineer in Preliminary and Location Surveys and in Construction. *Fourth Edition, revised and enlarged.* Illustrated. 12mo, cloth..... $1.50

CLEVENGER, S. R. A Treatise on the Method of Government Surveying as prescribed by the U. S. Congress and Commissioner of the General Land Office, with complete Mathematical, Astronomical, and Practical Instructions for the use of the United States Surveyors in the field. 16mo, morocco..... $2.50

CLOUTH, F. Rubber, Gutta-Percha, and Balata. First English Translation with Additions and Emendations by the Author. With numerous figures, tables, diagrams, and folding plates. 8vo, cloth, illustrated..... *net*, $5.00

COFFIN, J. H. C., Prof. Navigation and Nautical Astronomy. Prepared for the use of the U. S. Naval Academy. *New Edition.* Revised by Commander Charles Belknap. 52 woodcut illustrations. 12mo, cloth..... *net*, $3.50

COLE, R. S., M.A. A Treatise on Photographic Optics. Being an account of the Principles of Optics, so far as they apply to photography. 12mo, cloth, 103 illus. and folding plates..$2.50

COLLINS, J. E. The Private Book of Useful Alloys and Memoranda for Goldsmiths, Jewelers, etc. 18mo, cloth..... $0.50

COLLINS, T. B. The Steam Turbine, or the New Engine. 8vo, cloth, illustrated..... *In Press.*

COOPER, W. R., M.A. Primary Batteries: Their Construction and Use. With numerous figures and diagrams. 8vo, cloth, illustrated..... *net*, $4.00

COPPERTHWAITE, WM. C. Tunnel Shields, and the Use of Compressed Air in Subaqueous Works. With numerous diagrams and figures. 4to, cloth, illustrated..... *net*, $9.00

COREY, H. T. Water-supply Engineering. Fully illustrated. *In Press.*

CORNWALL, H. B., Prof. Manual of Blow-pipe Analysis, Qualitative and Quantitative. With a Complete System of Determinative Mineralogy. 8vo, cloth, with many illustrations..... $2.50

COWELL, W. B. Pure Air, Ozone and Water. A Practical Treatise of their Utilization and Value in Oil, Grease, Soap. Paint, Glue and other Industries. With tables and figures. 12mo, cloth, illustrated..... *net*, $2.00

CRAIG, B. F. Weights and Measures. An Account of the Decimal System, with Tables of Conversion for Commercial and Scientific Uses. Square 32mo, limp cloth..... .50

CROCKER, F. B., Prof. Electric Lighting. A Practical Exposition of the Art. For use of Engineers, Students, and others interested in the Installation or Operation of Electrical Plants. Vol. I. The Generating Plant. *New Edition, thoroughly revised and rewritten.* 8vo, cloth, illustrated..... $3.00
Vol. II. Distributing Systems and Lamps. *Fifth Edition.* 8vo, cloth, illustrated..... $3.00

—— and WHEELER, S. S. The Management of Electrical Machinery. Being a *thoroughly revised and rewritten edition* of the authors' "Practical Management of Dynamos and Motors." With a special chapter by H. A. Foster. 12mo, cloth, illustrated. *net*, $1.00

CROSSKEY, L. R. Elementary Perspective: Arranged to meet the requirements of Architects and Draughtsmen, and of Art Students preparing for the elementary examination of the Science and Art Department, South Kensington. With numerous full-page plates and diagrams. 8vo, cloth, illustrated.. $1.00

—— and THAW, J. Advanced Perspective, involving the Drawing of Objects when placed in Oblique Positions, Shadows and Reflections. Arranged to meet the requirements of Architects, Draughtsmen, and Students preparing for the Perspective Examination of the Education Department. With numerous full-page plates and diagrams. 8vo. cloth, illustrated..... $1.50

DAVIES, E. H. Machinery for Metalliferous Mines. A Practical Treatise for Mining Engineers, Metallurgists and Managers of Mines. With upwards of 400 illustrations. *Second Edition, rewritten and enlarged.* 8vo, cloth..... *net*, $8.00

DAVIES, D. C. A Treatise on Metalliferous Minerals and Mining. *Sixth Edition, thoroughly revised and much enlarged* by his son. 8vo, cloth..... *net*, $5.00

—— Mining Machinery..... *In Press.*

DAVISON, G. C., Lieut. Water-tube Boilers..... *In Press.*

DAY, C. The Indicator and its Diagrams. With Chap ters on Engine and Boiler Testing; including a Table of Piston Constants compiled by W. H. Fowler. 12mo, cloth. 125 illustrations..... $2.00

DEITE, Dr. C. Manual of Soapmaking, including medi- cated soaps, stain-removing soaps, metal polishing soaps, soap powders and detergents. With a treatise on perfumes for scented soaps, and their production and tests for purity and strength. Edited from the text of numerous experts. Translated from the original by S. I. King, F.C.S. With figures. 4to, cloth, illustrated. *net*, $5.00

DE LA COUX, H. The Industrial Uses of Water. With numerous tables, figures, and diagrams. Translated from the French and revised by Arthur Morris. 8vo, cloth..... *net*, $4.50

DENNY, G. A. Deep-level Mines of the Rand, and their future development, considered from the commercial point of view. With folding plates, diagrams, and tables. 4to, cloth, illustrated..... *net*, $10.00

DERR, W. L. Block Signal Operation. A Practical Manual. Pocket Size. Oblong, cloth. *Second Edition*.... $1.50

DIBDIN, W. J. Public Lighting by Gas and Electricity. With tables, diagrams, engravings and full-page plates. 8vo, cloth, illustrated..... *net*, $8.00

—— Purification of Sewage and Water. With tables, engravings, and folding plates. *Third Edition, revised and enlarged.* 8vo, cloth, illus. and numerous folding plates.... $6.50

DIETERICH, K. Analysis of Resins, Balsams, and Gum Resins: their Chemistry and Pharmacognosis. For the use of the Scientific and Technical Research Chemist. With a Bibliography. Translated from the German, by Chas. Salter. 8vo. cloth..... *net*, $3.00

DIXON, D. B. The Machinist's and Steam Engineer's Practical Calculator. A Compilation of Useful Rules and Problems arithmetically solved, together with General Information applicable to Shop-tools, Mill-gearing, Pulleys and Shafts, Steam-boilers and Engines. Embracing valuable Tables and Instruction in Screw-cutting, Valve and Link Motion, etc. *Third Edition.* 16mo, full morocco, pocket form..... $1.25

DOBLE, W. A. Power Plant Construction on the Pacific Coast..... *In Press.*

DODD, GEO. Dictionary of Manufactures, Mining, Machinery, and the Industrial Arts. 12mo, cloth..... $1.50

DORR, B. F. The Surveyor's Guide and Pocket Table-book. *Fifth Edition, thoroughly revised and greatly extended.* With a second appendix up to date. 16mo, morocco flaps.. $2.00

DRAPER, C. H. An Elementary Text-book of Light, Heat and Sound, with Numerous Examples. *Fourth Edition.* 12mo, cloth, illustrated..... $1.00

—— Heat and the Principles of Thermo-dynamics. With many illustrations and numerical examples. 12mo, cloth... $1.50

DYSON, S. S. Practical Testing of Raw Materials. A Concise Handbook for Manufacturers, Merchants, and Users of Chemicals, Oils, Fuels, Gas Residuals and By-products, and Paper-making Materials, with Chapters on Water Analysis and the Testing of Trade Effluents. 8vo, cloth, illustrations, 177 pages..... *net*, $5.00

ECCLES, R. G. (Dr.), and DUCKWALL, E. W. Food Preservatives: their Advantages and Proper Use; The Practical *versus* the Theoretical Side of the Pure Food Problem. 8vo, paper..... $0.50
Cloth..... 1.00

EDDY, H. T., Prof. Researches in Graphical Statics. Embracing New Constructions in Graphical Statics, a New General Method in Graphical Statics, and the Theory of Internal Stress in Graphical Statics. 8vo, cloth..... $1.50

—— Maximum Stresses under Concentrated Loads. Treated graphically. Illustrated. 8vo, cloth..... $1.50

EISSLER, M. The Metallurgy of Gold. A Practical Treatise on the Metallurgical Treatment of Gold-bearing Ores, including the Processes of Concentration and Chlorination, and the Assaying, Melting and Refining of Gold. *Fifth Edition, revised and greatly enlarged.* Over 300 illustrations and numerous folding plates. 8vo, cloth..... $7.50

—— The Hydro-Metallurgy of Copper. Being an Account of processes adopted in the Hydro-metallurgical Treatment of Cupriferous Ores, including the Manufacture of Copper Vitriol. With chapters on the sources of supply of Copper and the Roasting of Copper Ores. With numerous diagrams and figures. 8vo, cloth, illustrated..... *net*, $4.50

EISSLER, M. The Metallurgy of Silver. A Practical Treatise on the Amalgamation, Roasting and Lixiviation of Silver Ores, including the Assaying, Melting and Refining of Silver Bullion. 124 illustrations. *Second Edition, enlarged.* 8vo, cloth. $4.00

—— The Metallurgy of Argentiferous Lead. A Practical Treatise on the Smelting of Silver-Lead Ores and the Refining of Lead Bullion. Including Reports on Various Smelting Establishments and Descriptions of Modern Smelting Furnaces and Plants in Europe and America. With 183 illustrations. 8vo, cloth, $5.00

—— Cyanide Process for the Extraction of Gold and its Practical Application on the Witwatersrand Gold Fields in South Africa. *Third Edition, revised and enlarged.* Illustrations and folding plates. 8vo, cloth..... $3.00

—— A Handbook on Modern Explosives. Being a Prac- tical Treatise on the Manufacture and Use of Dynamite, Guncotton, Nitroglycerine, and other Explosive Compounds, including the manufacture of Collodion-cotton, with chapters on Explosives in Practical Application. *Second Edition, enlarged with 150 illustrations.* 12mo, cloth..... $5.00

ELIOT, C. W., and STORER, F. H. A Compendious. Manual of Qualitative Chemical Analysis. Revised with the cooperation of the authors, by Prof. William R. Nichols. Illustrated. *Twentieth Edition, newly revised by Prof. W. B. Lindsay.* 12mo, cloth..... *net*, $1.25

ELLIOT, G. H., Maj. European Light-house Systems. Being a Report of a Tour of Inspection made in 1873. 51 engravings and 21 woodcuts. 8vo, cloth..... $5.00

ERFURT, J. Dyeing of Paper Pulp. A Practical Treatise for the use of paper-makers, paper-stainers, students and others, With illustrations and 157 patterns of paper dyed in the pulp, with formulas for each. Translated into English and edited, with additions, by Julius Hübner, F.C.S. 8vo, cloth, illustrated..... *net*, $7.50

EVERETT, J. D. Elementary Text-book of Physics. Illustrated. *Seventh Edition.* 12mo, cloth..... $1.50

EWING, A. J., Prof. The Magnetic Induction in Iron and other metals. *Third Edition, revised.* 159 illustrations 8vo, cloth..... $4.00

FAIRIE, J., F.G.S. Notes on Lead Ores: Their Distribution and Properties. 12mo, cloth..... $1.00

—— Notes on Pottery Clays: The Distribution, Properties, Uses and Analysis of Ball Clays, China Clays and China Stone. With tables and formulæ. 12mo, cloth..... $1.50

FANNING, J. T. A Practical Treatise on Hydraulic and Water-supply Engineering. Relating to the Hydrology, Hydrodynamics and Practical Construction of Water-works in North America. 180 illus. 8vo, cloth. *Sixteenth Edition, revised, enlarged,* and *new tables and illustrations added.* 650 pp.....$5.00

FAY, I. W. The Coal-tar Colors: Their Origin and Chemistry. 8vo, cloth, illustrated.....*In Press.*

FERNBACH, R. L. Glue and Gelatine; a Practical Treatise on the Methods of Testing and Use. 12mo, cloth..*In Press.*

FISH, J. C. L. Lettering of Wo king Drawings. Thirteen plates, with descriptive text. Oblong, 9×12½, boards. $1.00

FISHER, H. K. C., and DARBY, W. C. Students' Guide to Submarine Cable Testing. *Third (new and enlarged) Edition.* 8vo, cloth, illustrated..... $3.50

FISHER, W. C. The Potentiometer and its Adjuncts. 8vo, cloth..... $2.25

FISKE, B. A., Lieut., U.S.N. Electricity in Theory and Practice; or, The Elements of Electrical Engineering. *Eighth Edition.* 8vo, cloth..... $2.50

FLEISCHMANN, W. The Book of the Dairy. A Manual of the Science and Practice of Dairy Work. Translated from the German, by C. M. Aikman and R. Patrick Wright. 8vo, cloth..... $4.00

FLEMING, J. A., Prof. The Alternate-current Transformer in Theory and Practice. Vol. I., The Induction of Electric Currents; 611 pages. *New Edition,* illustrated. 8vo, cloth, $5.00
Vol. II., The Utilization of Induced Currents. Illustrated. 8vo, cloth..... $5.00

—— Centenary of the El ctrical Current, 1799–1899. 8vo, paper, illustrated..... $0.50

FLEMING, J. A., Prof. Electric Lamps and Electric Lighting. Being a course of four lectures delivered at the Royal Institution, April–May, 1894. 8vo, cloth, fully illustrated.....$3.00

—— Electrical Laboratory Notes and Forms, Elementary and Advanced. 4to, cloth, illustrated..... $5.00

—— A Handbook for the Electrical Laboratory and Testing Room. 2 volumes. 8vo, cloth.... .....each $5.00

FLEURY, H. The Calculus Without Limits or Infinitesimals. Translated by C. O. Mailloux.....*In Press*

FOLEY, N., and PRAY, THOS., Jr. The Mechanical Engineers' Reference Book for Machine and Boiler Construction, in two parts. Part 1—General Engineering Data. Part 2—Boiler Construction. With 51 plates and numerous illustrations, specially drawn for this work. Folio, half morocco. *New Edition*.....*In Press.*

FORNEY, M. N. Catechism of the Locomotive. Second *Edition, revised and enlarged.* 46th thousand. 8vo, cloth..$3.50

FOSTER, H. A. Electrical Engineers' Pocket-book. With the Collaboration of Eminent Specialists. A handbook of useful data for Electricians and Electrical Engineers. With innumerable tables, diagrams, and figures. *Third Edition, revised* Pocket size, full leather, 1000 pp..... $5.00

FOSTER, J. G., Gen., U.S.A. Submarine Blasting in Boston Harbor, Massachusetts. Removal of Tower and Corwin Rocks. Illustrated with 7 plates. 4to, cloth..... $3.50

FOSTER, J. Treatise on the Evaporation of Saccharine, Chemical and other Liquids by the Multiple System in Vacuum and Open Air. *Third Edition.* Diagrams and large plates. 8vo, cloth..... $7.50

FOX, WM., and THOMAS, C. W., M.E. A Practical Course in Mechanical Drawing. *Second Edition, revised.* 12mo, cloth, with plates..... $1.25

FRANCIS, J. B., C.E. Lowell Hydraulic Experiments. Being a selection from experiments on Hydraulic Motors on the Flow of Water over Weirs, in Open Canals of uniform rectangular section, and through submerged Orifices and diverging Tubes. Made at Lowell, Mass. *Fourth Edition, revised and enlarged,* with many new experiments, and illustrated with 23 copper-plate engravings 4to, cloth..... $15.00

FRASER, R. H., and CLARK, C. H. Marine Engineering. *In Press.*

FULLER, G. W. Report on the Investigations into the Purification of the Ohio River Water at Louisville, Kentucky, made to the President and Directors of the Louisville Water Company. Published under agreement with the Directors. 3 full-page plates. 4to, cloth..... *net*, $10.00

FURNELL, J. Students' Manual of Paints, Colors, Oils and Varnishes. 8vo, cloth, illustrated..... *net*, $1.00

GARCKE, E., and FELLS, J. M. Factory Accounts: their principles and practice. A handbook for accountants and manufacturers, with appendices on the nomenclature of machine details, the rating of factories, fire and boiler insurance, the factory and workshop acts, etc., including also a large number of specimen rulings. *Fifth Edition, revised and extended.* 8vo, cloth, illustrated..... $3.00

GEIKIE, J. Structural and Field Geology, for Students of Pure and Applied Science. With figures, diagrams, and half-tone plates. 8vo, cloth, illustrated..... *net*, $4.00

GERBER, N. Chemical and Physical Analysis of Milk, Condensed Milk, and Infants' Milk-food. 8vo, cloth..... $1.25

GERHARD, WM. P. Sanitary Engineering. 12mo, cloth..... $1.25

GESCHWIND, L. Manufacture of Alum and Sulphates, and other Salts of Alumina and Iron; their uses and applications as mordants in dyeing and calico printing, and their other applications in the Arts, Manufactures, Sanitary Engineering, Agriculture and Horticulture. Translated from the French by Charles Salter. With tables, figures and diagrams. 8vo, cloth, illustrated..... *net*, $5.00

GIBBS, W. E. Lighting by Acetylene, Generators, Burners and Electric Furnaces. With 66 illustrations. *Second Edition, revised.* 12mo, cloth..... $1.50

GILLMORE, Q. A., Gen. Treatise on Limes, Hydraulic Cements and Mortars. Papers on Practical Engineering, United States Engineer Department, No. 9, containing Reports of numerous Experiments conducted in New York City during the years 1858 to 1861, inclusive. With numerous illustrations. 8vo, cloth..... $4.00

GILLMORE, Q. A., Gen. Practical Treatise on the Construction of Roads, Streets and Pavements. *Tenth Edition.* With 70 illustrations. 12mo, cloth..... $2.00

—— Report on Strength of the Building Stones in the United States, etc. 8vo, illustrated, cloth..... $1.00

GOLDING, H. A. The Theta-Phi Diagram. Practically Applied to Steam, Gas, Oil and Air Engines. 12mo, cloth, illustrated..... *net*, $1.25

GOODEVE, T. M. A Text-book on the Steam-engine. With a Supplement on Gas-engines. *Twelfth Edition, enlarged.* 143 illustrations. 12mo, cloth..... $2.00

GORE, G., F.R.S. The Art of Electrolytic Separation of Metals, etc. (Theoretical and Practical.) Illustrated. 8vo, cloth..... $3.50

GOULD, E. S. The Arithmetic of the Steam-engine. 8vo, cloth..... $1.00

—— Practical Hydrostatics and Hydrostatic Formulas. With numerous figures and diagrams. (*Van Nostrand's Science Series.*) 16mo, cloth, illustrated, 114 pp..... $0.50

GRAY, J., B.Sc. Electrical Influence Machines: Their Historical Development, and Modern Forms, with instructions for making them. With numerous figures and diagrams. *Second Edition, revised and enlarged.* 12mo, cloth, illus., 296 pp.... $2.00

GREENWOOD, E. Classified Guide to Technical and Commercial Books. Subject List of Principal British and American Works in print. 8vo, cloth..... *net*, $3.00

GRIFFITHS, A. B., Ph.D. A Treatise on Manures, or the Philosophy of Manuring. A Practical Handbook for the Agriculturist, Manufacturer, and Student. 12mo, cloth... $3.00

—— Dental Metallurgy. A Manual for Students and Dentists. 8vo, cloth, illustrated, 208 pp..... *net*, $3.50

GROSS, E. Hops, in their Botanical, Agricultural and Technical Aspect, and as an article of Commerce. Translated from the German by Charles Salter. With tables, diagrams, and illustrations. 8vo, cloth, illustrated..... *net*, $4.50

GROVER, F. Practical Treatise on Modern Gas and Oil Engines. 8vo, cloth, illustrated..... *net*, $2.00

GRUNER, A. Power-loom Weaving and Yarn Numbering, according to various systems, with conversion tables. An auxiliary and text-book for pupils of weaving schools, as well as for self-instruction, and for general use by those engaged in the weaving industry. Illustrated with colored diagrams. 8vo, cloth..... *net*, $3.00

GURDEN, R. L. Traverse Tables: Computed to Four-place Decimals for every single minute of angle up to 100 of Distance. For the use of Surveyors and Engineers. *New Edition.* Folio, half morocco..... $7.50

GUY, A. E. Experiments on the Flexure of Beams, resulting in the Discovery of New Laws of Failure by Buckling. Reprinted from the "American Machinist." With diagrams and folding plates. 8vo, cloth, illustrated, 122 pages..... *net*, $1.25

—— A. F. Electric Light and Power: Giving the Result of Practical Experience in Central-station Work. 8vo, cloth, illustrated..... $2.50

HAEDER, H., C.E. A Handbook on the Steam-engine. With especial reference to small and medium-sized engines. For the use of Engine-makers, Mechanical Draughtsmen, Engineering Students and Users of Steam Power. Translated from the German, with considerable additions and alterations, by H. H. P. Powles. *Third English Edition, revised.* 8vo, cloth, illustrated, 458 pages..... $3.00

HALL, C. H. Chemistry of Paints and Paint Vehicles. 8vo, cloth..... *net*, $2.00

—— W. S., Prof. Elements of the Differential and In-tegral Calculus. *Sixth Edition, revised.* 8vo, cloth, illustrated. *net*, $2.25

—— Descriptive Geometry, with Numerous Problems and Practical Applications. Comprising an 8vo volume of 76 pages of text and a 4to atlas of 31 plates. 2 vols., cloth.... *net*, $3.50

HALSEY, F. A. Slide-valve Gears. An Explanation of the Action and Construction of Plain and Cut-off Slide Valves. Illustrated. *Seventh Edition.* 12mo, cloth $1.50

—— The Use of the Slide Rule. With illustrations and folding plates. *Second Edition.* 16mo, boards. (*Van Nostrand's Science Series*, No. 114.) ..... .50

—— The Locomotive Link Motion, with Diagrams and Tables. 8vo, cloth, illustrated $1.00

—— Worm and Spiral Gearing. Revised and Enlarged *Edition.* 16mo, cloth (*Van Nostrand's Science Series*, No. 116.) Illustrated ..... .50

—— The Metric Fallacy, and "The Metric Failure in the Textile Industry," by Samuel S. Dale. 8vo, cloth, illustrated $1.00

HAMILTON, W. G. Useful Information for Railway Men. *Tenth Edition, revised and enlarged.* 562 pages, pocket form. Morocco, gilt $2.00

HAMMER, W. J. Radium, and Other Radio-active Sub- stances; Polonium, Actinium and Thorium. With a consideration of Phosphorescent and Fluorescent Substances, the Properties and Applications of Selenium, and the treatment of disease by the Ultra-Violet Light. *Second Edition.* With engravings and photographic plates. 8vo, cloth, illustrated, 72 pp... $1.00

HANCOCK, H. Text-book of Mechanics and Hydro- statics, with over 500 diagrams. 8vo, cloth ..... *net*, $1.50

HARDY, E. Elementary Principles of Graphic Statics. Containing 192 diagrams. 8vo, cloth, illustrated $1.50

HARRISON, W. B. The Mechanics' Tool-book. With Practical Rules and Suggestions for use of Machinists, Iron-workers and others. With 44 engravings. 12mo, cloth $1.50

HART, J. W. External Plumbing Work. A Treatise on Lead Work for Roofs. With numerous figures and diagrams. 8vo, cloth, illustrated ..... *net*, $3.00

—— Hints to Plumbers on Joint Wiping, Pipe Bending, and Lead Burning. Containing 184 figures and diagrams. 8vo, cloth, illustrated ..... *net*, $3.00

HART, J. W. Principles of Hot-water Supply. With numerous illustrations. 8vo, cloth.....*net*, $3.00

—— Sanitary Plumbing and Drainage. With numerous diagrams and figures. 8vo, cloth, illustrated..... *net*, $3.00

HASKINS, C. H. The Galvanometer and its Uses. A Manual for Electricians and Students. *Fourth Edition.* 12mo. cloth..... $1.50

HAUFF, W. A. American Multiplier: Multiplications and Divisions of the largest numbers rapidly performed. With index giving the results instantly of all numbers to $1000 \times 1000 = 1{,}000{,}000$; also tables of circumferences and areas of circles. Cloth, $6\frac{1}{2} \times 15\frac{1}{2}$..... $5.00

HAUSBRAND, E. Drying by Means of Air and Steam. With explanations, formulas, and tables, for use in practice. Translated from the German by A. C. Wright, M.A. 12mo, cloth, illustrated.....*net*, $2.00

—— Evaporating, Condensing and Cooling Apparatus: Explanations, Formulæ, and Tables for Use in Practice. Translated from the Second Revised German Edition by A. C. Wright, M.A. With numerous figures, tables and diagrams. 8vo, cloth, illustrated, 400 pages..... *net*, $5.00

HAUSNER, A. Manufacture of Preserved Foods and Sweetmeats. A Handbook of all the Processes for the Preservation of Flesh, Fruit, and Vegetables, and for the Preparation of Dried Fruit, Dried Vegetables, Marmalades, Fruit-syrups, and Fermented Beverages, and of all kinds of Candies, Candied Fruit Sweetmeats, Rocks, Drops, Dragees, Pralines, etc. Translated from the Third Enlarged German Edition, by Arthur Morris and Herbert Robson, B.Sc. 8vo, cloth, illustrated.....*net*, $3.00

HAWKE, W. H. The Premier Cipher Telegraphic Code, containing 100,000 Words and Phrases. The most complete and most useful general code yet published. 4to, cloth..... $5.00

—— 100,000 Words Supplement to the Premier Code. All the words are selected from the official vocabulary. Oblong quarto, cloth..... $5.00

HAWKESWORTH, J. Graphical Handbook for Rein- forced Concrete Design. A series of plates, showing graphically, by means of plotted curves, the required design for slabs, beams, and columns under various conditions of external loading, together with practical examples showing the method of using each plate. 8vo, cloth.....*In Press.*

HAWKINS, C. C., and WALLIS, F. The Dynamo: its Theory, Design, and Manufacture. 190 illustrations. 12mo, cloth.....*net*, $3.00

HAY, A. Alternating Currents; Their Theory, Generation, and Transformation. 8vo, cloth, illustrated.....*net*, $2.50

—— Principles of Alternate-current Working. 12mo, cloth, illustrated..... $2.00

HEAP, D. P., Major, U.S.A. Electrical Appliances of the Present Day. Report of the Paris Electrical Exposition of 1881. 250 illustrations. 8vo, cloth..... $2.00

HEAVISIDE, O. Electromagnetic Theory. 8vo, cloth. two volumes.....each, $5.00

HECK, R. C. H. Steam-Engine and Other Steam Motors. A text-book for engineering colleges and a treatise for engineers. Vol. I. The Thermodynamics and the Mechanics of the Engine. With numerous figures, diagrams, and tables. 8vo, cloth, illustrated.....*net*, $3.50
Vol. II. Form, Construction, and Working of the Engine: The Steam Turbine. 8vo, cloth, illustrated.....*net*, $3.50

HEDGES, K. Modern Lightning Conductors. An Illus-trated Supplement to the Report of the Lightning Research Committee of 1905, with Notes as to Methods of Protection and Specifications. With figures, half-tones, and folding tables. 8vo, cloth, illustrated.....*net*, $3.00

HEERMANN, P. Dyers' Materials. An Introduction to the Examination, Valuation, and Application of the most important substances used in Dyeing, Printing, Bleaching and Finishing. Translated by Arthur C. Wright, M.A. 12mo, cloth, illustrated.....*net*, $2.50

HENRICI, O. Skeleton Structures, Applied to the Build-ing of Steel and Iron Bridges. 8vo, cloth, illustrated..... $1.50

HERMANN, F. Painting on Glass and Porcelain and Enamel Painting. On the basis of Personal Practical Experience of the Condition of the Art up to date. Translated by Charles Salter. *Second greatly enlarged edition.* 8vo, cloth, illustrated, *net*, $3.50

HERRMANN, G. The Graphical Statics of Mechanism. A Guide for the Use of Machinists, Architects and Engineers; and also a Text-book for Technical Schools. Translated and annotated by A. P. Smith, M.E. *Fourth Edition.* 12mo, cloth, 7 folding plates..... $2.00

HERZFELD, J., Dr. The Technical Testing of Yarns and Textile Fabrics, with reference to official specifications. Translated by Chas. Salter. With 69 illustrations. 8vo, cloth *net*, $3.50

HEWSON, W. Principles and Practice of Embanking Lands from River Floods, as applied to the Levees of the Mississippi. 8vo, cloth..... $2.00

HILL, J. W. The Purification of Public Water Supplies. Illustrated with valuable tables, diagrams, and cuts. 8vo, cloth..... $3.00

HIROI, I. Statically-Indeterminate Stresses in Frames Commonly Used for Bridges. With figures, diagrams, and examples. 12mo, cloth, illustrated.....*net*, $2.00

HOBBS, W. R. P. The Arithmetic of Electrical Measurements, with numerous examples, fully worked. Revised by Richard Wormell, M.A. *Ninth Edition.* 12mo, cloth..... .50

HOFF, J. N. Paint and Varnish Facts and Formulas. A hand-book for the maker, dealer, and user of paints and varnishes. Containing over 600 recipes. 8vo, cloth.....*net*, $3.00

HOFF, WM. B., Com., U.S.N. The Avoidance of Collisions at Sea. 18mo, morocco..... .75

HOLLEY, A. L. Railway Practice. American and Euro- pean Railway Practice in the Economical Generation of Steam, including the Materials and Construction of Coal-burning Boilers, Combustion, the Variable Blast, Vaporization, Circulation, Superheating, Supplying and Heating Feed Water, etc., and the Adaptation of Wood and Coke-burning Engines to Coal-burning; and in Permanent Way, including Road-bed, Sleepers, Rails, Joint Fastenings, Street Railways, etc. With 77 lithographed plates. Folio, cloth..... $12.00

HOLMES, A. B. The Electric Light Popularly Explained. *Fifth Edition.* Illustrated. 12mo, paper..... .40

HOPKINS, N. M. Experimental Electrochemistry: Theo- retically and Practically Treated. With 132 figures and diagrams. 8vo, cloth, illustrated.....*net*, $3.00

—— Model Engines and Small Boats. New Methods of Engine and Boiler Making, with a chapter on Elementary Ship Design and Construction. 12mo, cloth..... $1.25

HORNER, J. Engineers' Turning, in Principle and Practice. A Handbook for Working Engineers, Technical Students, and Amateurs. With 488 figures and diagrams. 8vo, cloth, illustrated. ..... *net*, $3.50

HOUSTON, E. J., and KENNELLY, A. E. Algebra Made Easy. Being a clear explanation of the Mathematical Formulæ found in Prof. Thompson's "Dynamo-electric Machinery and Polyphase Electric Currents." With figures and examples. 8vo, cloth, illustrated. ..... .75

—— The Interpretation of Mathematical Formulæ. With figures and examples. 8vo, cloth, illustrated. $1.25

HOWARD, C. R. Earthwork Mensuration on the Basis of the Prismoidal Formulæ. Containing Simple and Labor-saving Methods of obtaining Prismoidal Contents directly from End Areas. Illustrated by Examples and accompanied by Plain Rules for Practical Use. Illustrated. 8vo, cloth. $1.50

HOWORTH, J. Art of Repairing and Riveting Glass, China and Earthenware. *Second Edition.* 8vo, pamphlet, illustrated. ..... *net*, $0.50

HUBBARD, E. The Utilization of Wood-waste. A Complete Account of the Most Advantageous Methods of Working Up Wood Refuse, especially Sawdust, Exhausted Dye Woods and Tan as Fuel, as a Source of Chemical Products for Artificial Wood Compositions, Explosives, Manures, and many other Technical Purposes. Translated from the German of the *second revised and enlarged edition.* 8vo, cloth, illustrated, 192 pages.. *net*, $2.50

HUMBER, W., C.E. A Handy Book for the Calculation of Strains in Girders, and Similar Structures, and their Strength; consisting of Formulæ and Corresponding Diagrams, with numerous details for practical application, etc. *Fourth Edition.* 12mo, cloth. $2.50

HUMPHREYS, A. C. (Stevens Institute). Lecture Notes on some of the Business Features of Engineering Practice. 8vo, cloth, with supplement. ..... *net*, $1.25

HURST, G. H., F.C.S. Color. A Handbook of the Theory of Color. A practical work for the Artist, Art Student, Painter, Dyer and Calico Printer, and Others. Illustrated with 10 colored plates and 72 illustrations. 8vo, cloth. ..... *net*, $2.50

—— Dictionary of Chemicals and Raw Products Used in the Manufacture of Paints, Colors, Varnishes and Allied Preparations. 8vo, cloth. ..... *net*, $3.00

HURST, G.H., F.C.S. Lubricating Oils, Fats and Greases: Their Origin, Preparation, Properties, Uses and Analysis. 313 pages, with 65 illustrations. 8vo, cloth..... *net*, $3.00

—— **Soaps. A Practical Manual of the Manufacture of** Domestic, Toilet and other Soaps. Illustrated with 66 engravings. 8vo, cloth..... *net*, $5.00

—— **Textile Soaps and Oils: A Handbook on the Preparation,** Properties, and Analysis of the Soaps and Oils Used in Textile Manufacturing, Dyeing and Printing. With tables and illustrations. 8vo, cloth..... *net*, $2.50

HUTCHINSON, R. W., Jr. Long Distance Electric Power Transmission: being a Treatise on the Hydro-electric Generation of Energy; its Transformation, Transmission, and Distribution. 12mo, cloth, illustrated..... *In Press.*

—— **and IHLSENG, M. C. Electricity in Mining; being a** Theoretical and Practical Treatise on the Construction, Operation, and Maintenance of Electrical Mining Machinery. 12mo, cloth, illustrated..... *In Press.*

—— **W. B. Patents and How to Make Money out of** Them. 12mo, cloth..... $1.25

HUTTON, W. S. Steam-boiler Construction. A Practical Handbook for Engineers, Boiler-makers and Steam-users. Containing a large collection of rules and data relating to recent practice in the design, construction and working of all kinds of stationary, locomotive and marine steam-boilers. With upwards of 540 illustrations. *Fourth Edition, carefully revised and much enlarged.* 8vo, cloth..... $6.00

—— **Practical Engineer's Handbook, comprising a Treatise** on Modern Engines and Boilers, Marine, Locomotive and Stationary. *Fourth Edition, carefully revised, with additions.* With upwards of 570 illustrations. 8vo, cloth..... $7.00

—— **The Works' Manager's Handbook of Modern Rules,** Tables and Data for Civil and Mechanical Engineers, Millwrights and Boiler-makers, etc., etc. With upwards of 150 illustrations. *Fifth Edition, carefully revised, with additions.* 8vo, cloth..... $6.00

INGLE, H. Manual of Agricultural Chemistry. 8vo, cloth, illustrated, 388 pages..... *net*, $3.00

INNES, C. H. Problems in Machine Design. For the use of Students, Draughtsmen and others. *Second Edition*, 12mo, cloth..... *net*, $2.00

—— Centrifugal Pumps, Turbines and Water Motors. Including the Theory and Practice of Hydraulics. *Fourth and enlarged edition.* 12mo, cloth..... *net*, $2.00

ISHERWOOD, B. F. Engineering Precedents for Steam Machinery. Arranged in the most practical and useful manner for Engineers. With illustrations. Two volumes in one. 8vo, cloth..... $2.50

JAMIESON, A., C.E. A Text-book on Steam and Steam-engines. Specially arranged for the use of Science and Art, City and Guilds of London Institute, and other Engineering Students. *Thirteenth Edition.* Illustrated. 12mo, cloth..... $3.00

—— Elementary Manual on Steam and the Steam-engine. Specially arranged for the use of First-year Science and Art, City and Guilds of London Institute, and other Elementary Engineering Students. *Third Edition.* 12mo, cloth..... $1.50

JANNETTAZ, E. A Guide to the Determination of Rocks: being an Introduction to Lithology. Translated from the French by G. W. Plympton, Professor of Physical Science at Brooklyn Polytechnic Institute. 12mo, cloth..... $1.50

JEHL, F., Mem. A.I.E.E. The Manufacture of Carbons for Electric Lighting and Other Purposes. A Practical Handbook, giving a complete description of the art of making carbons, electros, etc. The various gas generators and furnaces used in carbonizing, with a plan for a model factory. Illustrated with numerous diagrams, tables, and folding plates. 8vo, cloth, illustrated. *net*, $4.00

JENNISON, F. H. The Manufacture of Lake Pigments from Artificial Colors. A useful handbook for color manufacturers, dyers, color chemists, paint manufacturers, drysalters, wallpaper-makers, enamel and surface-paper makers. With 15 plates illustrating the various methods and errors that arise in the different processes of production. 8vo, cloth..... *net*, $3.00

JEPSON, G. Cams, and the Principles of their Construction. With figures, half-tones, full-page and folding plates. 8vo, cloth, illustrated..... *net*, $1.50

JOCKIN, WM. Arithmetic of the Gold and Silversmith. Prepared for the use of Jewelers, Founders, Merchants, etc., especially for those engaged in the conversion and alloying of gold or other metals, the mixing of various substances, etc. With numerous examples. 12mo, cloth..... $1.25

JOHNSON, W. McA. "The Metallurgy of Nickel." *In Press.*

JOHNSTON, J. F. W., Prof., and CAMERON, Sir Chas. Elements of Agricultural Chemistry and Geology. *Seventeenth Edition.* 12mo, cloth..... $2.60

JONES, H. C. Outlines of Electrochemistry. With tables and diagrams. 4to, cloth, illustrated..... $1.50

—— Electrical Nature of Matter and Radioactivity. 12mo, cloth.....*net*, $2.00

JONES, M. W. The Testing and Valuation of Raw Materials used in Paint and Color Manufacture. 12mo, cloth,.*net*, $2.00

JOYNSON, F. H. The Metals Used in Construction. Iron, Steel, Bessemer Metal, etc. Illustrated. 12mo, cloth... .75

—— Designing and Construction of Machine Gearing. Illustrated. 8vo, cloth..... $2.00

JÜPTNER, H. F. V. Siderology: The Science of Iron. (The Constitution of Iron Alloys and Iron.) Translated from the German. 8vo, cloth, 345 pages, illustrated..... *net*, $5.00

KANSAS CITY BRIDGE, THE. With an Account of the Regimen of the Missouri River and a Description of the Methods used for Founding in that River, by O. Chanute, Chief Engineer, and George Morison, Assistant Engineer. Illustrated with 5 lithographic views and 12 plates of plans. 4to, cloth. $6.00

KAPP, G., C.E. Electric Transmission of Energy and its Transformation, Subdivision and Distribution. A practical handbook. *Fourth Edition, revised.* 12mo, cloth..... $3.50

—— Dynamos, Motors, Alternators and Rotary Converters. Translated from the *third German edition*, by Harold H. Simmons, A.M.I.E.E. With numerous diagrams and figures. 8vo, cloth, 507 pages..... $4.00

KEIM, A. W. Prevention of Dampness in Buildings. With Remarks on the Causes, Nature and Effects of Saline Efflorescences and Dry Rot. For Architects, Builders, Overseers, Plasterers, Painters and House Owners. Translated from the *second revised, German edition.* With colored plates and diagrams. 8vo, cloth, illustrated, 115 pages..... *net*, $2.00

KELSEY, W. R. Continuous-current Dynamos and Motors, and their Control: being a series of articles reprinted from *The Practical Engineer*, and completed by W. R. Kelsey. With many figures and diagrams. 8vo, cloth, illustrated. . .$2.50

KEMP, J. F., A.B., E.M. (Columbia Univ.). A Handbook of Rocks. For use without the microscope. With a glossary of the names of rocks and of other lithological terms. *Third Edition, revised.* 8vo, cloth, illustrated..... $1.50

KEMPE, H. R. The Electrical Engineer's Pocket-book of Modern Rules, Formulæ, Tables and Data. Illustrated. 32mo, morocco, gilt..... $1.75

KENNEDY, R. Modern Engines and Power Generators. A Practical Work on Prime Movers and the Transmission of Power: Steam, Electric, Water, and Hot-air. With tables, figures, and full-page engravings. 6 vols. 8vo, cloth, illustrated. $15.00
Single volumes, each..... $3.00

—— **Electrical Installations of Electric Light, Power,** Traction, and Industrial Electrical Machinery. With numerous diagrams and engravings.

—— **Vol. I. The Electrical Circuit, Measurement, Elements** of Motors, Dynamos, Electrolysis. 8vo, cloth, illus.. $3.50

—— **Vol. II. Instruments, Transformers, Installation Wiring,** Switches and Switchboards. 8vo, cloth, illustrated.... $3.50

—— **Vol. III. Production of Electrical Energy, Prime** Movers, Generators and Motors. 8vo, cloth, illustrated... $3.50

—— **Vol. IV. Mechanical Gearing; Complete Electric Installations;** Electrolytic, Mining and Heating Apparatus; Electric Traction; Special Applications of Electric Motors. 8vo, cloth, illustrated..... $3.50

KENNEDY, R. Vol. V. Apparatus and Machinery used in Telegraphs, Telephones, Signals, Wireless Telegraph, X-Rays, and Medical Science. 8vo, cloth, illustrated.....$3.50
Complete sets of the five volumes..... $15.00

KENNELLY, A. E. Theoretical Elements of Electro-dynamic Machinery. 8vo, cloth..... $1.50

KINGDON, J. A. Applied Magnetism. An Introduction to the Design of Electromagnetic Apparatus. 8vo, cloth.. $3.00

KINZBRUNNER, C. Alternate Current Windings; Their Theory and Construction. A Handbook for Students, Designers, and all Practical Men. 8vo, cloth, illustrated.....*net*, $1.50

—— Continuous Current Armatures; Their Winding and Construction. A Handbook for Students, Designers, and all Practical Men. 8vo, cloth, illustrated.....*net*, $1.50

KIRKALDY, W. G. Illustrations of David Kirkaldy's System of Mechanical Testing, as Originated and Carried on by him during a Quarter of a Century. Comprising a Large Selection of Tabulated Results, showing the Strength and other Properties of Materials used in Construction, with Explanatory Text and Historical Sketch. Numerous engravings and 25 lithographed plates. 4to, cloth..... $10.00

KIRKBRIDE, J. Engraving for Illustration: Historical and Practical Notes, with illustrations and 2 plates by ink photo process. 8vo, cloth, illustrated..... *net*, $1.50

KIRKWOOD, J. P. Report on the Filtration of River Waters for the Supply of Cities, as practised in Europe, made to the Board of Water Commissioners of the City of St. Louis. Illustrated by 30 double-page engravings. 4to, cloth $7.50

KLEIN, J. F. Design of a High-speed Steam-engine. With notes, diagrams, formulas and tables. *Second Edition, revised and enlarged.* 8vo, cloth, illustrated.. .....*net*, $5.00

KLEINHANS, F. B. Boiler Construction. A Practical ex-planation of the best modern methods of Boiler Construction, from the laying out of sheets to the completed Boiler. With diagrams and full-page engravings. 8vo, cloth, illustrated..$3.00

KNIGHT, A. M., Lieut.-Com. U.S.N. Modern Seaman-ship. Illustrated with 136 full-page plates and diagrams. 8vo, cloth, illustrated. *Second Edition, revised*..... *net*, $6.00
Half morocco..... $7.50

KNOTT, C. G., and MACKAY, J. S. Practical Mathematics. With numerous examples, figures and diagrams. *New Edition.* 8vo, cloth, illustrated..... $2.00

KOLLER, T. The Utilization of Waste Products. A Treatise on the Rational Utilization, Recovery and Treatment of Waste Products of all kinds. Translated from the German *second revised edition.* With numerous diagrams. 8vo, cloth, illustrated..... *net*, $3.50

KOLLER, T. Cosmetics. A Handbook of the Manufacture, Employment and Testing of all Cosmetic Materials and Cosmetic Specialties. Translated from the German by Chas. Salter. 8vo. cloth..... *net*, $2.50

KRAUCH, C., Dr. Testing of Chemical Reagents for Purity. Authorized translation of the *Third Edition*, by J. A. Williamson and L. W. Dupre. With additions and emendations by the author. 8vo, cloth..... *net*, $4.50

LAMBERT, T. Lead, and its Compounds. With tables, diagrams and folding plates. 8vo, cloth..... *net*, $3.50

—— **Bone Products and Manures. An Account of** the most recent improvements in the manufacture of Fat, Glue, Animal Charcoal, Size, Gelatine and Manures. With plans and diagrams. 8vo, cloth, illustrated..... *net*, $3.00

LAMBORN, L. L. Cottonseed Products: A Manual of the Treatment of Cottonseed for its Products and Their Utilization in the Arts. With Tables, figures, full-page plates, and a large folding map. 8vo, cloth, illustrated..... *net*, $3.00

—— **Modern Soaps, Candles, and Glycerin. A practical** manual of modern methods of utilization of Fats and Oils in the manufacture of Soaps and Candles, and the recovery of Glycerin. 8vo, cloth, illustrated..... *net*, $7.50

LAMPRECHT, R. Recovery Work after Pit Fires. A description of the principal methods pursued, especially in fiery mines, and of the various appliances employed, such as respiratory and rescue apparatus, dams, etc. With folding plates and diagrams. Translated from the German by Charles Salter. 8vo, cloth, illustrated..... *net*, $4.00

LARRABEE, C. S. Cipher and Secret Letter and Tele- graphic Code, with Hog's Improvements. The most perfect Secret Code ever invented or discovered. Impossible to read without the key. 18mo, cloth..... .60

LASSAR-COHN, Dr. An Introduction to Modern Scien- tific Chemistry, in the form of popular lectures suited to University Extension Students and general readers. Translated from the author's corrected proofs for the *second German edition*, by M. M. Pattison Muir, M.A. 12mo, cloth, illustrated..... $2.00

LATTA, M. N. Gas Engineering Practice. With figures, diagrams and tables. 8vo, cloth, illustrated.....*in Press.*

LEASK, A. R. Breakdowns at Sea and How to Repair Them. With 89 illustrations. *Second Edition.* 8vo, cloth. $2.00

—— Triple and Quadruple Expansion Engines and Boilers and their Management. With 59 illustrations. *Third Edition, revised.* 12mo, cloth..... $2.00

—— Refrigerating Machinery: Its Principles and Management. With 64 illustrations. 12mo, cloth..... $2.00

LECKY, S. T. S. "Wrinkles" in Practical Navigation. With 130 illustrations. 8vo, cloth. *Fourteenth Edition, revised and enlarged*..... $8.00

LEFÈVRE, L. Architectural Pottery: Bricks, Tiles, Pipes, Enameled Terra-Cottas, Ordinary and Incrusted Quarries, Stoneware Mosaics, Faiences and Architectural Stoneware. With tables, plates and 950 cuts and illustrations. With a preface by M. J.-C. Formigé. Translated from the French, by K. H. Bird, M.A., and W. Moore Binns. 4to, cloth, illustrated..... *net*, $7.50

LEHNER, S. Ink Manufacture: including Writing, Copy- ing, Lithographic, Marking, Stamping and Laundry Inks. Translated from the *fifth German edition*, by Arthur Morris and Herbert Robson, B.Sc. 8vo, cloth, illustrated.....*net*, $2.50

LEMSTROM, Dr. Electricity in Agriculture and Horticul- ture. Illustrated.....*net*, $1.50

LEVY, C. L. Electric-light Primer. A simple and com- prehensive digest of all the most important facts connected with the running of the dynamo, and electric lights, with precautions for safety. For the use of persons whose duty it is to look after the plant. 8vo, paper..... .50

LIVERMORE, V. P., and WILLIAMS, J. How to Become a Competent Motorman. Being a Practical Treatise on the Proper Method of Operating a Street Railway Motor Car; also giving details how to overcome certain defects. 16mo, cloth, illustrated, 132 pages..... $1.00

LOBBEN, P., M.E. Machinists' and Draftsmen's Hand-book, containing Tables, Rules, and Formulas, with numerous examples, explaining the principles of mathematics and mechanics, as applied to the mechanical trades. Intended as a reference book for all interested in Mechanical work. Illustrated with many cuts and diagrams. 8vo, cloth..... $2.50

LOCKE, A. G. and C. G. A Practical Treatise on the Manufacture of Sulphuric Acid. With 77 constructive plates, drawn to scale measurements, and other illustrations. Royal 8vo, cloth. $10.00

LOCKERT, L. Petroleum Motor-cars. 12mo, cloth, $1.50

LOCKWOOD, T. D. Electricity, Magnetism, and Electro-telegraphy. A Practical Guide for Students, Operators, and Inspectors. 8vo, cloth. *Third Edition*..... $2.50

—— Electrical Measurement and the Galvanometer: its Construction and Uses. *Second Edition.* 32 illustrations. 12mo, cloth..... $1.50

LODGE, O. J. Elementary Mechanics, including Hydro-statics and Pneumatics. *Revised Edition.* 12mo, cloth ... $1.50

—— Signalling Across Space, Without Wires: being a description of the work of Hertz and his successors. With numerous diagrams and half-tone cuts, and additional remarks concerning the application to Telegraphy and later developments. *Third Edition.* 8vo, cloth, illustrated..... *net*, $2.00

LORD, R. T. Decorative and Fancy Fabrics. A Valuable Book with designs and illustrations for manufacturers and designers of Carpets, Damask, Dress and all Textile Fabrics. 8vo, cloth, illustrated..... *net*, $3.50

LORING, A. E. A Handbook of the Electro-magnetic Telegraph. 16mo, cloth, boards. *New and enlarged edition*.. .50

LUCE, S. B. (Com., U. S. N.). Text-book of Seamanship. The Equipping and Handling of Vessels under Sail or Steam. For the use of the U. S. Naval Academy. *Revised and enlarged edition*, by Lieut. Wm. S. Benson. 8vo, cloth, illustrated. $10.00

LUCKE, C. E. Gas Engine Design. With figures and diagrams. *Second Edition, revised.* 8vo, cloth, illustrated. *net*, $3.00

—— Power, Cost and Plant Designs and Construction. *In Press.*

LUCKE, C. E. Power Plant Papers. Form I. The Steam Power Plant. Pamphlet (8×13)..... *net*, $1.50

LUNGE, G., Ph.D. Coal-tar and Ammonia: being the third and enlarged edition of "A Treatise on the Distillation of Coal-tar and Ammoniacal Liquor," with numerous tables, figures and diagrams. Thick 8vo, cloth, illustrated..... *net*, $15.00

—— **A Theoretical and Practical Treatise on the Manufacture** of Sulphuric Acid and Alkali with the Collateral Branches.

—— **Vol. I. Sulphuric Acid. In two parts, not sold separately.** *Second Edition, revised and enlarged.* 342 illus. 8vo, cloth.. $15.00

—— **Vol. II. Salt Cake, Hydrochloric Acid and Leblanc** Soda. *Second Edition, revised and enlarged.* 8vo, cloth... $15.00

—— **Vol. III. Ammonia Soda, and various other processes** of Alkali-making, and the preparation of Alkalis, Chlorine and Chlorates, by Electrolysis. 8vo, cloth. *New Edition*, 1896.. $15.00

—— **and HURTER, F. The Alkali Maker's Handbook.** Tables and Analytical Methods for Manufacturers of Sulphuric Acid, Nitric Acid, Soda, Potash and Ammonia. *Second Edition.* 12mo, cloth..... $3.00

LUPTON, A., PARR, G. D. A., and PERKIN, H. Electricity as Applied to Mining. With tables, diagrams and folding plates. *Second Edition, revised and enlarged.* 8vo, cloth, illustrated..... *net*, $4.50

LUQUER, L. M., Ph.D. (Columbia Univ.). Minerals in Rock Sections. The Practical Method of Identifying Minerals in Rock Sections with the Microscope. Especially arranged for Students in Technical and Scientific Schools. *Revised Edition.* 8vo, cloth, illustrated..... *net*, $1.50

MACKIE, JOHN. How to Make a Woolen Mill Pay. 8vo, cloth..... *net*, $2.00

MACKROW, C. The Naval Architect's and Ship-builder's Pocket-book of Formulæ, Rules, and Tables; and Engineers' and Surveyors' Handy Book of Reference. *Eighth Edition, revised and enlarged.* 16mo, limp leather, illustrated..... $5.00

MAGUIRE, E., Capt., U.S.A. The Attack and Defence of Coast Fortifications With maps and numerous illustrations, 8vo, cloth..... $2.50

MAGUIRE, WM. R. Domestic Sanitary Drainage and Plumbing Lectures on Practical Sanitation. 332 illustrations. 8vo..... $4.00

MAILLOUX, C. O. Electro-traction Machinery. 8vo, cloth, illustrated.....*In Press.*

MARKS, E. C. R. Notes on the Construction of Cranes and Lifting Machinery. With numerous diagrams and figures. *New and enlarged edition.* 12mo, cloth.....*net*, $1.50

——Notes on the Construction and Working of Pumps. With figures, diagrams and engravings. 12mo, cloth, illustrated.....*net*, $1.50

——G. C. Hydraulic Power Engineering. A Practical Manual on the Concentration and Transmission of Power by Hydraulic Machinery. With over 200 diagrams and tables 8vo, cloth, illustrated..... $3.50

MARSH, C. F. Reinforced Concrete. With full-page and folding plates, and 512 figures and diagrams. 4to, cloth, illustrated.....*net*, $7.00

MAVER, W. American Telegraphy: Systems, Apparatus, Operation. 450 illustrations. 8vo, cloth..... $5.00

MAYER, A. M., Prof. Lecture Notes on Physics. 8vo, cloth..... $2.00

McCULLOCH, R. S., Prof. Elementary Treatise on the Mechanical Theory of Heat, and its application to Air and Steam-engines. 8vo, cloth..... $3.50

McINTOSH, J. G. Technology of Sugar. A Practical Treatise on the Manufacture of Sugar from the Sugar-cane and Sugar-beet. With diagrams and tables. 8vo, cloth, illustrated.*net*, $4.50

——Manufacture of Varnishes and Kindred Industries. Based on and including the "Drying Oils and Varnishes," of Ach. Livache. Volume I. Oil Crushing, Refining and Boiling, Manufacture of Linoleum, Printing and Lithographic Inks, and India-rubber Substitutes. *Second greatly enlarged English Edition.* 8vo, cloth, illustrated.....*net*, $3.50
(To be complete in three volumes.)

McNEILL, B. McNeill's Code. Arranged to meet the requirements of Mining, Metallurgical and Civil Engineers, Directors of Mining, Smelting and other Companies, Bankers, Stock and Share Brokers, Solicitors, Accountants, Financiers and General Merchants. Safety and Secrecy. 8vo, cloth. ... $6.00

McPHERSON, J. A., A. M. Inst. C. E. Waterworks Distribution. A practical guide to the laying out of systems of distributing mains for the supply of water to cities and towns With tables, folding plates and numerous full-page diagrams 8vo, cloth, illustrated. $2.50

MERCK, E. Chemical Reagents: Their Purity and Tests. *In Press.*

MERRITT, WM. H. Field Testing for Gold and Silver. A Practical Manual for Prospectors and Miners. With numerous half-tone cuts, figures and tables. 16mo, limp leather, illustrated. $1.50

METAL TURNING. By a Foreman Pattern-maker. Illus- trated with 81 engravings. 12mo, cloth. $1.50

MICHELL, S. Mine Drainage: being a Complete Prac- tical Treatise on Direct-acting Underground Steam Pumping Machinery. Containing many folding plates, diagrams and tables. *Second Edition, rewritten and enlarged.* Thick 8vo, cloth, illustrated. $10.00

MIERZINSKI, S., Dr. Waterproofing of Fabrics. Trans- lated from the German by Arthur Morris and Herbert Robson. With diagrams and figures. 8vo, cloth, illustrated... *net*, $2.50

MILLER, E. H. (Columbia Univ.). Quantitative Analysis for Mining Engineers. 8vo, cloth. ..... *net*, $1.50

MINIFIE, W. Mechanical Drawing. A Text-book of Geometrical Drawing for the use of Mechanics and Schools, in which the Definitions and Rules of Geometry are familiarly explained; the Practical Problems are arranged from the most simple to the more complex, and in their description technicalities are avoided as much as possible. With illustrations for drawing Plans, Sections, and Elevations of Railways and Machinery; an Introduction to Isometrical Drawing, and an Essay on Linear Perspective and Shadows. Illustrated with over 200 diagrams engraved on steel. *Tenth Thousand, revised.* With an Appendix on the Theory and Application of Colors. 8vo, cloth.. $4.00

MINIFIE, W. Geometrical Drawing. Abridged from the octavo edition, for the use of schools. Illustrated with 48 steel plates. *Ninth Edition.* 12mo, cloth..... $2.00

MODERN METEOROLOGY. A Series of Six Lectures, delivered under the auspices of the Meteorological Society in 1870. Illustrated. 12mo, cloth..... $1.50

MOORE, E. C. S. New Tables for the Complete Solu-tion of Ganguillet and Kutter's Formula for the flow of liquids in open channels, pipes, sewers and conduits. In two parts. Part I, arranged for 1080 inclinations from 1 over 1 to 1 over 21,120 for fifteen different values of (n). Part II, for use with all other values of (n). With large folding diagram. 8vo, cloth, illustrated..... *net*, $5.00

MOREING, C. A., and NEAL, T. New General and Mining Telegraph Code. 676 pages, alphabetically arranged. For the use of mining companies, mining engineers, stock brokers, financial agents, and trust and finance companies. *Eighth Edition.* 8vo, cloth..... $5.00

MOSES, A. J. The Characters of Crystals. An Intro-duction to Physical Crystallography, containing 321 illustrations and diagrams. 8vo..... *net*, $2.00

—— and PARSONS, C. L. Elements of Mineralogy, Crystallography and Blowpipe Analysis from a Practical Standpoint. *Third Enlarged Edition.* 8vo, cloth, 336 illustrations, *net*, $2.50

MOSS, S. A. Elements of Gas Engine Design. Reprint of a Set of Notes accompanying a Course of Lectures delivered at Cornell University in 1902. 16mo, cloth, illustrated. (Van Nostrand's Science Series)..... $0.50

MOSS, S. A. The Lay-out of Corliss Valve Gears. (*Van Nostrand's Science Series.*) 16mo, cloth, illustrated..... $0.50

MULLIN, J. P., M.E. Modern Moulding and Pattern-making. A Practical Treatise upon Pattern-shop and Foundry Work: embracing the Moulding of Pulleys, Spur Gears, Worm Gears, Balance-wheels, Stationary Engine and Locomotive Cylinders, Globe Valves, Tool Work, Mining Machinery, Screw Propellers, Pattern-shop Machinery, and the latest improvements in English and American Cupolas; together with a large collection of original and carefully selected Rules and Tables for every-day use in the Drawing Office, Pattern-shop and Foundry. 12mo, cloth, illustrated..... $2.50

MUNRO, J., C.E., and JAMIESON, A., C.E. A Pocket-book of Electrical Rules and Tables for the use of Electricians and Engineers. *Fifteenth Edition, revised and enlarged.* With numerous diagrams. Pocket size. Leather..... $2.50

MURPHY, J. G., M.E. Practical Mining. A Field Manual for Mining Engineers. With Hints for Investors in Mining Properties. 16mo, cloth..... $1.00

NAQUET, A. Legal Chemistry. A Guide to the Detection of Poisons, Falsification of Writings, Adulteration of Alimentary and Pharmaceutical Substances, Analysis of Ashes, and Examination of Hair, Coins, Arms and Stains, as applied to Chemical Jurisprudence, for the use of Chemists, Physicians, Lawyers, Pharmacists and Experts. Translated, with additions, including a list of books and memoirs on Toxicology, etc., from the French, by J. P. Battershall, Ph.D., with a Preface by C. F. Chandler, Ph.D., M.D., LL.D. 12mo, cloth..... $2.00

NASMITH, J. The Student's Cotton Spinning. Third *Edition, revised and enlarged.* 8vo, cloth, 622 pages, 250 illustrations..... $3.00

NEUBURGER, H., and NOALHAT, H. Technology of Petroleum. The Oil Fields of the World: their History, Geography and Geology. Annual Production, Prospection and Development; Oil-well Drilling; Transportation of Petroleum by Land and Sea. Storage of Petroleum. With 153 illustrations and 25 plates. Translated from the French, by John Geddes McIntosh. 8vo, cloth, illustrated..... *net*, $10.00

NEWALL, J. W. Plain Practical Directions for Drawing, Sizing and Cutting Bevel-gears, showing how the Teeth may be cut in a Plain Milling Machine or Gear Cutter so as to give them a correct shape from end to end; and showing how to get out all particulars for the Workshop without making any Drawings. Including a Full Set of Tables of Reference. Folding plates. 8vo, cloth..... $1.50

NEWLANDS, J. The Carpenters' and Joiners' Assistant: being a Comprehensive Treatise on the Selection, Preparation and Strength of Materials, and the Mechanical Principles of Framing, with their application in Carpentry, Joinery and Hand-railing; also, a Complete Treatise on Sines; and an Illustrated Glossary of Terms used in Architecture and Building. Illustrated. Folio, half morocco..... $15.00

NIPHER, F. E., A.M. Theory of Magnetic Measurements, with an Appendix on the Method of Least Squares. 12mo, cloth..... $1.00

NOLL, AUGUSTUS. How to Wire Buildings: A Manual of the Art of Interior Wiring. With many illustrations. *Fourth Edition.* 8vo, cloth, illustrated..... $1.50

NUGENT, E. Treatise on Optics; or, Light and Sight Theoretically and Practically Treated, with the Application to Fine Art and Industrial Pursuits. With 103 illustrations. 12mo, cloth..... $1.50

O'CONNOR, H. The Gas Engineer's Pocket-book. Com- prising Tables, Notes and Memoranda relating to the Manufacture, Distribution and Use of Coal-gas and the Construction of Gas-works. *Second Edition, revised.* 12mo, full leather, gilt edges..... $3.50

OLSEN, J. C., Prof. Text-book of Quantitative Chemical Analysis by Gravimetric, Electrolytic, Volumetric and Gasometric Methods. With Seventy-two Laboratory Exercises giving the Analysis of Pure Salts, Alloys, Minerals and Technical Products. With numerous figures and diagrams. *Second Edition, revised.* 8vo, cloth..... *net*, $4.00

OSBORN, F. C. Tables of Moments of Inertia, and Squares of Radii of Gyration; supplemented by others on the Ultimate and Safe Strength of Wrought-iron Columns, Safe Strength of Timber Beams, and Constants for readily obtaining the Shearing Stresses, Reactions and Bending Moments in Swing Bridges. *Fifth Edition.* 12mo, leather..... *net*, $3.00

OUDIN, M. A. Standard Polyphase Apparatus and Systems. With many diagrams and figures. *Third Edition, thoroughly revised.* Fully illustrated..... $3.00

PALAZ, A., Sc.D. A Treatise on Industrial Photometry, with special application to Electric Lighting. Authorized translation from the French by George W. Patterson, Jr. *Second Edition, revised.* 8vo, cloth, illustrated..... $4.00

PAMELY, C. Colliery Manager's Handbook. A Compre- hensive treatise on the Laying-out and Working of Collieries. Designed as a book of reference for colliery managers and for the use of coal-mining students preparing for first-class certificates. *Fifth Edition, revised and enlarged.* Containing over 1,000 diagrams, plans, and engravings. 8vo, cloth, illustrated..*net*, $10.00

PARR, G. D. A. Electrical Engineering Measuring Instruments, for Commercial and Laboratory Purposes. With 370 diagrams and engravings. 8vo, cloth, illustrated.....*net*, $3.50

PARRY, E. J., B.Sc. The Chemistry of Essential Oils and Artificial Perfumes. Being an attempt to group together the more important of the published facts connected with the subject; also giving an outline of the principles involved in the preparation and analysis of Essential Oils. With numerous diagrams and tables. 8vo, cloth, illustrated.....*net*, $5.00

—— **and COSTE, J. H. Chemistry of Pigments.** With tables and figures. 8vo, cloth.....*net*, $4.50

PARRY, L. A., M.D. The Risks and Dangers of Various Occupations and their Prevention. A book that should be in the hands of manufacturers, the medical profession, sanitary inspectors, medical officers of health, managers of works, foremen and workmen. 8vo, cloth.....*net*, $3.00

PARSHALL, H. F., and HOBART, H. M. Armature Windings of Electric Machines. With 140 full-page plates, 65 tables and 165 pages of descriptive letter-press. 4to, cloth. $7.50

—— **and PARRY, E. Electrical Equipment of Tramways.** *In Press.*

PASSMORE, A. C. Handbook of Technical Terms used in Architecture and Building, and their Allied Trades and Subjects. 8vo, cloth.....*net*, $3.50

PATERSON, D., F.C.S. The Color Printing of Carpet Yarns. A useful manual for color chemists and textile printers. With numerous illustrations. 8vo, cloth, illustrated...*net*, $3.50

—— **Color Matching on Textiles. A Manual intended for** the use of Dyers, Calico Printers, and Textile Color Chemists. Containing colored frontispiece and 9 illustrations, and 14 dyed patterns in appendix. 8vo, cloth, illustrated.....*net*, $3.00

—— **The Science of Color Mixing. A Manual intended** for the use of Dyers, Calico Printers, and Color Chemists. With figures, tables, and colored plate. 8vo, cloth, illustrated.*net*, $3.00

PATTEN, J. A Plan for Increasing the Humidity of the Arid Region and the Utilization of Some of the Great Rivers of the United States for Power and other Purposes. A paper communicated to the National Irrigation Congress, Ogden, Utah. Sept. 12, 1903. 4to, pamphlet, 20 pages, with 7 maps. .. $1.00

PATTON, H. B. Lecture Notes on Crystallography *Revised Edition, largely rewritten.* Prepared for use of the students at the Colorado School of Mines. With blank pages for note-taking. 8vo, cloth. ..... *net* $1.25

PAULDING, C. P. Practical Laws and Data on the Condensation of Steam in Covered and Bare Pipes; to which is added a translation of Péclet's "Theory and Experiments on the Transmission of Heat Through Insulating Materials." 8vo, cloth, illustrated, 102 pages. ..... *net*, $2.00

—— **Transmission of Heat through Cold-storage Insulation:** Formulas, Principles, and Data Relating to Insulation of Every Kind. A Manual for refrigerating engineers. With tables and diagrams. 12mo, cloth, illustrated..... *net*, $1.00

PEIRCE, B. System of Analytic Mechanics. 4to, cloth. $10.00

PERRINE, F. A. C., A.M., D.Sc. Conductors for Electrical Distribution: their Manufacture and Materials, the Calculation of Circuits, Pole Line Construction, Underground Working and other Uses. With numerous diagrams and engravings. 8vo, cloth, illustrated, 287 pages. ..... *net*, $3.50

PERRY, J. Applied Mechanics. A Treatise for the Use of students who have time to work experimental, numerical, and graphical exercises illustrating the subject. 8vo, cloth, 650 pages..... *net*, $2.50

PHILLIPS, J. Engineering Chemistry. A Practical Treatise for the use of Analytical Chemists, Engineers, Iron Masters, Iron Founders, students and others. Comprising methods of Analysis and Valuation of the principal materials used in Engineering works, with numerous Analyses, Examples, and Suggestions. Illustrated. *Third Edition, revised and enlarged.* 8vo, cloth..... *net*, $4.50

—— **Gold Assaying. A Practical Handbook giving the** Modus Operandi for the Accurate Assay of Auriferous Ores and Bullion, and the Chemical Tests required in the Processes of Extraction by Amalgamation, Cyanidation, and Chlorination. With an appendix of tables and statistics and numerous diagrams and engravings. 8vo, cloth, illustrated..... *net*, $2.50

PHIN, J. Seven Follies of Science. A Popular Account of the most famous scientific impossibilities and the attempts which have been made to solve them; to which is added a small Budget of Interesting Paradoxes, Illusions, and Marvels. With numerous illustrations. 8vo, cloth, illustrated..... *net*, $1.25

PICKWORTH, C. N. The Indicator Handbook. A Practical Manual for Engineers. Part I. The Indicator: its Construction and Application. 81 illustrations. 12mo, cloth. $1.50

—— The Indicator Handbook. Part II. The Indicator Diagram: its Analysis and Calculation. With tables and figures. 12mo, cloth, illustrated..... $1.50

—— Logarithms for Beginners. 8vo, boards..... $0.50

—— The Slide Rule. A Practical Manual of Instruction for all Users of the Modern Type of Slide Rule, containing Succinct Explanation of the Principle of Slide-rule Computation, together with Numerous Rules and Practical Illustrations, exhibiting the Application of the Instrument to the Every-day Work of the Engineer—Civil, Mechanical and Electrical. *Seventh Edition.* 12mo, flexible cloth..... $1.00

Plane Table, The. Its Uses in Topographical Surveying. From the Papers of the United States Coast Survey. Illustrated. 8vo, cloth..... $2.00
"This work gives a description of the Plane Table employed at the United States Coast Survey office, and the manner of using it."

PLATTNER'S Manual of Qualitative and Quantitative Analysis with the Blow-pipe. *Eighth Edition, revised.* Translated by Henry B. Cornwall, E.M., Ph.D., assisted by John H. Caswell, A.M. From the sixth German edition, by Prof. Friederich Kolbeck. With 87 woodcuts. 463 pages. 8vo, cloth.....*net*, $4.00

PLYMPTON, GEO. W., Prof. The Aneroid Barometer: its Construction and Use. Compiled from several sources. *Eighth Edition, revised and enlarged.* 16mo, boards, illustrated..... $0.50

POCKET LOGARITHMS, to Four Places of Decimals, including Logarithms of Numbers, and Logarithmic Sines and Tangents to Single Minutes. To which is added a Table of Natural Sines, Tangents, and Co-tangents. 16mo, boards. $0.50

POPE, F. L. Modern Practice of the Electric Telegraph. A Technical Handbook for Electricians, Managers and Operators. *Fifteenth Edition, rewritten and enlarged, and fully illustrated.* 8vo, cloth..... $1.50

POPPLEWELL, W. C. Elementary Treatise on Heat and Heat Engines. Specially adapted for engineers and students of engineering. 12mo, cloth, illustrated..... $3.00

POPPLEWELL, W. C. Prevention of Smoke, combined with the Economical Combustion of Fuel. With diagrams, figures and tables. 8vo, cloth illustrated..... *net*, $3.50

Practical Compounding of Oils, Tallow and Grease, for Lubrication, etc. By an Expert Oil Refiner. 8vo, cloth. *net*, $3.50

Practical Iron Founding. By the Author of "Pattern Making," etc. Illustrated with over 100 engravings. *Third Edition.* 12mo, cloth..... $1.50

PRAY, T., Jr. Twenty Years with the Indicator: being a Practical Text-book for the Engineer or the Student, with no complex Formulæ. Illustrated. 8vo, cloth..... $2.50

—— **Steam Tables and Engine Constant. Compiled from** Regnault, Rankine and Dixon directly, making use of the exact records. 8vo, cloth..... $2.00

PREECE, W. H. Electric Lamps..... *In Press.*

—— **and STUBBS, A. T. Manual of Telephony. Illus-** trations and plates. 12mo, cloth..... $4.50

PRELINI, C., C.E. Earth and Rock Excavation. A Manual for Engineers, Contractors, and Engineering Students. With tables and many diagrams and engravings. 8vo, cloth, illustrated. *net*, $3.00

—— **Retaining Walls and Dams. 8vo, cloth, illustrated.** *In Press.*

—— **Tunneling. A Practical Treatise containing 149** Working Drawings and Figures. With additions by Charles S. Hill, C.E., Associate Editor "Engineering News." 311 pages. *Second Edition, revised.* 8vo, cloth, illustrated..... $3.00

PREMIER CODE. (See Hawke, Wm. H.)

PRESCOTT, A. B., Prof. Organic Analysis. A Manual of the Descriptive and Analytical Chemistry of certain Carbon Compounds in Common Use; a Guide in the Qualitative and Quantitative Analysis of Organic Materials in Commercial and Pharmaceutical Assays, in the Estimation of Impurities under Authorized Standards, and in Forensic Examinations for Poisons, with Directions for Elementary Organic Analysis. *Fifth Edition.* 8vo, cloth..... $5.00

PRESCOTT, A. B., Prof. Outlines of Proximate Organic Analysis, for the Identification, Separation and Quantitative Determination of the more commonly occurring Organic Compounds. *Fourth Edition.* 12mo, cloth..... $1.75

—— and JOHNSON, O. C. Qualitative Chemical Analysis. A Guide in Qualitative Work, with Data for Analytical Operations, and Laboratory Methods in Inorganic Chemistry. *Sixth revised and enlarged Edition, entirely rewritten,* with an appendix by H. H. Willard, containing a few improved methods of analysis. 8vo, cloth.....*net,* $3.50

—— and SULLIVAN, E. C. (University of Michigan). First Book in Qualitative Chemistry. For Studies of Water Solution and Mass Action. *Twelfth Edition, entirely rewritten.* 12mo, cloth.....*net,* $1.50

PRITCHARD, O. G. The Manufacture of Electric-light Carbons. Illustrated. 8vo, paper..... $0.60

PROST, E. Manual of Chemical Analysis as Applied to the Assay of Fuels, Ores, Metals, Alloys, Salts, and other Mineral Products. Translated from the original by J. C. Smith. Part I, Fuels, Waters, Ores, Salts, and other mineral industrial products; Part II, Metals; Part III, Alloys. 8vo, cloth...*net,* $4.50

PULLEN, W. W. F. Application of Graphic Methods to the Design of Structures. Specially prepared for the use of Engineers. A Treatment by Graphic Methods of the Forces and Principles necessary for consideration in the Design of Engineering Structures, Roofs, Bridges, Trusses, Framed Structures, Wells, Dams, Chimneys, and Masonry Structures. 12mo, cloth, profusely illustrated.....*net,* $2.50

PULSIFER, W. H. Notes for a History of Lead. 8vo, cloth, gilt top..... $4.00

PUTSCH, A. Gas and Coal-dust Firing. A Critical Review of the Various Appliances Patented in Germany for this Purpose since 1885. With diagrams and figures. Translated from the German by Charles Salter. 8vo, cloth, illustrated.....*net,* $3.00

PYNCHON, T. R., Prof. Introduction to Chemical Physics, designed for the use of Academies, Colleges and High Schools. Illustrated with numerous engravings, and containing copious experiments, with directions for preparing them. *New Edition, revised and enlarged,* and illustrated by 269 wood engravings. 8vo, cloth..... $3.00

RADFORD, C. S., Lieut. Handbook on Naval Gunnery. Prepared by Authority of the Navy Department. For the use of U. S. Navy, U. S. Marine Corps, and U. S. Naval Reserves. Revised and enlarged, with the assistance of Stokely Morgan, Lieut. U. S. N. *Third Edition, revised and enlarged.* 12mo, flexible leather.....*net*, $2.00

RAFTER, G. W. Treatment of Septic Sewage (*Van Nostrand's Science Series, No.* 118). 16mo, cloth..... $0.50

—— **Tables for Sewerage and Hydraulic Engineers.** *In Press.*

—— **and BAKER, M. N. Sewage Disposal in the United** States. Illustrations and folding plates. *Third Edition.* 8vo, cloth..... $6.00

RAM, G. S. The Incandescent Lamp and its Manufac- ture. 8vo, cloth.....*net*, $3.00

RAMP, H. M. Foundry Practice.....*In Press.*

RANDALL, J. E. A Practical Treatise on the Incan- descent Lamp. 16mo, cloth, illustrated..... $0.50

RANDALL, P. M. Quartz Operator's Handbook. New *Edition, revised and enlarged, fully illustrated.* 12mo, cloth, $2.00

RANDAU, P. Enamels and Enamelling. An introduction to the preparation and application of all kinds of enamels for technical and artistic purposes. For enamel-makers, workers in gold and silver, and manufacturers of objects of art. *Third German Edition.* Translated by Charles Salter. With figures, diagrams and tables. 8vo, cloth, illustrated..... *net*, $4.00

RANKINE, W. J. M. Applied Mechanics. Comprising the Principles of Statics and Cinematics, and Theory of Structures, Mechanism, and Machines. With numerous diagrams. *Seventeenth Edition*, thoroughly revised by W. J. Millar. 8vo, cloth..... $5.00

—— **Civil Engineering. Comprising Engineering Sur-** veys, Earthwork, Foundations, Masonry, Carpentry, Metalwork, Roads, Railways, Canals, Rivers, Water-works, Harbors, etc. With numerous tables and illustrations. *Twenty-first Edition*, thoroughly revised by W. J. Millar. 8vo, cloth.... $6.50

RANKINE, W. J. M. Machinery and Millwork. Comprising the Geometry, Motions, Work, Strength, Construction, and Objects of Machines, etc. Illustrated with nearly 300 woodcuts. *Seventh Edition*, thoroughly revised by W. J. Millar. 8vo, cloth. $5.00

—— **The Steam-engine and Other Prime Movers. With** diagram of the Mechanical Properties of Steam. Folding plates, numerous tables and illustrations. *Fifteenth Edition*, thoroughly revised by W. J. Millar. 8vo, cloth..... $5.00

—— **Useful Rules and Tables for Engineers and Others.** With Appendix, Tables, Tests and Formulæ for the use of Electrical Engineers. Comprising Submarine Electrical Engineering, Electric Lighting and Transmission of Power. By Andrew Jamieson, C.E., F.R.S.E. *Seventh Edition*, thoroughly revised by W. J. Millar. 8vo, cloth..... $4.00

—— **and BAMBER, E. F., C.E. A Mechanical Text-book.** With numerous illustrations. *Fifth Edition.* 8vo, cloth.. $3.50

RAPHAEL, F. C. Localization of Faults in Electric Light and Power Mains, with chapters on Insulation Testing. With figures and diagrams. *Second Edition, revised.* 8vo, cloth, illustrated..... *net*, $3.00

RATEAU, A. Experimental Researches on the Flow of Steam through Nozzles and Orifices, to which is added a note on the Flow of Hot Water. (Extrait des Annales des Mines, January, 1902.) Authorized translation by H. Boyd Brydon. With figures, tables, and folding plates. 8vo, cloth, illustrated. *net*, $1.50

RAUTENSTRAUCH, Prof. W. Syllabus of Lectures and Notes on the Elements of Machine Design. With blank pages for note-taking. 8vo, cloth, illustrated.....*net*, $1.50

RAYMOND, E. B. Alternating-current Engineering Prac- tically Treated. With numerous diagrams and figures. *Second Edition.* 12mo, cloth.....*net*, $2.50

RAYNER, H. Silk Throwing and Waste Silk Spinning. With numerous diagrams and figures. 8vo, cloth, illustrated, *net*, $2.50

RECIPES for the Color, Paint, Varnish, Oil, Soap and Drysaltery Trades. Compiled by an Analytical Chemist. 8vo, cloth..... $3.50

RECIPES FOR FLINT GLASS MAKING. Being Leaves from the mixing-book of several experts in the Flint Glass Trade. Containing up-to-date recipes and valuable information as to Crystal, Demi-crystal, and Colored Glass in its many varieties. It contains the recipes for cheap metal suited to pressing, blowing, etc., as well as the most costly Crystal and Ruby. British manufacturers have kept up the quality of this glass from the arrival of the Venetians to Hungry Hill, Stourbridge, up to the present time. The book also contains remarks as to the result of the metal as it left the pots by the respective metal mixers, taken from their own memoranda upon the originals. Compiled by a British Glass Master and Mixer. 12mo, cloth..... *net*, $4.50

REED'S ENGINEERS' HANDBOOK to the Local Marine Board Examinations for Certificates of Competency as First and Second Class Engineers. By W. H. Thorn. With the answers to the Elementary Questions. Illustrated by 358 diagrams and 37 large plates. *Seventeenth Edition, revised and enlarged.* 8vo, cloth..... $5.00

—— Key to the Seventeenth Edition of Reed's Engineers' Handbook to the Board of Trade Examination for First and Second Class Engineers, and containing the workings of all the questions given in the examination papers. By W. H. Thorn. 8vo, cloth..... $3.00

REED. Useful Hints to Sea-going Engineers, and How to Repair and Avoid "Breakdowns"; also appendices containing Boiler Explosions, Useful Formulæ, etc. With 42 diagrams and 8 plates. *Third Edition, revised and enlarged.* 12mo, cloth..... $1.50

—— Marine Boilers. A Treatise on the Causes and Pre- vention of their Priming, with Remarks on their General Management. 12mo, cloth, illustrated..... $2.00

REINHARDT, C. W. Lettering for Draftsmen, Engineers, and Students. A Practical System of Free-hand Lettering for Working Drawings. *Revised and enlarged edition. Eighteenth Thousand.* Oblong boards..... $1.00

—— The Technic of Mechanical Drafting. A Practical guide to neat, correct and legible drawing, containing many illustrations, diagrams and full-page plates. 4to, cloth, illus... $1.00

REISER, F. Hardening and Tempering of Steel, in Theory and Practice. Translated from the German of the *third and enlarged edition*, by Arthur Morris and Herbert Robson. 8vo, cloth, 120 pages..... $2.50

REISER, N. Faults in the Manufacture of Woolen Goods, and their Prevention. Translated from the *second German edition*, by Arthur Morris and Herbert Robson. 8vo, cloth, illustrated ..... *net*, $2.50

—— Spinning and Weaving Calculations with Special reference to Woolen Fabrics. Translated from the German by Chas. Salter. 8vo, cloth illustrated. ..... *net*, $5.00

RICE, J. M., and JOHNSON, W. W. On a New Method of Obtaining the Differential of Functions, with especial reference to the Newtonian Conception of Rates or Velocities. 12mo, paper..... $0.50

RIDEAL, S., D.Sc. Glue and Glue Testing, with figures and tables. 8vo, cloth, illustrated..... *net*, $4.00

RIPPER, W. A Course of Instruction in Machine Drawing and Design for Technical Schools and Engineer Students. With 52 plates and numerous explanatory engravings. Folio, cloth, *net*, $6.00

ROBERTSON, L. S. Water-tube Boilers. Based on a short course of Lectures delivered at University College, London. With upward of 170 illustrations and diagrams. 8vo, cloth, illustrated..... $3.00

ROBINSON, S. W. Practical Treatise on the Teeth of Wheels, with the theory and the use of Robinson's Odontograph. *Third Edition, revised, with additions.* 16mo, cloth, illustrated. (Van Nostrand's Science Series.)..... $0.50

ROEBLING, J. A. Long and Short Span Railway Bridges. Illustrated with large copper-plate engravings of plans and views. Imperial folio, cloth..... $25.00

ROLLINS, W. Notes on X-Light. With 152 full-page plates. 8vo, cloth, illustrated..... *net*, $7.50

ROSE, J., M.E. The Pattern-makers' Assistant. Embrac- ing Lathe Work, Branch Work, Core Work, Sweep Work and Practical Gear Constructions, the Preparation and Use of Tools, together with a large collection of useful and valuable Tables. *Ninth Edition.* With 250 engravings. 8vo, cloth..... $2.50

ROSE, J., M.E. Key to Engines and Engine-running. A Practical Treatise upon the Management of Steam-engines and Boilers for the use of those who desire to pass an examination to take charge of an engine or boiler With numerous illustrations, and Instructions upon Engineers' Calculations, Indicators, Diagrams, Engine Adjustments and other Valuable Information necessary for Engineers and Firemen 12mo. cloth Illus.. $2 50

ROWAN, F. J. The Practical Physics of the Modern Steam-boiler. With an Introduction by Prof R. H Thurston With numerous illustrations and diagrams 8vo, cloth, illustrated..... $7.50

SABINE, R. History and Progress of the Electric Tele-graph. With descriptions of some of the apparatus *Second Edition, with additions* 12mo cloth $1 25

SAELTZER, A. Treatise on Acoustics in Connection with Ventilation. 12mo. cloth. $1.00

SALOMONS, Sir D., M.A. Electric-light Installations. A Practical Handbook. With numerous illustrations. Vol. I., The Management of Accumulators. *Ninth Edition, revised and mostly rewritten.* 12mo cloth. $1.50
Vol. II. *Seventh Edition, revised and enlarged.* Apparatus. 296 illustrations. 12mo, cloth..... $2.25
Vol. III. *Seventh Edition, revised and enlarged.* Applications. 12mo, cloth..... $1.50

SANFORD, P. G. Nitro-explosives. A Practical Treatise concerning the Properties, Manufacture and Analysis of Nitrated Substances, including the Fulminates, Smokeless Powders and Celluloid. 8vo, cloth, 270 pages..... $3.00

SAUNDERS, C. H. Handbook of Practical Mechanics for use in the Shop and Draughting-room; containing Tables, Rules and Formulæ, and Solutions of Practical Problems by Simple and Quick Methods. 16mo, limp cloth..... $1.00

SAUNNIER, C. Watchmaker's Handbook. A Workshop Companion for those engaged in Watchmaking and allied Mechanical Arts. Translated by J. Tripplin and E. Rigg. *Second Edition, revised, with appendix.* 12mo, cloth..... $3.50

SCHELLEN, H., Dr. Magneto-electric and Dynamo-electric Machines: their Construction and Practical Application to Electric Lighting, and the Transmission of Power. Translated from the *third German edition* by N. S. Keith and Percy Neymann, Ph.D. With very large additions and notes relating to American Machines, by N. S. Keith. Vol. 1, with 353 illustrations. *Second Edition.* 8vo, cloth $5.00

SCHERER, R. Casein: its Preparation and Technical Utilization. Translated from the German. 8vo, cloth, illustrated. . . ..*net*, $3.00

SCHMALL, C. N. First Course in Analytical Geometry, Plane and Solid, with Numerous Examples. Containing figures and diagrams. 12mo, cloth, illustrated.....*net*, $1.75

SCHMALL, C. N., and SHACK, S. M. Elements of Plane Geometry. An Elementary Treatise. With many examples and diagrams. 12mo, half leather, illustrated.....*net*, $1.25

SCHMEER, LOUIS. Flow of Water: A New Theory of the Motion of Water under Pressure, and in Open Conduits. 8vo, cloth, illustrated.....*In Press.*

SCHUMANN, F. A Manual of Heating and Ventilation in its Practical Application, for the use of Engineers and Architects. Embracing a Series of Tables and Formulæ for Dimensions of Heating, Flow and Return Pipes for Steam and Hot-water Boilers, Flues, etc. 12mo, illustrated, full roan..... $1.50

SCHWEIZER, V. Distillation of Resins, Resinate Lakes and Pigments; Carbon Pigments and Pigments for Typewriting Machines, Manifolders, etc. A description of the proper methods of distilling resin-oils, the manufacture of resinates, resin-varnishes, resin-pigments and enamel paints, the preparation of all kinds of carbon pigments, and printers' ink, lithographic inks and chalks, and also inks for typewriters, manifolders, and rubber stamps. With tables and 68 figures and diagrams. 8vo, cloth, illustrated.....*net*, $3.50

SCIENCE SERIES, The Van Nostrand. (Follows end of this list.)

SCRIBNER, J. M. Engineers' and Mechanics' Com- panion. Comprising United States Weights and Measures, Mensuration of Superfices and Solids, Tables of Squares and Cubes, Square and Cube Roots, Circumference and Areas of Circles, the Mechanical Powers, Centres of Gravity, Gravitation of Bodies, Pendulums, Specific Gravity of Bodies, Strength, Weight and Crush of Materials, Water-wheels, Hydrostatics, Hydraulics, Statics, Centres of Percussion and Gyration, Friction Heat, Tables of the Weight of Metals, Scantling, etc., Steam and Steam-engine. *Twenty-first Edition, revised.* 16mo, full morocco..... $1.50

SEATON, A. E. A Manual of Marine Engineering. Comprising the Designing, Construction and Working of Marine Machinery. With numerous tables and illustrations reduced from Working Drawings. *Fifteenth Edition*, revised throughout, with an additional chapter on Water-tube Boilers. 8vo, cloth. $6.00

—— and ROUNTHWAITE, H. M. A Pocket-book of Marine Engineering Rules and Tables. For the use of Marine Engineers and Naval Architects, Designers, Draughtsmen, Superintendents and all engaged in the design and construction of Marine Machinery, Naval and Mercantile. *Seventh Edition, revised and enlarged.* Pocket size. Leather, with diagrams. $3.00

SEIDELL, A. Handbook of Solubilities. 12mo, cloth. *In Press.*

SEVER, G. F., Prof. Electrical Engineering Experiments and Tests on Direct-current Machinery. With diagrams and figures. 8vo pamphlet, illustrated..... *net*, $1.00

—— and TOWNSEND, F. Laboratory and Factory Tests in Electrical Engineering. *Second Edition.* 8vo, cloth, illustrated..... *net*, $2.50

SEWALL, C. H. Wireless Telegraphy. With diagrams and engravings. *Second Edition, corrected.* 8vo, cloth, illustrated..... *net*, $2.00

—— Lessons in Telegraphy. For use as a text-book in schools and colleges, or for individual students. Illustrated. 12mo, cloth..... $1.00

SEWELL, T. Elements of Electrical Engineering. A First Year's Course for Students. *Second Edition, revised*, with additional chapters on Alternating-current Working and Appendix of Questions and Answers. With many diagrams, tables and examples. 8vo, cloth, illustrated, 432 pages..... *net*, $3.00

SEXTON, A. H. Fuel and Refractory Materials. 8vo, cloth..... $2.00

—— Chemistry of the Materials of Engineering. A Hand-book for Engineering Students. With tables, diagrams and illustrations. 12mo, cloth, illustrated..... $2.50

SEYMOUR, A. Practical Lithography. With figures and engravings. 8vo, cloth, illustrated..... *net*, $2.50

SHAW, S. The History of the Staffordshire Potteries, and the Rise and Progress of the Manufacture of Pottery and Porcelain; with references to genuine specimens, and notices of eminent potters. A re-issue of the original work published in 1829. 8vo, cloth, illustrated..... *net*, $3.00

—— Chemistry of the Several Natural and Artificial Heterogeneous Compounds used in Manufacturing Porcelain, Glass and Pottery. Re-issued in its original form, published in 1837. 8vo, cloth..... *net*, $5.00

SHELDON, S., Ph.D., and MASON, H., B.S. Dynamo-electric Machinery: its Construction, Design and Operation, Direct-current Machines. *Fifth Edition, revised.* 8vo, cloth, illustrated..... *net*, $2.50

—— Alternating-current Machines: being the second volume of the author's "Dynamo-electric Machinery: its Construction, Design and Operation." With many diagrams and figures. (Binding uniform with volume I.) *Fourth Edition.* 8vo, cloth, illustrated..... *net*, $2.50

SHIELDS, J. E. Notes on Engineering Construction. Embracing Discussions of the Principles involved, and Descriptions of the Material employed in Tunneling, Bridging, Canal and Road Building, etc. 12mo, cloth..... $1.50

SHOCK, W. H. Steam Boilers: their Design, Construc-tion and Management. 4to, half morocco..... $15.00

SHREVE, S H. A Treatise on the Strength of Bridges and Roofs. Comprising the determination of algebraic formulas for strains in Horizontal, Inclined or Rafter, Triangular, Bowstring, Lenticular and other Trusses, from fixed and moving loads, with practical applications and examples, for the use of Students and Engineers. 87 woodcut illustrations. *Fourth Edition.* 8vo, cloth..... $3.50

SHUNK, W. F. The Field Engineer. A Handy Book of practice in the Survey, Location and Track-work of Railroads, containing a large collection of Rules and Tables, original and selected, applicable to both the Standard and Narrow Gauge, and prepared with special reference to the wants of the young Engineer. *Sixteenth Edition, revised and enlarged.* With addenda 12mo, morocco, tucks..... $2.50

SIMMS, F. W. A Treatise on the Principles and Practice of Leveling. Showing its application to purposes of Railway Engineering, and the Construction of Roads, etc. Revised and corrected, with the addition of Mr. Laws' Practical Examples for setting out Railway Curves. Illustrated. 8vo, cloth..... $2.50

—— **Practical Tunneling. Fourth Edition, Revised and** greatly extended. With additional chapters illustrating recent practice by D. Kinnear Clark. With 36 plates and other illustrations. Imperial 8vo, cloth..... $8.00

SIMPSON, G. The Naval Constructor. A Vade Mecum of Ship Design, for Students, Naval Architects, Ship Builders and Owners, Marine Superintendents, Engineers and Draughtsmen. 12mo, morocco, illustrated, 500 pages.....*net*, $5.00

SLATER, J. W. Sewage Treatment, Purification and Utilization. A Practical Manual for the Use of Corporations, Local Boards, Medical Officers of Health, Inspectors of Nuisances, Chemists, Manufacturers, Riparian Owners, Engineers and Ratepayers. 12mo, cloth..... $2.25

SMITH, F. E. Handbook for Mechanics. 12mo, cloth, illustrated.....*In Press.*

—— **Mechanics for Practical Men. 8vo, cloth, about 400** pp., illustrated.....*In Press.*

—— **I. W., C.E. The Theory of Deflections and of Lati-** tudes and Departures. With special applications to Curvilinear Surveys, for Alignments of Railway Tracks. Illustrated. 16mo, morocco, tucks..... $3.00

—— **J. C. Manufacture of Paint. A Practical Handbook** for Paint Manufacturers, Merchants and Painters With 60 illustrations and one large diagram. 8vo, cloth.....*net*, $3.00

SNELL, A. T. Electric Motive Power: The Transmission and Distribution of Electric Power by Continuous and Alternate Currents. With a Section on the Applications of Electricity to Mining Work. *Second Edition.* 8vo, cloth, illustrated..*net*, $4.00

SNOW, W. G., and NOLAN, T. Ventilation of Buildings. 16mo, cloth. (Van Nostrand's Science Series.)..... $0.50

SODDY, F. Radio-Activity: An elementary treatise from the standpoint of the disintegration theory. With 40 figures and diagrams. 8vo, cloth, illustrated.....*net*, $3.00

SOXHLET, D. H. Art of Dyeing and Staining Marble, Artificial Stone, Bone, Horn, Ivory and Wood, and of imitating all sorts of Wood. A practical Handbook for the use of Joiners, Turners, Manufacturers of Fancy Goods, Stick and Umbrella Makers, Comb Makers, etc. Translated from the German by Arthur Morris and Herbert Robson, B.Sc. 8vo, cloth, 170 pages..... *net*, $2.50

SPANG, H. W. A Practical Treatise on Lightning Protection. With figures and diagrams. 12mo, cloth..... $1.00

SPEYERS, C. L. Text-book of Physical Chemistry. 8vo, cloth..... $2.25

STAHL, A. W., and WOODS, A. T. Elementary Mechanism. A Text-book for Students of Mechanical Engineering. *Fifteenth Edition.* 12mo, cloth..... $2.00

STALEY, C., and PIERSON, G. S. The Separate System of Sewerage: its Theory and Construction. *Third Edition, revised and enlarged.* With chapter on Sewage Disposal. With maps, plates and illustrations. 8vo, cloth..... $3.00

STANDAGE, H. C. Leatherworkers' Manual: being a Compendium of Practical Recipes and Working Formulæ for Curriers, Boot-makers, Leather Dressers, Blacking Manufacturers, Saddlers, Fancy Leather Workers, and all persons engaged in the manipulation of leather. 8vo, cloth..... *net*, $3.50

—— Sealing Waxes, Wafers, and Other Adhesives. For the Household, Office, Workshop and Factory. 8vo, cloth, 96 pages..... *net*, $2.00

STEWART, R. W. Text-book of Heat. Illustrated. 8vo, cloth..... $1.00

—— Text-book of Magnetism and Electricity. 160 Illustrations and numerous examples. 12mo, cloth..... $1.00

STILES, A. Tables for Field Engineers. Designed for Use in the Field. Tables containing all the Functions of a One Degree Curve, from which a corresponding one can be found for any required Degree. Also, Tables of Natural Sines and Tangents. 12mo, morocco, tucks..... $2.00

STILLMAN, P. Steam-engine Indicator and the Improved Manometer Steam and Vacuum Gauges; their Utility and Application. *New edition.* 12mo, flexible cloth..... $1.00

STODOLA, Dr. A. Steam Turbines. With an appendix on Gas Turbines, and the future of Heat Engines. Authorized translation by Dr. Louis C. Loewenstein (Lehigh University). With 241 cuts and 3 lithographed tables. 8vo, cloth, illustrated. *net*, $4.50

STONE, R., Gen'l. New Roads and Road Laws in the United States. 200 pages, with numerous illustrations. 12mo, cloth..... $1.00

STONEY, B. D. The Theory of Stresses in Girders and Similar Structures. With Observations on the Application of Theory to Practice, and Tables of Strength and other Properties of Materials. *New revised edition*, with numerous additions on Graphic Statics, Pillars, Steel, Wind Pressure, Oscillating Stresses, Working Loads, Riveting, Strength and Tests of Materials. 777 pages, 143 illus. and 5 folding-plates. 8vo, cloth.... $12.50

SUFFLING, E. R. Treatise on the Art of Glass Painting. Prefaced with a Review of Ancient Glass. With engravings and colored plates. 8vo, cloth..... *net*, $3.50

SWEET, S. H. Special Report on Coal, Showing its Dis- tribution, Classification, and Costs delivered over Different Routes to Various Points in the State of New York and the Principal Cities on the Atlantic Coast. With maps. 8vo, cloth....$3.00

SWOOPE, C. W. Practical Lessons in Electricity: Prin- ciples, Experiments, and Arithmetical Problems. An Elementary Text-book. With numerous tables, formulæ, and two large instruction plates. 8vo, cloth, illustrated. *Seventh Edition..net*, $2.00

TAILFER, L. Practical Treatise on the Bleaching of Linen and Cotton Yarn and Fabrics. With tables and diagrams. Translated from the French by John Geddes McIntosh. 8vo, cloth, illustrated..... *net*, $5.00

TEMPLETON, W. The Practical Mechanic's Workshop Companion. Comprising a great variety of the most useful rules and formulæ in Mechanical Science, with numerous tables of practical data and calculated results facilitating mechanical operations. Revised and enlarged by W. S. Hutton. 12mo, morocco..... $2.00

THOM, C., and JONES, W. H. Telegraphic Connections: embracing Recent Methods in Quadruplex Telegraphy. 20 full-page plates, some colored. Oblong, 8vo, cloth..... $1.50

THOMAS, C. W. Paper-makers' Handbook. A Practical Treatise. Illustrated.. ..... *In Press.*

THOMPSON, A. B. Oil Fields of Russia and the Russian Petroleum Industry. A Practical Handbook on the Exploration, Exploitation, and Management of Russian Oil Properties, including Notes on the Origin of Petroleum in Russia, a Description of the Theory and Practice of Liquid Fuel, and a Translation of the Rules and Regulations concerning Russian Oil Properties. With numerous illustrations and photographic plates and a map of the Balakhany-Saboontchy-Romany Oil Field. 8vo, cloth, illustrated ..... *net*, $7.50

THOMPSON, E. P., M.E. How to Make Inventions; or, Inventing as a Science and an Art. A Practical Guide for Inventors. *Second Edition.* 8vo, boards..... $0.50

—— **Roentgen Rays and Phenomena of the Anode and** Cathode. Principles, Applications, and Theories. For Students, Teachers, Physicians, Photographers, Electricians and others. Assisted by Louis M. Pignolet, N. D. C. Hodges and Ludwig Gutmann, E.E. With a chapter on Generalizations, Arguments, Theories, Kindred Radiations and Phenomena. By Professor Wm. Anthony. 50 diagrams, 40 half-tones. 8vo, cloth..... $1.00

THOMPSON, W. P. Handbook of Patent Law of All Countries. *Thirteenth Edition, completely revised*, March, 1905. 16mo, cloth..... $1.50

THORNLEY, T. Cotton Combing Machines. With Nu-merous tables, engravings and diagrams. 8vo, cloth, illustrated, 343 pages..... *net*, $3.00

THURSO, J. W. Modern Turbine Practice and Water-Power Plants. With eighty-eight figures and diagrams. 8vo, cloth, illustrated..... *net*, $4.00

TOCH, M. Chemistry and Technology of Mixed Paints. 8vo, cloth..... *In Press.*

TODD, J., and WHALL, W. B. Practical Seamanship for Use in the Merchant Service: including all ordinary subjects; also Steam Seamanship, Wreck Lifting, Avoiding Collision. Wire Splicing, Displacement and everything necessary to be known by seamen of the present day. *Fifth Edition*, with 247 illustrations and diagrams. 8vo, cloth..... *net*, $7.50

TOMPKINS, A. E. Text-book of Marine Engineering. *Second Edition, entirely rewritten, rearranged, and enlarged.* With over 250 figures, diagrams, and full-page plates. 8vo, cloth, illustrated..... *net*, $6.00

TOOTHED GEARING. A Practical Handbook for Offices and Workshops. By a Foreman Patternmaker. 184 illustrations. 12mo, cloth..... $2.25

TRATMAN, E. E. R. Railway Track and Track-work. With over 200 illustrations. 8vo, cloth..... $3.00

TRAVERSE TABLE, Showing Latitude and Departure for each Quarter Degree of the Quadrant, and for Distances from 1 to 100, to which is appended a Table of Natural Sines and Tangents for each five minutes of the Quadrant. (Reprinted from Scribner's Pocket Table Book.) *Van Nostrand's Science Series.* 16mo, cloth..... $0.50
Morocco..... $1.00

TRINKS, W., and HOUSUM, C. Shaft Governors. 16mo, cloth, illustrated. (Van Nostrand's Science Series.)..... $0.50

TUCKER, J. H., Dr. A Manual of Sugar Analysis, including the Applications in General of Analytical Methods to the Sugar Industry. With an Introduction on the Chemistry of Cane-sugar, Dextrose, Levulose, and Milk-sugar. *Sixth Edition.* 8vo, cloth, illustrated..... $3.50

TUMLIRZ, O., Dr. Potential and its Application to the Explanation of Electrical Phenomena, Popularly Treated. Translated from the German by D. Robertson. 12mo, cloth, ill. $1.25

TUNNER, P. A. Treatise on Roll-turning for the Manufacture of Iron. Translated and adapted by John B. Pearse, of the Pennsylvania Steel Works, with numerous engravings, woodcuts. 8vo, cloth, with folio atlas of plates..... $10.00

TURBAYNE, A. A. Alphabets and Numerals. With 27 plates. 4to, boards..... $2.00

UNDERHILL, C. R. The Electro-magnet. New and *revised edition.* 8vo, cloth, illustrated. ..... *net,* $1.50

URQUHART, J. W. Electric Light Fitting. Embodying Practical Notes on Installation Management. A Handbook for Working Electrical Engineers. With numerous illustrations. 12mo, cloth. $2.00

—— **Electro-plating. A Practical Handbook on the Depo-**sition of Copper, Silver, Nickel, Gold, Brass, Aluminium, Platinum, etc. *Fourth Edition.* 12mo. $2.00

—— **Electrotyping. A Practical Manual Forming a New** and Systematic Guide to the Reproduction and Multiplication of Printing Surfaces, etc. 12mo. $2.00

—— **Electric Ship Lighting. A Handbook on the Practical** Fitting and Running of Ship's Electrical Plant. For the Use of Ship Owners and Builders, Marine Electricians and Sea-going Engineers-in-Charge. Illustrated. 12mo, cloth. $3.00

UNIVERSAL TELEGRAPH CIPHER CODE. Arranged for General Correspondence. 12mo, cloth. $1.00

VAN NOSTRAND'S Chemical Annual, based on Bieder-mann's "Chemiker Kalender." Edited by Prof. J. C. Olsen, with the co-operation of Eminent Chemists. First year of issue 1906. 12mo, cloth, illustrated. ..... *In Press.*

—— **Engineering Magazine. Complete Sets, 1869 to 1886** inclusive. 35 vols., in cloth. $60.00
" " in half morocco. $100.00

—— **Year Book of Mechanical Engineering Data. With** many tables and diagrams. (First Year of issue 1906.) *In Press.*

VAN WAGENEN, T. F. Manual of Hydraulic Mining. For the Use of the Practical Miner. *Revised and enlarged edition.* 18mo, cloth. $1.00

VILLON, A. M. Practical Treatise on the Leather Industry. With many tables and illustrations and a copious index. A translation of Villon's "Traite Pratique de la Fabrication des Cuirs et du Travail des Peaux," by Frank T. Addyman, B.Sc. 8vo, cloth, illustrated. ..... *net,* $10.00

VINCENT, C. Ammonia and its Compounds: their Manufacture and Uses. Translated from the French by M. J. Salter. 8vo, cloth, illustrated..... *net*, $2.00

VOLK, C. Haulage and Winding Appliances Used in Mines. With plates and engravings. Translated from the German. 8vo, cloth, illustrated..... *net*, $4.00

VON GEORGIEVICS, G. Chemical Technology of Textile Fibres: their Origin, Structure, Preparation, Washing, Bleaching, Dyeing, Printing, and Dressing. Translated from the German by Charles Salter. With many diagrams and figures. 8vo, cloth, illustrated. 306 pages..... *net*, $4.50

Contents.—The Textile Fibres; Washing, Bleaching, and Carbonizing; Mordants and Mordanting; Dyeing, Printing, Dressing and Finishing; Index.

—— **Chemistry of Dyestuffs. Translated from the Second** *German edition* by Chas. Salter. 8vo, cloth, 412 pages... *net*, $4.50

WABNER, R. Ventilation in Mines. Translated from the German by Charles Salter. With plates and engravings. 8vo, cloth, illustrated, 240 pages..... *net*, $4.50

WADE, E. J. Secondary Batteries: their Theory, Construction and Use. With innumerable diagrams and figures. 8vo, cloth, illustrated, 492 pages..... *net*, $4.00

WALKER, F., C.E. Aërial Navigation. A Practical Handbook on the Construction of Dirigible Balloons, Aërostats, Aëroplanes and Aëromotors. With diagrams, tables and illustrations. 8vo, cloth, illustrated, 151 pages..... *net*, $3.00

WALKER, S. F. Electrical Engineering in Our Homes and Workshops. A Practical Treatise on Auxiliary Electrical Apparatus. *Third Edition, revised, with numerous illustrations.* 8vo, cloth $2.00

—— **Electric Lighting for Marine Engineers, or How to** Light a Ship by the Electric Light and How to Keep the Apparatus in Order. *Second Edition.* 103 illus., 8vo, cloth..... $2.00

WALKER, W. H. Screw Propulsion. Notes on Screw Propulsion; its Rise and History. 8vo, cloth..... $0.75

WALLING, B. T., Lieut. Com. U.S.N., and MARTIN, JULIUS. Electrical Installations of the United States Navy. With many diagrams and engravings. 8vo, cloth, illustrated..... *In Press.*

WALLIS TAYLER, A. J. Bearings and Lubrication. A Handbook for Every user of Machinery. Fully illustrated. 8vo, cloth..... $1.50

—— **Modern Cycles, a Practical Handbook on Their Construction and Repair.** With 300 illustrations. 8vo, cloth. $4.00

—— **Motor Cars, or Power Carriages for Common Roads.** With numerous illustrations. 8vo, cloth..... $1.80

—— **Motor Vehicles for Business Purposes. 8vo, cloth,** illustrated.....*net.* $3.50.

—— **Refrigerating and Ice-making Machinery. A Descriptive** Treatise for the use of persons employing refrigerating and ice-making installations, and others. 8vo, cloth, illustrated. $3.00

—— **Refrigeration and Cold Storage: being a Complete** practical treatise on the art and science of refrigeration. 600 pages, 361 diagrams and figures. 8vo, cloth ..... *net*, $4.50

—— **Sugar Machinery. A Descriptive Treatise, devoted** to the Machinery and Apparatus used in the Manufacture of Cane and Beet Sugars. 12mo, cloth, illustrated..... $2.00

WANKLYN, J. A. A Practical Treatise on the Examination of Milk and its Derivatives, Cream, Butter and Cheese. 12mo, cloth..... $1.00

—— **Water Analysis. A Practical Treatise on the Exam**ination of Potable Water. *Tenth Edition.* 12mo, cloth.... $2.00

WANSBROUGH, W. D. The A B C of the Differential Calculus. 12mo, cloth..... $1.50

WARD, J. H. Steam for the Million. A Popular Treatise on Steam, and its application to the Useful Arts, especially to Navigation. 8vo, cloth..... $1.00

WARING, G. E., Jr. Sewerage and Land Drainage. Illustrated with woodcuts in the text, and full-page and folding plates. *New Edition*.....*In Press.*

—— **Modern Methods of Sewage Disposals for Towns,** Public Institutions and Isolated Houses. *Second Edition, revised and enlarged.* 260 pages. Illustrated. Cloth..... $2.00

WARING, G. E., Jr. How to Drain a House. Practical Information for Householders. *Third Edition, enlarged.* 12mo, cloth..... $1.25

WARREN, F. D. Handbook on Reinforced Concrete. 16mo, cloth, illustrated.....*net*, $2.50

WATSON, E. P. Small Engines and Boilers. A Manual of Concise and Specific Directions for the Construction of Small Steam-engines and Boilers of Modern Types from five Horse-power down to model sizes. Illustrated with Numerous Diagrams and Half-tone Cuts. 12mo, cloth..... $1.25

WATT, A. Electro-plating and Electro-refining of Metals: being a new edition of Alexander Watts' "Electro-Deposition." *Revised and largely rewritten* by Arnold Philip, B.Sc. With numerous figures and engravings. 8vo, cloth, illustrated, 680 pages.....*net*, $4.50

—— **Electro-metallurgy Practically Treated. Eleventh** *Edition, considerably enlarged.* 12mo, cloth..... $1.00

—— **The Art of Soap-making. A Practical Handbook of** the Manufacture of Hard and Soft Soaps, Toilet Soaps, etc. Including many New Processes, and a Chapter on the Recovery of Glycerine from Waste Lyes. With illustrations. *Fifth Edition, revised and enlarged.* 8vo, cloth..... $3.00

—— **The Art of Leather Manufacture: being a Practical** Handbook, in which the Operations of Tanning, Currying and Leather Dressing are Fully Described, and the Principles of Tanning Explained, and many Recent Processes Introduced. With numerous illustrations. *New Edition*.....*In Press.*

WEALE, J. A Dictionary of Terms Used in Architecture, Building, Engineering, Mining, Metullargy, Archæology, the Fine Arts, etc., with explanatory observations connected with applied Science and Art. *Fifth Edition, revised and corrected.* 12mo, cloth..... $2.50

WEBB, H. L. A Practical Guide to the Testing of Insu- lated Wires and Cables. Illustrated. 12mo, cloth..... $1.00

—— **The Telephone Handbook. 128 Illustrations. 146** pages. 16mo, cloth..... $1.00

WEEKES, R. W. The Design of Alternate Current Transformers. Illustrated. 12mo, cloth..... $1.00

WEISBACH, J. A Manual of Theoretical Mechanics. *Ninth American edition.* Translated from the fourth augmented and improved German edition, with an Introduction to the Calculus by Eckley B. Coxe, A.M., Mining Engineer. 1,100 pages and 902 woodcut illustrations. 8vo, cloth..... $6.00
Sheep..... $7.50

—— and HERRMANN, G. Mechanics of Air Machinery. Authorized translation, with an appendix on American practice by A. Trowbridge. With figures, diagrams, and folding plates. 8vo, cloth, illustrated.....*net*, $3.75

WESTON, E. B. Tables Showing Loss of Head Due to Friction of Water in Pipes. *Second Edition.* 12mo, cloth.. $1.50

WEYMOUTH, F. M. Drum Armatures and Commutators. (Theory and Practice.) A complete Treatise on the Theory and Construction of Drum Winding, and of commutators for closed-coil armatures, together with a full résumé of some of the principal points involved in their design, and an exposition of armature reactions and sparking. 8vo, cloth..... $3.00

WHEELER, J. B., Prof. Art of War. A Course of Instruction in the Elements of the Art and Science of War, for the Use of the Cadets of the United States Military Academy, West Point, N. Y. 12mo, cloth..... $1.75

—— Field Fortifications. The Elements of Field Fortifications, for the Use of the Cadets of the United States Military Academy, West Point, N. Y. 12mo, cloth..... $1.75

WHIPPLE, S., C.E. An Elementary and Practical Treatise on Bridge Building. 8vo, cloth..... $3.00

WHITE, W. H., K.C.B. A Manual of Naval Architecture, for use of Officers of the Royal Navy, Officers of the Mercantile Marine, Yachtsmen, Shipowners and Shipbuilders. Containing many figures, diagrams and tables. Thick, 8vo, cloth, illustrated..... $9.00

WILKINSON, H. D. Submarine Cable-laying, Repairing, and Testing. 8vo, cloth. *New Edition*.....*In Press.*

WILLIAMSON, R. S. On the Use of the Barometer on Surveys and Reconnoissances. Part I. Meteorology in its Connection with Hypsometry. Part II. Barometric Hypsometry. With illustrative tables and engravings. 4to, cloth.... $15.00

—— Practical Tables in Meteorology and Hypsometry, in connection with the use of the Barometer. 4to, cloth..... $2.50

WILSON, G. Inorganic Chemistry, with New Notation. Revised and enlarged by H. G. Madan. *New edition.* 12mo, cloth..... $2.00

WILLSON, F. N. Theoretical and Practical Graphics. An Educational Course on the Theory and Practical Applications of Descriptive Geometry and Mechanical Drawing. Prepared for students in General Science, Engraving, or Architecture. *Third Edition, revised.* 4to, cloth, illustrated..... *net*, $4.00

—— Note-taking, Dimensioning and Lettering. 4to, Cloth, illustrated..... *net*, $1.25

—— Third Angle Method of Making Working Drawings. 4to, cloth, illustrated..... *net*, $1.25

—— Some Mathematical Curves, and Their Graphical Construction. 4to, cloth, illustrated..... *net*, $1.50

—— Practical Engineering, Drawing, and Third Angle Projection. 4to, cloth, illustrated..... *net*, $2.80

—— Shades, Shadows, and Linear Perspective. 4to, Cloth. illustrated..... *net*, $1.00

—— Descriptive Geometry — Pure and Applied, with a chapter on Higher Plane Curves, and the Helix. 4to, cloth illustrated..... *net*, $3.00

WINKLER, C., and LUNGE, G. Handbook of Technical Gas-Analysis. With figures and diagrams. *Second English edition.* Translated from the third greatly enlarged German edition, with some additions by George Lunge, Ph.D. 8vo, cloth, illustrated, 190 pages..... $4.00

WOODBURY, D. V. Treatise on the Various Elements of Stability in the Well-proportioned Arch. With numerous tables of the Ultimate and Actual Thrust. 8vo, half morocco. Illustrated..... $4.00

WRIGHT, A. C. Analysis of Oils and Allied Substances. 8vo, cloth, illustrated, 241 pages..... *net*, $3.50

—— Simple Method for Testing Painters' Materials. 8vo, cloth, 160 pages..... *net*, $2.50

WRIGHT, T. W., Prof. (Union College.) Elements of Mechanics; including Kinematics, Kinetics and Statics. With applications. *Third Edition, revised and enlarged.* 8vo, cloth..$2.50

—— and HAYFORD, J. F. Adjustment of Observations by the Method of Least Squares, with applications to Geodetic Work. *Second Edition, rewritten.* 8vo, cloth, illustrated. *net*, $3.00

YOUNG, J. E. Electrical Testing for Telegraph Engineers. With Appendices consisting of Tables. 8vo, cloth, illus... $4.00

YOUNG SEAMAN'S MANUAL. Compiled from Various Authorities, and Illustrated with Numerous Original and Select Designs, for the Use of the United States Training Ships and the Marine Schools. 8vo, half roan..... $3.00

ZEUNER, A., Dr. Technical Thermodynamics. Translated from the German, by Prof. J. F. Klein, Lehigh University. 8vo, cloth, illustrated.....*In Press.*

ZIMMER, G. F. Mechanical Handling of Material. Being a treatise on the handling of material, such as coal, ore, timber, etc., by automatic and semi-automatic machinery, together with the various accessories used in the manipulation of such plant, also dealing fully with the handling, storing, and warehousing of grain. With 542 figures, diagrams, full-page and folding plates. Royal 8vo, cloth, illustrated.....*net*, $10.00

ZIPSER, J. Textile Raw Materials, and Their Conversion into Yarns. The study of the Raw Materials and the Technology of the Spinning Process. A Text-book for Textile, Trade and higher Technical Schools, as also for self-instruction. Based upon the ordinary syllabus and curriculum of the Imperial and Royal Weaving Schools. Translated from the German by Chas. Salter. 8vo, cloth, illustrated.....*net*, $5.00

Catalogue of the Van Nostrand Science Series.

THEY are put up in a uniform, neat, and attractive form. 18mo, boards. Price 50 cents per volume. The subjects are of an eminently scientific character and embrace a wide range of topics, and are amply illustrated when the subject demands.

No. 1. CHIMNEYS FOR FURNACES AND STEAM BOILERS. By R. Armstrong, C.E. Third American Edition. Revised and partly rewritten, with an Appendix on "Theory of Chimney Draught," by F. E. Idell, M.E.

No. 2. STEAM-BOILER EXPLOSIONS. By Zerah Colburn. New Edition, revised by Prof. R. H. Thurston.

No. 3. PRACTICAL DESIGNING OF RETAINING-WALLS. Fourth edition, by Prof. W. Cain.

No. 4. PROPORTIONS OF PINS USED IN BRIDGES. By Charles E. Bender, C.E. Second edition, with Appendix.

No. 5. VENTILATION OF BUILDINGS. By Wm. G. Snow, S.B., and Thos. Nolan, A.M.

No. 6. ON THE DESIGNING AND CONSTRUCTION OF STORAGE Reservoirs. By Arthur Jacob, B.A. Third American edition, revised, with additions by E. Sherman Gould.

No. 7. SURCHARGED AND DIFFERENT FORMS OF RETAINING-walls. By James S. Tate, C.E.

No. 8. A TREATISE ON THE COMPOUND STEAM-ENGINE. By John Turnbull, Jr. 2nd edition, revised by Prof. S. W. Robinson.

No. 9. A TREATISE ON FUEL. By Arthur V. Abbott, C.E. Founded on the original treatise of C. William Siemens, D.C.L. Third ed.

No. 10. COMPOUND ENGINES. Translated from the French of A. Mallet. Second edition, revised with results of American Practice, by Richard H. Buel, C.E.

No. 11. THEORY OF ARCHES. By Prof. W. Allan.

No. 12. THEORY OF VOUSSOIR ARCHES. By Prof. Wm. Cain. Third edition, revised and enlarged.

No. 13. GASES MET WITH IN COAL MINES. By J. J. Atkinson. Third edition, revised and enlarged, to which is added The Action of Coal Dusts by Edward H. Williams, Jr.

No. 14. FRICTION OF AIR IN MINES. By J. J. Atkinson. Second American edition.

No. 15. SKEW ARCHES. By Prof. E. W. Hyde, C.E. Illustrated. Second edition.

No. 16. GRAPHIC METHOD FOR SOLVING CERTAIN QUESTIONS in Arithmetic or Algebra. By Prof. G. L. Vose. Second edition.

No. 17. WATER AND WATER-SUPPLY. By Prof. W. H. Corfield, of the University College, London. Second American edition.

No. 18. SEWERAGE AND SEWAGE PURIFICATION. By M. N. Baker, Associate Editor "Engineering News." Second edition, revised and enlarged.

No. 19. STRENGTH OF BEAMS UNDER TRANSVERSE LOADS. By Prof. W. Allan, author of "Theory of Arches." Second edition, revised.

No. 20. BRIDGE AND TUNNEL CENTRES. By John B. McMaster, C.E. Second edition.

No. 21. SAFETY VALVES. By Richard H. Buel, C.E. Third edition.

No. 22. HIGH MASONRY DAMS. By E. Sherman Gould, M. Am. Soc. C. E.

No. 23. THE FATIGUE OF METALS UNDER REPEATED STRAINS. With various Tables of Results and Experiments. From the German of Prof. Ludwig Spangenburg, with a Preface by S. H. Shreve, A.M.

No. 24. A PRACTICAL TREATISE ON THE TEETH OF WHEELS. By Prof. S. W. Robinson. 2nd edition, revised, with additions.

No. 25. THEORY AND CALCULATION OF CANTILEVER BRIDGES. By R. M. Wilcox.

No. 26. PRACTICAL TREATISE ON THE PROPERTIES OF CONtinuous Bridges. By Charles Bender, C.E.

No. 27. BOILER INCRUSTATION AND CORROSION. By F. J. Rowan. New edition. Revised and partly rewritten by F. E. Idell.

No. 28. TRANSMISSION OF POWER BY WIRE ROPES. By Albert W. Stahl, U.S.N. Second edition, revised.

No. 29. STEAM INJECTORS, THEIR THEORY AND USE. Translated from the French of M. Leon Pochet.

No. 30. MAGNETISM OF IRON VESSELS AND TERRESTRIAL Magnetism. By Prof. Fairman Rogers.

No. 31. **THE SANITARY CONDITION OF CITY AND COUNTRY** Dwelling-houses. By George E. Waring, Jr. Second edition, revised.

No. 32. **CABLE-MAKING FOR SUSPENSION BRIDGES.** By W. Hildenbrand, C.E.

No. 33. **MECHANICS OF VENTILATION.** By George W. Rafter, C.E. Second edition, revised.

No. 34. **FOUNDATIONS.** By Prof. Jules Gaudard, C.E. Translated from the French. Second edition.

No. 35. **THE ANEROID BAROMETER: ITS CONSTRUCTION AND USE.** Compiled by George W. Plympton. Ninth edition, revised and enlarged.

No. 36. **MATTER AND MOTION.** By J. Clerk Maxwell, M.A. Second American edition.

No. 37. **GEOGRAPHICAL SURVEYING: ITS USES, METHODS,** and Results. By Frank De Yeaux Carpenter, C.E.

No. 38. **MAXIMUM STRESSES IN FRAMED BRIDGES.** By Prof. William Cain, A.M., C.E. New and revised edition.

No. 39. **A HANDBOOK OF THE ELECTRO-MAGNETIC TELEGRAPH.** By A. E. Loring. Fourth edition, revised.

No. 40. **TRANSMISSION OF POWER BY COMPRESSED AIR.** By Robert Zahner, M.E. New edition, in press.

No. 41. **STRENGTH OF MATERIALS.** By William Kent, C.E., Assoc. Editor "Engineering News." Second edition.

No. 42. **THEORY OF STEEL-CONCRETE ARCHES, AND OF** Vaulted Structures. By Prof. Wm. Cain. Third edition, thoroughly revised.

No. 43. **WAVE AND VORTEX MOTION.** By Dr. Thomas Craig, of Johns Hopkins University.

No. 44. **TURBINE WHEELS.** By Prof. W. P. Trowbridge, Columbia College. Second edition. Revised.

No. 45. **THERMO-DYNAMICS.** By Prof. H. T. Eddy, University of Cincinnati. New edition, in press.

No. 46. **ICE-MAKING MACHINES.** From the French of M. Le Doux. Revised by Prof. J. E. Denton, D. S. Jacobus, and A. Riesenberger. Fifth edition, revised.

No. 47. **LINKAGES: THE DIFFERENT FORMS AND USES OF** Articulated Links. By J. D. C. De Roos.

No. 48. **THEORY OF SOLID AND BRACED ELASTIC ARCHES** By William Cain, C.E.

No. 49. **MOTION OF A SOLID IN A FLUID.** By Thomas Craig, Ph.D.

No. 50. DWELLING-HOUSES: THEIR SANITARY CONSTRUCTION and Arrangements. By Prof. W. H. Corfield.

No. 51. THE TELESCOPE: OPTICAL PRINCIPLES INVOLVED IN the Construction of Refracting and Reflecting Telescopes, with a new chapter on the Evolution of the Modern Telescope, and a Bibliography to date. With diagrams and folding plates. By Thomas Nolan. Second edition, revised and enlarged.

No. 52. IMAGINARY QUANTITIES: THEIR GEOMETRICAL INterpretation. Translated from the French of M. Argand by Prof. A. S. Hardy.

No. 53. INDUCTION COILS: HOW MADE AND HOW USED. Eleventh American edition.

No. 54. KINEMATICS OF MACHINERY. By Prof. Alex. B. W. Kennedy. With an introduction by Prof. R. H. Thurston.

No. 55. SEWER GASES: THEIR NATURE AND ORIGIN. By A. de Varona. Second edition, revised and enlarged.

No. 56. THE ACTUAL LATERAL PRESSURE OF EARTHWORK. By Benj. Baker, M. Inst., C.E.

No. 57. INCANDESCENT ELECTRIC LIGHTING. A Practical Description of the Edison System. By L. H. Latimer. To which is added the Design and Operation of Incandescent Stations, by C. J. Field; and the Maximum Efficiency of Incandescent Lamps, by John W. Howell.

No. 58. VENTILATION OF COAL MINES. By W. Fairley, M.E., and Geo. J. André.

No. 59. RAILROAD ECONOMICS; OR, NOTES WITH COMMENTS. By S. W. Robinson, C.E.

No. 60. STRENGTH OF WROUGHT-IRON BRIDGE MEMBERS. By S. W. Robinson, C.E.

No. 61. POTABLE WATER, AND METHODS OF DETECTING Impurities. By M. N. Baker. Second ed., revised and enlarged.

No. 62. THEORY OF THE GAS-ENGINE. By Dougald Clerk. Third edition. With additional matter. Edited by F. E. Idell, M.E.

No. 63. HOUSE-DRAINAGE AND SANITARY PLUMBING. By W. P. Gerhard. Tenth edition.

No. 64. ELECTRO-MAGNETS. By A. N. Mansfield.

No. 65. POCKET LOGARITHMS TO FOUR PLACES OF DECIMALS. Including Logarithms of Numbers, etc.

No. 66. DYNAMO-ELECTRIC MACHINERY. By S. P. Thompson. With an Introduction by F. L. Pope. Third edition, revised.

No. 67. HYDRAULIC TABLES FOR THE CALCULATION OF THE Discharge through Sewers, Pipes, and Conduits. Based on "Kutter's Formula." By P. J. Flynn.

No. 68. **STEAM-HEATING.** By Robert Briggs. Third edition, revised, with additions by A. R. Wolff.

No. 69. **CHEMICAL PROBLEMS.** By Prof. J. C. Foye. Fourth edition, revised and enlarged.

No. 70. **EXPLOSIVE MATERIALS.** By Lieut. John P. Wisser.

No. 71. **DYNAMIC ELECTRICITY.** By John Hopkinson, J. A. Shoolbred, and R. E. Day.

No. 72. **TOPOGRAPHICAL SURVEYING.** By George J. Specht, Prof. A. S. Hardy, John B. McMaster, and H. F. Walling. Third edition, revised.

No. 73. **SYMBOLIC ALGEBRA; OR, THE ALGEBRA OF ALGEbraic Numbers.** By Prof. William Cain.

No. 74. **TESTING MACHINES: THEIR HISTORY, CONSTRUCtion and Use.** By Arthur V. Abbott.

No. 75. **RECENT PROGRESS IN DYNAMO-ELECTRIC MACHINES.** Being a Supplement to "Dynamo-electric Machinery." By Prof. Sylvanus P. Thompson.

No. 76. **MODERN REPRODUCTIVE GRAPHIC PROCESSES.** By Lieut. James S. Pettit, U.S.A.

No. 77. **STADIA SURVEYING.** The Theory of Stadia Measurements. By Arthur Winslow. Sixth edition.

No. 78. **THE STEAM-ENGINE INDICATOR AND ITS USE.** By W. B. Le Van.

No. 79. **THE FIGURE OF THE EARTH.** By Frank C. Roberts, C.E.

No. 80. **HEALTHY FOUNDATIONS FOR HOUSES.** By Glenn Brown.

No. 81. **WATER METERS: COMPARATIVE TESTS OF ACCURACY,** Delivery, etc. Distinctive features of the Worthington, Kennedy, Siemens, and Hesse meters. By Ross E. Browne.

No. 82. **THE PRESERVATION OF TIMBER BY THE USE OF ANTIseptics.** By Samuel Bagster Boulton, C.E.

No. 83. **MECHANICAL INTEGRATORS.** By Prof. Henry S. H. Shaw, C.E.

No. 84. **FLOW OF WATER IN OPEN CHANNELS, PIPES, CONduits, Sewers, etc.** With Tables. By P. J. Flynn, C.E.

No. 85. **THE LUMINIFEROUS ÆTHER.** By Prof. De Volson Wood.

No. 86. **HANDBOOK OF MINERALOGY: DETERMINATION,** Description, and Classification of Minerals Found in the United States. By Prof. J. C. Foye. Fifth edition, revised.

No. 87. TREATISE ON THE THEORY OF THE CONSTRUCTION of Helicoidal Oblique Arches. By John L. Culley, C.E.

No. 88. BEAMS AND GIRDERS. Practical Formulas for their Resistance. By P. H. Philbrick.

No. 89. MODERN GUN COTTON: ITS MANUFACTURE, PROPerties, and Analyses. By Lieut. John P. Wisser, U.S.A.

No. 90. ROTARY MOTION AS APPLIED TO THE GYROSCOPE. By Major J. G. Barnard.

No. 91. LEVELING: BAROMETRIC, TRIGONOMETRIC, AND Spirit. By Prof. I. O. Baker. Second edition.

No. 92. PETROLEUM: ITS PRODUCTION AND USE. By Boverton Redwood, F.I.C., F.C.S.

No. 93. RECENT PRACTICE IN THE SANITARY DRAINAGE OF Buildings. With Memoranda on the Cost of Plumbing Work. Second edition, revised and enlarged. By William Paul Gerhard, C.E.

No. 94. THE TREATMENT OF SEWAGE. By Dr. C. Meymott Tidy.

No. 95. PLATE-GIRDER CONSTRUCTION. By Isami Hiroi, C.E. Fourth edition, revised.

No. 96. ALTERNATE CURRENT MACHINERY. By Gisbert Kapp, Assoc. M. Inst., C.E.

No. 97. THE DISPOSAL OF HOUSEHOLD WASTES. By W. Paul Gerhard, Sanitary Engineer.

No. 98. PRACTICAL DYNAMO-BUILDING FOR AMATEURS. HOW to Wind for Any Output. By Frederick Walker. Fully illustrated. Third edition.

No. 99. TRIPLE-EXPANSION ENGINES AND ENGINE TRIALS. By Prof. Osborne Reynolds. Edited with notes, etc., by F. E. Idell, M.E.

No. 100. HOW TO BECOME AN ENGINEER; or, The Theoretical and Practical Training necessary in Fitting for the Duties of the Civil Engineer. By Prof. Geo. W. Plympton.

No. 101. THE SEXTANT, and Other Reflecting Mathematical Instruments. With Practical Hints for their Adjustment and Use. By F. R. Brainard, U. S. Navy.

No. 102. THE GALVANIC CIRCUIT INVESTIGATED MATHEmatically By Dr. G. S. Ohm, Berlin, 1827. Translated by William Francis. With Preface and Notes by the Editor, Thomas D. Lockwood, M.I.E.E.

No. 103. THE MICROSCOPICAL EXAMINATION OF POTABLE Water. With Diagrams. By Geo. W. Rafter. Second edition.

No. 104. VAN NOSTRAND'S TABLE-BOOK FOR CIVIL AND MEchanical Engineers. Compiled by Prof. Geo. W. Plympton.

No. 105. DETERMINANTS. An Introduction to the Study of, with Examples and Applications. By Prof. G. A. Miller.

No. 106. COMPRESSED AIR. Experiments upon the Transmission of Power by Compressed Air in Paris. (Popp's System.) By Prof. A. B. W. Kennedy. The Transmission and Distribution of Power from Central Stations by Compressed Air. By Prof. W. C. Unwin. Edited by F. E. Idell. Third edition.

No. 107. A GRAPHICAL METHOD FOR SWING BRIDGES. A Rational and Easy Graphical Analysis of the Stresses in Ordinary Swing Bridges. With an Introduction on the General Theory of Graphical Statics, with Folding Plates. By Benjamin F. La Rue.

No. 108. SLIDE-VALVE DIAGRAMS. A French Method for Constructing Slide-valve Diagrams. By Lloyd Bankson, B.S., Assistant Naval Constructor, U. S. Navy. 8 Folding Plates.

No. 109. THE MEASUREMENT OF ELECTRIC CURRENTS. Electrical Measuring Instruments. By James Swinburne. Meters for Electrical Energy. By C. H. Wordingham. Edited, with Preface, by T. Commerford Martin. With Folding Plate and Numerous Illustrations.

No. 110. TRANSITION CURVES. A Field-book for Engineers, Containing Rules and Tables for Laying out Transition Curves. By Walter G. Fox, C.E.

No. 111. GAS-LIGHTING AND GAS-FITTING. Specifications and Rules for Gas-piping. Notes on the Advantages of Gas for Cooking and Heating, and Useful Hints to Gas Consumers. Third edition. By Wm. Paul Gerhard, C.E.

No. 112. A PRIMER ON THE CALCULUS. By E. Sherman Gould, M. Am. Soc. C. E. Third edition, revised and enlarged.

No. 113. PHYSICAL PROBLEMS and Their Solution. By A. Bourgougnon, formerly Assistant at Bellevue Hospital. Second ed.

No. 114. MANUAL OF THE SLIDE RULE. By F. A. Halsey, of the "American Machinist." Third edition, corrected.

No. 115. TRAVERSE TABLE. Showing the Difference of Latitude and Departure for Distances Between 1 and 100 and for Angles to Quarter Degrees Between 1 Degree and 90 Degrees. (Reprinted from Scribner's Pocket Table Book.)

No. 116. WORM AND SPIRAL GEARING. Reprinted from "American Machinist." By F. A. Halsey. Second revised and enlarged edition.

No. 117. PRACTICAL HYDROSTATICS, AND HYDROSTATIC FORmulas. With Numerous Illustrative Figures and Numerical Examples. By E. Sherman Gould.

No. 118. TREATMENT OF SEPTIC SEWAGE, with Diagrams and Figures. By Geo. W. Rafter.

No. 119. LAY-OUT OF CORLISS VALVE GEARS. With Folding Plates and Diagrams. By Sanford A. Moss, M.S, Ph.D Reprinted from "The American Machinist," with revisions and additions. Second edition.

No. 120. ART OF GENERATING GEAR TEETH. By Howard A. Coombs. With Figures, Diagrams and Folding Plates. Reprinted from the "American Machinist."

No. 121. ELEMENTS OF GAS ENGINE DESIGN. Reprint of a Set of Notes accompanying a Course of Lectures delivered at Cornell University in 1902. By Sanford A. Moss. Illustrated.

No. 122. SHAFT GOVERNORS. By W. Trinks and C. Housum. Illustrated.

No. 123. FURNACE DRAFT; ITS PRODUCTION BY MECHANICAL Methods. A Handy Reference Book, with figures and tables. By William Wallace Christie. Illustrated.

CABOT SCIENCE LIBRARY
CABOT
AUG 1 9 2002
BOOK DUE

3 2044 056 194 665

This book should be returned to the Library on or before the last date stamped below.

A fine of five cents a day is incurred by retaining it beyond the specified time.

Please return promptly.

3 2044 056 194 665